ÉCOLE NATIONALE DE LA STATISTIQUE ET DE L'ADMINISTRATION ÉCONOMIQUE
3, avenue Pierre Larousse-92245 Malakoff CEDEX, FRANCE

Actuarial Thesis - Year 2008

Modelling commodities for commodity derivatives pricing purposes

Louis MARGUERITTE
Jean-Baptiste NESSI

KEYWORDS : commodities, futures, term structure, implied volatility, skew

---

1 Purpose of this study

1.1 A hot topic

Commodities are subject to many economic, geopolitical and financial speculations. On a regular basis, most major newspapers report that the Brent oil price hits new records (+100 % over the last past year), that the supply of certain oil-producing countries suddenly decreases, or that the demand in developing countries, where growth seems to be strong and steady, significantly increases.

1.2 Subject matter

Driven by an increase in the demand worldwide, the energetic crisis has created a significant shift in the variation of spot prices and derivatives : futures contracts and forwards (with delivery of the underlying asset at a certain date set in the contract), vanilla options, future options and exotic options. Valuation methods and financial models have to integrate these features. The specificities of commodities make them different from other kinds of financial assets such as equities, and should also be taken into account in our models. Modelling commodities in accordance with Black-Scholes' approach with constant volatility would not be very realistic. Indeed, moments' study of logarithmic returns shows a leptokurtic distribution. Additionally, statistical based on time series of spot prices and future contracts shows a mean reversion reflecting the assets' trend to oscillate around an average value. Volatility also appears higher on a short-term basis than on a long-term basis illustrating what is known as the Samuelson effect. Data related to the natural gas traded on the Henry Hub market shows a certain seasonality, which may be explained by a higher demand during wintertime. The purpose of this actuarial study is to list various methods used for modelling commodities and valuating commodities derivatives. This study deals with two types of models :

- spot models : they model spot prices of physical products and stochastic convenience yields, which drive the future curve.
- curve models : they directly specify in a HJM framework (used for interest rate valuations) the variations of the futures price curve. Studies we have undertaken establish that a single factor accounts for more than 95 % of the curve variations, which warrants the use of such models with few factors. We propose several extensions as well as various methods for calibration used in our pricing tools.

2 Modelling commodities with spot models

This kind of model consists in specifying stochastic processes for the spot-price dynamics. Our study analyzes Schwartz-Smith's model, which has a two-factor approach of price logarithms : a short-term deviation toward zero average following a Ornstein-Uhlenbeck dynamics and a long-term deviation following a standard dynamics. Mathematics allow us to obtain two closed forms for futures contracts and vanilla options. We propose two extensions, which, to our knowledge, do not exist in academic research : adding a seasonality parameter with regard to natural gas, and adding a stochastic volatility in accordance with the Heston model. In both cases, we obtain closed forms for futures contracts and vanilla options, which are the basic hedging instruments. This type of models has the benefit of taking into account all behaviors retained during the statistical tests, although certain parameters are unobservable. Furthermore, prices effectively observed are those of futures contracts, since a commodity cannot be delivered immediately, which makes the interpretation of spot prices difficult.

3 Modeling commodities with curve models

These models directly specify the variations of the futures price curve on the basis of today's futures price curve (contract price based on their maturity). This study describes a one-factor model giving the dynamics of the curve following a log normal distribution. Futures may be valuated in accordance with the Black formula. The spot price dynamics can be obtained by considering the delivery occurs immediately at the time of the valuation (maturity is zero in this case). We suggest two new possible developments in order to take into account the entire volatility surface. The first development solves the smile problem (implied volatility) vis-à-vis its dependency toward maturity. The method considers a piecewise constant volatility with regard to each interval set by futures maturities. The result of this extension is that arbitrage is avoided after calibration on the smile's term structure. The second development deals with the skew risk (dependency of implied volatility towards strike). On the basis of the previous model, we propose a log-shifted term model in order to fit the skew and to avoid arbitrage regarding the smile risk. Finally, one could further this study with the proposed two-factor curve model.

4 Calibration

A market operator, typically an investment bank, must valuate a product fulfilling an investor's specific needs and must transfer its risk to the financial market. In order to do so, such an operator has to estimate the parameters of the models subject to this study and analyze their stability in order to minimize the cost of his hedging strategy. We have itemized the various calibration models that may be used. For each type of model, we have selected a type of calibration. In each case, we studied algorithm stability : we start from various given initial values of the parameters and see if the final values differ following iterations. Then we study the stability of the parameters on each trading date with regard to the variations of market data.

The maximum likelihood principle has the advantage to minimize a simple function, although it is of no use when a large number of parameters is available, in particular for Schwartz-Smith two-factors spot models.

4.1 The quadratic approach

This is the standard method used on trading floors. Basically, it compares implied volatilities as perceived by the markets with implied volatilities as obtained by inverting the Black-Scholes formula. One limit of such a method is the growing instability as the number of parameters increases.

4.2 The Kalman filter approach

Kalman filtering is a recursive method of valuation of parameters in a model, whether financial or not. For instance, in the Schwartz-Smith model we try to estimate short-term and long-term deviations that explain spot price. Each price cotation brings its updated value for state variables. A comparison between this estimate and the value generated by the model enables us to create a gaussian model and maximize its likelihood. This method is theoretically efficient and allows the obtention of the values for each paramater, although it is time-consuming and strongly relies on the ultimate optimization tool.

4.3 Implied estimation from curves features

This approach consists in taking the most of various curves' features obtained via a given model. With regard to the Schwartz-Smith model, it consists in analyzing the futures curves and volatility term structure. The values of these curves at $t = 0$ and $t = +\infty$ give these parameters.

5 Models summary

Model	**Calibration**		**Recommendations**
	Futures	Options	
2-factors spot	++	++	May be used for back tests and investment opportunities tracking. Allows interpolation for non traded contracts.
2-factors spot + seasonality			A model which takes into account seasonality. May be used for gaz market.
3-factors spot			
1-factor curve	+++	+	Future curve becomes arbitrage free. May be used for monostrike, weakly path dependent, derivatives pricing.
1-factor curve first extension	+++	++	Future curve remains arbitrage free. Volatility term structure becomes arbitrage free as well. May be used for path dependent derivatives pricing, but not skew dependent derivatives.
1-factor curve second extension	+++	+++	The future curve and the whole volatility surface (term structure and smile) are arbitrage free. May be used for path dependent and skew dependent derivatives pricing.
2-factors curve			Adds a risk factor. May be used for correlation derivatives pricing.

6 Conclusion and further developments

Choosing a model and its calibration may depend on the product intended to be priced and the hedging needs of the market operator considered. For instance, we priced asian and bermudan options and highlighted model risks. Modeling commodities requires to be carried out with carefulness, because of the specific behaviors at stake and the speculations going on. The recent deregulation of European markets and the fear of an oil-shortage explain important yields, which do not confirm, at least for recent periods, the Schwartz-Smith model. One has to keep an eye on the parameters' stability and test the underlying assumptions on a regular basis.

We suggest several possible further developments of our study. We could calibrate spot-model with seasonality and stochastic volatility in accordance with the Heston model. The same study should be done with two-factor curve models. Otherwise a study of hedging strategies allowing an operator to hedge himself against a specific risk could be done. It would be particularly useful to analyze the hedge of more complex options that we evaluated such as Asian and Bermudan options. Finally, one could study and create more complex models best fitting the features of other types of commodities. Spikes and jumps may be added to the modeling of electricity prices, but research could be extended to other types of commodities such as gold or other metals.