

**NEW ACCOUNTING STANDARDS: THE FAIR
VALUE OF LIFE INSURANCE LIABILITIES**

by

ENRICO BIFFIS

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Department of Actuarial Science and Statistics
Cass Business School, City of London

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Abstract

In this dissertation we present an overview of the proposals of the International Accounting Standards Board for measurement of insurance assets and liabilities at market value. The Board has not yet finalised a standard for insurance contracts, but a Draft Statement of Principles has been published, providing indication of how fair value accounting will be implemented. We focus on life insurance liabilities and discuss many implementation issues from an actuarial perspective. First, we cover the underpinnings of the Board's proposals, underlining the differences that they present with other accounting standards and reporting methodologies. Then, we deal with measurement issues, outlining the key features of the valuation approaches proposed. Finally, we present an example based on an insurance product sold in the Italian market to show the differences between prudential, embedded value and fair value reporting.

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1. INTRODUCTION

In this dissertation, we present a survey of the proposals of the International Accounting Standard Board (IASB) for measurement of insurance liabilities at market value. We concentrate on life insurance business and look at the issues arising from the implementation of the IASB's principles by examining them from an actuarial perspective. In particular, we consider market value accounting in the context of the experience the actuarial community has matured over the years in embedded value and prudential reporting.

The dissertation is organised as follows: In Section 2, we give an overview of the trends which are leading the insurance industry towards market value based accounting. In section 3, we present the most relevant principles proposed by the IASB, paying particular attention to the debated issues of: definition of insurance contract and of insurance risk; distinction between fair value and entity-specific value; adoption of an asset and liability measurement approach as opposed to a deferral-and-matching approach.

In Section 4 we deal with measurement issues. Namely, we examine the IASB's views on: estimation of future cash flows; adjustments for risk and uncertainty; replicating portfolio approach; choice of the discount rate to reflect the time value of money. Non-profit business is mainly covered, because some issues concerning performance-linked contracts have not been settled yet. We give some indications for with-profit and unit-linked business consistent with the meetings the IASB has been holding recently.

In Section 5, we propose a numerical example for a unit-linked deferred annuity product sold in the Italian market. We try to apply the IASB's proposals to determine the fair value of the product. Particular emphasis is posed on the estimation of market value margins, i.e. the adjustments for risk and uncertainty to the estimation of future cash flows. Results are used to compare fair value reporting with embedded value and prudential reporting.

Finally, section 6 draws some conclusions on the practical implementation of the IASB's proposals.

2. BACKGROUND

Insurance is an important and increasingly international industry which has no official international standard for financial reporting. Indeed, a great diversity in accounting practices for insurers currently exists and consistency

with regulation of other sectors, such as banking or securities, is questionable at least.

In the last few years, however, significant changes in the standards for insurance companies have been taking place, with regard to statements prepared both in accordance with Generally Accepted Accounting Principles (GAAP) and in accordance with regulatory principles (see, for example, Vanderhoof and Altman (1998), Gutterman (2001) and Acutis *et al.* (2002)). These changes are driven by major trends, such as, for example, the increasing importance of capital markets, globalisation of business and convergence of accounting standards worldwide. In particular, multinational enterprises are becoming more and more important and the products offered by the financial services industry are increasingly similar. The traditional distinctiveness of the insurance industry is under pressure.

The need for transparency is another key driving factor. Many users of financial statements feel that the current system has not responded sufficiently to market changes and cannot cope with their needs. Several investors and analysts simply do not understand insurers' reports and often complain that current insurance accounting is an impenetrable 'black box'. That does not help the market in valuing appropriately insurance business and has led to lower price/earnings multiples for the insurance industry over the years (Gutterman (2001)).

The boom of derivatives market and the recent techniques developed in modern financial theory have affected and enriched current actuarial practice. The primary approaches used for measurement purposes (namely historical cost and deferral-and-matching: see section 3) are now being questioned by many. There is an increasing awareness of the limits they present in picturing a company's profitability and many believe their popularity is due more to practical reasons rather than sound financial underpinnings.

Changes in financial reporting for insurance companies are characterised by a switch towards market value accounting. Standards regarding the asset side of the balance sheet have witnessed a more readily and effective implementation of fair value principles,¹ putting insurers in the awkward situation of marking only one side of the balance sheet to market, thus distorting equity and earnings of the company.

Steps toward fair valuation of the liability side of the balance sheet have been more slow and difficult, since a true market for insurance liabilities does

¹In 1993, for example, issuance of the Statement of Financial Accounting Standards No. 115 in the US.

not exist or is very thin at least. In January 1994, the American Academy of Actuaries appointed a Fair Valuation of Liabilities Task Force to address the issue. The aim was to study and catalog the methodologies which capture the economics of insurance liabilities, intentionally ignoring GAAP. The Task Force assembled a ‘white paper’ (see Doll *et al.* (1998)) that set the basis for further study and discussion.

In 1997, the Board (IASB) of the former International Accounting Standards Committee (IASC)² set up a Steering Committee to carry out the initial work on an ‘Insurance Contracts’ project. In December 1999, the Steering Committee published an Issues Paper (see IASC (1999)) which attracted 138 comment letters from financial institutions, supervisory authorities and insurance companies worldwide. The Steering Committee reviewed the comment letters and developed a report to the IASB in the form of a Draft Statement of Principles (IASB (2001), referred to as DSOP henceforth), which is available on the IASB’s web site.³

In June 2000, the IASB’s work was given great motivation by the communication by the European Community that all listed EU companies should prepare IAS consolidated accounts by 2005 at the latest and that EU Member States would be free to extend the requirement to unlisted companies and individual accounts. However, in 2002 the IASB recognised that an International Financial Reporting Standard (IFRS)⁴ will not be in place by 2005 (see the IASB’s website and Wright (2002)). In May 2002, the Board decided to split its project on insurance contracts into two phases. Phase I is an interim solution which will enable insurers to implement part of the proposals by 2005. Phase II, the definitive solution, is meant to be completed by the end of 2007.

3. FAIR VALUE ACCOUNTING: PRINCIPLES

In this section we go through the underpinnings of the IASB’s proposals on market value accounting of (life) insurance liabilities. The proposals undergo

²The International Accounting Standards Board (IASB), known as International Accounting Standards Committee (IASC) until early 2001, is an independent privately-funded body based in London, UK. Its objectives are the formulation and publication in the public interest of accounting standards for financial statements and the promotion of their acceptance and observance worldwide.

³www.iasb.org.uk

⁴The standards formulated by the IASB, once called International Accounting Standards (IAS), are now being published as International Financial Reporting Standards (IFRS).

continue discussion and improvement, so we try to use the most up-to-date information available to the public at the time of writing. When needed, we specify whether a principle implementation concerns the IASB's Insurance Project as a whole or just Phase I of the project.

The Insurance Project of the IASB is aimed at issuing an IFRS to be used in general purpose financial statements directed toward the common information needs of a wide range of users. It will cover insurance contracts of *all* enterprises and will not deal with the treatment of assets held by insurers, other than assets arising under insurance contracts.

3.1. Insurance Contracts. The DSOP proposes a single recognition and measurement approach for all forms of insurance contracts, regardless of the type of risk underwritten (principle 2.1). In particular, it is argued that the only helpful distinction between general and life insurance is, for financial reporting purposes, the length of the insurer's price commitment. Insurance is treated as general insurance if the insurer is committed to a pricing structure for less than twelve months, as life insurance otherwise.

The definition of insurance contract given by the DSOP goes as follows (principle 1.2):

An **insurance contract** is a contract under which one party (the insurer) accepts significant insurance risk by agreeing with another party (the policyholder) to compensate the policyholder or other beneficiary if a specified uncertain future event (the insured event) adversely affects the policyholder or other beneficiary.

The definition is necessary in order to distinguish insurance contracts from financial instruments covered by IAS 39 (Financial Instruments: Recognition and Measurement) or by a successor standard resulting from the JWG Draft,⁵ and from other assets, such as provisions (covered by IAS 37) or intangible assets (covered by IAS 38).

The key feature of the definition is the reference to a *significant insurance risk* taken over by the insurer. The DSOP recognises ***uncertainty*** (or risk)

⁵In December 2000, the Joint Working Group of Standard Setters (JWG) published a Draft Standard and Basis for Conclusions, Financial Instruments and Similar Items (referred to as JWG Draft henceforth). While IAS 39 prescribes measurement at fair value for a portion of financial assets and liabilities, a successor standard could introduce fair value measurement for the substantial majority of financial assets and liabilities. This has relevant implications for the choice between Fair Value and Entity-specific Value in measurement of insurance liabilities (see section 3.3).

as the essence of an insurance contract, and assumes that at least one of the following must be uncertain at the inception of a contract:

- whether a future event specified in the contract will occur;
- when the specified event will occur;
- how much the insurer will need to pay if the specified future event occurs.

However, risk must be of an insurance kind. *Insurance risk* is “risk other than financial risk”, where financial risk is the risk of a possible future change in one or more of a specified interest rate, security price, commodity price, foreign exchange rate, index of prices or rates, a credit rating or credit index or similar variable (DSOP, paragraph 1.28).

So, when is insurance risk significant? The DSOP does not propose any quantitative guidance, but states:

A contract creates sufficient insurance risk to qualify as an insurance contract if, and only if, there is a reasonable possibility that an event affecting the policyholder or other beneficiary will cause a significant change in the present value of the insurer’s net cash flows arising from that contract. In considering whether there is a reasonable possibility of such significant change, it is necessary to consider both the probability of the event and the magnitude of its effect.

It is worth noting that the DSOP defines the magnitude of an insured event by its significance in relation to the individual contract. The test for insurance risk is thus performed on a *contract-by-contract basis* and risk can therefore be present even in those cases when, for a book of contracts as a whole, there is a minimal risk of significant changes in the present value of payments. Moreover, a contract that qualifies as an insurance contract at inception or later remains an insurance contract until all rights and obligations are extinguished or expire. That should clarify any doubts regarding contracts under which the sum at risk varies considerably over the policy term.

Examples of (life) insurance contracts are annuities and pensions, whole life and term assurances.⁶ Contracts that have the legal form of insurance, but that do not expose the insurer to insurance risk or pass all significant insurance risk back to the policyholder (e.g. through performance-linking

⁶Endowment contracts can give rise to different insurance risk exposures, depending on the size of the sum at risk.

mechanisms) are non-insurance financial instruments. Examples are given by unit-linked products which provide a death benefit not significantly higher than the account balance at the time of death.

Among the items that may meet the DSOP's definition of insurance contracts, but are excluded from the scope of the standard, we single out employers' assets and liabilities under employee benefit plans and retirement benefit obligations reported by defined benefit retirement benefit plans (covered respectively by IAS 19 and IAS 26).

3.2. Asset and Liability Measurement Approach. There are two broad types of approach to accounting for insurance contracts, referred to as "deferral and matching approach" and "asset and liability measurement approach" (see IASC (1999)). The IASB favours the latter.

- A *deferral and matching* approach aims at associating claim costs with premium revenue. Revenue and expenses from an insurance contract are recognised progressively over time as services are provided.
- An *asset and liability measurement* approach requires the recognition of insurance assets and liabilities that meet specified definitions and recognition criteria. Income and expenses are defined in terms of changes in measurement of insurance assets and liabilities, while items that do not meet those definitions are excluded.

Both approaches enable the insurer to recognise income as it is released from risk, but the implementation of this feature is different. Moreover, the deferral and matching approach may lead to recognition of items that are not assets or liabilities according to the draft's definition (see below).

To get a better understanding of the two approaches, it is useful to take a look at one of US GAAP standards (which are representative of the first approach; see, for example, Vanderhoof and Altman (1998)). Financial Accounting Standard No. 60 (FAS 60),⁷ suited for traditional non-profit contracts, works in the following way for long duration policies. Revenue is defined as earned investment income and premium, where premium is recognised in proportion to performance under the contract. Acquisition costs are deferred and amortised in proportion to premium revenue over the life of the block of contracts. This leads to recognition of a deferred acquisition costs (DAC) asset. Liabilities are valued using assumptions reflecting best-estimates plus a provision for adverse deviation (PAD). Those assumptions, once chosen, are 'locked in' unless severe adverse experience develops

⁷See FASB (1982).

in the future. ‘Unlocking’ of assumptions is allowed only if a gross premium valuation shows that premium deficiency exists, thus triggering a ‘loss recognition’.

Contrary to US GAAP, the DSOP adopts an asset and liability approach. It defines insurance assets and insurance liabilities as assets and liabilities arising under an insurance contract. In particular, an insurer or policyholder should recognise (principle 2.2):

- an **insurance asset** when, and only when, it has contractual rights under an insurance contract that result in an asset; and
- an **insurance liability** when, and only when, it has contractual obligations under an insurance contract that result in a liability.

Many items that are currently found in the insurers’ financial statements in many countries remain excluded from recognition. Namely, deferred acquisition costs or catastrophe and equalisation provisions (see sections 4.1.1 and 4.1.3). Note that the exclusion of the latter two items is a Phase I proposal.

Although separate disclosure might be needed, the DSOP considers the contractual rights and obligations under a book of contracts as components of a single net asset or liability, rather than separate assets and liabilities. As a consequence, the *measurement* of insurance assets and liabilities is based on books of insurance contracts. Their *recognition*, however, happens on an individual contract-by-contract basis, accordingly to principle 2.2.

The book measurement must account only for the contracts in force at the reporting date (***closed book*** approach), since an open book approach is inconsistent with the DSOP definitions of assets and liabilities, which require the existence, as a result of past events, of a resource or present obligation. That leads to the issue of determining whether possible future renewals of an existing contract are part of an existing contract or separate, future contracts (see section 4.1.2).

3.3. Fair Value and Entity-specific Value. The IASB is favourable to a measurement of both assets and liabilities at market value, but it recognises that a successor of IAS 39 might not be in place by the time the Board finalises a standard on insurance contracts. Thus, the DSOP proposes two measures for an insurance liability: Entity-specific Value and Fair Value, the former to be used in case IAS 39 is still in place, the latter otherwise.

They are defined as follows (principle 3.1):

Entity-specific value represents the value of an asset or liability to the enterprise that holds it, and may reflect factors that are not available (or not relevant) to other market participants. In particular, the entity-specific value of an insurance liability is the present value⁸ of the costs that the enterprise will incur in settling the liability with policyholder or other beneficiaries in accordance with its contractual terms over the life of the liability.

Fair value is the amount for which an asset could be exchanged or a liability settled between knowledgeable, willing parties in an arm's length transaction. In particular, the fair value of a liability is the amount that the enterprise would have to pay a third party at the balance sheet date to take over the liability.

The definition adopted for the fair value of a liability is the so called *fair value in exchange*, as opposed to “fair value as an asset” (i.e. the amount at which others are willing to hold the liability as an asset) and “fair value in settlement with a creditor” (i.e. the amount that the insurer would have to pay to the creditor to extinguish the liability). In particular, the definition refers to a hypothetical transaction with a party other than the policyholder.

Fair value is also an *exit value*, as opposed to an *entry value*, which is the amount of the premium that the insurer would charge in current market conditions if it were to issue new contracts that created the same remaining contractual rights and obligations.

The determination of both entity-specific and fair value relies on a prospective direct method (see section 4.1) based on the expected present value of future cash flows. As a result, the two measures could disagree because the insurer:

- has a superior management or other skills that enables him to maximise cash inflows or minimise cash outflows; or, on the opposite, is more prone (e.g., for competitive reasons) to accept a higher level of cash outflows than other market participants;
- has the same ability as the market to generate cash inflows or propensity to accept cash outflows, but still makes different estimates about those cash flows;

⁸The methodology proposed by the DSOP is based on an expected present value approach. Details concerning entity-specific expectation and discounting will be given in section 4.

- has different views about the amount of risk associated with the cash flows;
- has different risk preferences and prices that risk accordingly;
- has different views about the insurer's own credit standing in measuring insurance liabilities;
- has different liquidity needs.

As we will see in sections 3 and 4, only some of these sources of disagreement may be reflected under the DSOP's proposal. In practice, the two measures will be very close: since the definition of fair value refers to *knowledgeable parties*, the insurer and the market may be assumed to have identical knowledge about the characteristics of the liability for the purpose of determining fair value. It follows that in most cases both entity-specific and fair value will reflect the actual knowledge of the insurer.

3.4. Prospective Approach and Direct Method. Both entity-specific and fair value determination rely on a prospective approach.

Retrospective approaches focus on an accumulation of past transactions between policyholders and insurers. A life office, for example, measures the insurance liability initially on the basis of the premium received, and defers acquisition costs, considering them as an asset or as a liability reduction. The expected profit margins on the contract impact the measurement gradually over the life of the contract, in a way that depends on the basis assumed and on the accounting standard followed.

It is typical of these approaches to use the policyholder account or the surrender value as a basis of measurement for contracts with an explicit surrender value or an explicit account balance (e.g. contracts described as universal life, unit-linked, variable or indexed).

Prospective approaches focus on the future cash inflows and outflows from the closed book of insurance contracts. The insurance liability is valued through an estimate of the present value of all future net cash flows arising from the contract. That estimate is defined as *provision for unexpired risk* (IASC (1999)), and can be more or less than the premium already paid by the policyholder, thus giving rise (unlike in the retrospective case) to a possible net profit on initial recognition.

Note that, in some countries, accounting rules require the net profit arising under a profitable contract to be recognised over the period of premium payment. Under the DSOP, recognition occurs over the period when the

insurer is at risk (i.e. beyond the period of premium payment for many contracts).

The approach proposed by the DSOP is also a *direct method*, since it requires the direct measurement of the liability through discounting of the cash flows arising from the book of contracts. The possibility to use embedded value techniques, which are indirect methods (or 'deductive' methods: Doll *et al.*(1998)), is therefore ruled out.

Indirect methods discount all cash flows from the book of contracts *and* the assets backing the book, to derive a quantity which is then subtracted to the measurement of the assets, giving the value of the book as a result. Direct and indirect methods *can* produce the same results, provided that a consistent set of assumptions is used.⁹ The DSOP, however, takes the view that direct methods should be favoured because of their greater transparency.

The Embedded Value method, for example, measures liabilities on a supervisory basis and then recognises (e.g. UK banks with life insurance subsidiaries) or discloses (e.g. UK life insurers) an asset, the embedded value, which represents the amounts that will be released from the book of contracts as experience unfolds and liabilities are paid. In particular, the embedded value is given by the sum of the shareholders' net assets backing the book and the value of the business in-force at the valuation date. The value of in-force business is the present value of future profits expected to emerge on the supervisory basis from policies already written. See Fine and Geddes (1988), Simpson and Wells (2000) and section 5 for more details.

Embedded value is not consistent with the DSOP proposal, because it includes the present value of estimated future cash flows from investments representing the insurance liability. In particular, it attributes an amount other than fair value to assets held, because it does not discount the cash flows from those investments at a rate equal to the estimated return on those assets. Indeed, the risk discount rate used is generally meant to reflect not only the risk associated with the business, but also the cost of capital locked

⁹Girard (2000) shows this result in comparing the Actuarial Appraisal Method (AAM) and the Option Pricing Method (OPM). The first method aims at determining the Appraisal Value (i.e. embedded value plus goodwill) of a book of contracts. While the AAM is based on the discounting of free cash flows (distributable earnings), and gives the liability value as the difference between assets and appraisal value, the OPM discounts the liability cash flows directly. If in the AAM distributable earnings are discounted at the cost of capital, then, in equilibrium, the two methods give the same result if the expected total return of the assets net of the total return of the liability is equal to the expected (or required) return of capital.

in by capital requirements. This is inconsistent with the neutrality purpose of fair measurement (see below).

3.5. Backing Assets. Principle 3.2 of the DSOP states that the *type* of assets held by the insurer or the *return* on those assets should NOT affect the entity-specific or the fair value of an insurance liability. The only exception is given by those contracts in which the benefits paid to policyholders are directly influenced by the return on specified assets (e.g. unit-linked products).

The principle is based on the argument that neither insurers' investment decisions nor the characteristics of the assets held have much to do with the value of the liabilities. However, if entity-specific value and fair value are independent of the carrying amount of the insurer's assets, a significant asset-liability *mismatch risk* can arise, particularly from the interaction of IAS 39 and entity-specific value.

The DSOP suggests that mismatch risk would be minimised by a consistent choice of measurement basis for both assets and liabilities and by a proper use of the accounting options allowed for by IAS 39. Further discussion is going on, and the IASB will have to consider the issue in its project on performance reporting.

3.6. Neutrality. The DSOP does not allow the overstatement of insurance liabilities to impose implicit solvency or capital adequacy requirements.

The exercise of prudence, which is essential to enterprises that take on risks from policyholders, should not bias the financial statements. In particular, the DSOP prohibits the creation of hidden reserves or excessive provisions, the deliberate understatement of assets or income, or overstatement of liabilities and expenses.

3.7. Bundled Contracts. Some insurance contracts include an insurance element together with a non-derivative investment (e.g. returns linked to the investment performance of the insurer) or an embedded derivative (e.g. investment guarantees). The DSOP does not allow the unbundling of the investment component from the contract.

Unbundling would be more consistent with some of the JWG Draft proposals and would probably reduce the need for guidance on the level of insurance risk present in a contract to qualify for inclusion. However, unbundling would have several disadvantages in the DSOP's framework.

First of all, it would lead to retrospective measurement of the investment element (under IAS 39) and prospective measurement of the insurance element. Future investment management fees charged by the insurer would not decrease the liabilities, but would be recognised as revenue in future periods. Future premium receipts for the insurance element would be recognised at inception, while those for the investment element would be recognised as movements in the insurance liability reported in the balance sheet.

Additionally, it would often be difficult to unbundle complex products and sometimes even arbitrary (e.g. in variable policies with minimum insurance benefit guarantees, where insurance risk can be inexistent as well as huge depending on the underlying fund performance).

4. FAIR VALUE ACCOUNTING: MEASUREMENT ISSUES

In section 3, we described the principles underlying the DSOP's proposal for a measurement of insurance liabilities at market value. In this section, we focus on the implementation of fair value accounting, paying attention to the estimation of the amount and timing of cash flows from assets and liabilities, and of the price for bearing the uncertainty affecting those cash flows.

In what follows, we will always refer to fair value, meaning that the same conclusions or methodologies apply for entity-specific value, provided that the typical market participants' views on the cash flows concerned are replaced by those of the specific insurer. When the two entities give rise to particular differences, the distinction will be made clear.

4.1. Amount and Timing of Cash Flows. The estimation of the amount and timing of the cash flows included in fair value measurement relies on the use of the *expected present value*, i.e. the estimated probability-weighted arithmetic average of the present value arising from each possible scenario affecting those cash flows.

If properly used, the method gives all the advantages of a stochastic approach, such as great flexibility in dealing with uncertainties of the cash flows and in capturing possible correlations between cash flows and interest rates.

Note that the DSOP requires the use of an expected present value approach regardless of the size of the book being valued, i.e. even if the book consists of a single contract.

4.1.1. **Cash Flows Included.** The expected present value computation should encompass *all future pre-income tax cash flows* arising from the contractual rights and obligations associated with the closed book of insurance contracts. Those cash flows include the following items:

Payments to policyholders under existing contracts. They include claim payments and maturity or termination payments.

The DSOP does not adopt a *deposit floor* for contracts with an explicit account balance (that would impose a lower bound on the insurance liability equal to the amount of the account balance). The level of the account balance will be reflected indirectly by the probability-weighted estimates of account-related benefits.

Claim handling expenses, i.e. costs incurred in processing claim payments. Note that for entity-specific value, these costs will reflect the way in which the insurer expects to settle the related claim liabilities. In particular, if the insurer expects to carry out the settlement by transfer to another party, any transaction costs that would be incurred on the transfer should be included. That constitutes an exception to the exclusion of transaction costs (see section 4.1.3).

Future premium receipts from policyholders under existing contracts. Their expected present value will reflect the probability of lapses by policyholders. It may be necessary to use option pricing methods to estimate the impact of lapse options. The issues of renewals and lapses are treated in section 4.1.2.

Future policy loans to policyholders, and repayments by policyholders of principal and interest on current and future policy loans. The loan drawn down from the insurer by a policyholder has traditionally been regarded as a separate financial asset of the insurer. Under the DSOP, however, the policy loan is: a separate financial asset if the loan exceeds the carrying amount of the related insurance liability; a prepayment of the insurance liability otherwise.

Policy administration and maintenance costs. They include administrative costs and renewal commissions for the sales force or for brokers.

Overheads should be included to the extent that they can be directly attributed to the book of contracts or allocated to it in a reasonable and consistent basis. These overheads should include a reasonable charge for the consumption of all assets used to generate the cash flows concerned. Such

charge could be: the opportunity cost¹⁰ to the enterprise of the asset, for entity-specific value; the price that the market would charge for the use of the asset, for fair value.

Acquisition costs. Unlike in the deferral and matching approach, they should be recognised as an expense when they are incurred.

Transaction-based tax and levies. They are taxes and levies that arise directly from the insurance contract (e.g. premium and value added taxes). Capital taxes, which can be levied in some jurisdictions, must also be included in the cash flows, unless they relate to a separately recognised asset or liability (e.g. a financial asset held).

4.1.2. **Renewals.** Future premium receipts and termination payments are closely related to policy persistence. Under the DSOP, much emphasis is placed on the renewal concept. A long term contract with a policyholder cancellation option can indeed be seen as a short term contract containing a renewal option.

Two criteria are proposed to decide to what extent cash flows from future renewals should be included:

- their inclusion would increase the measurement of the insurer's liability;
- policyholders hold uncancellable renewal options that are potentially valuable to them.

A renewal option is potentially valuable if there is a reasonable possibility that it will significantly constrain the insurer's ability to reprice the contract at rates that would apply for new policyholders who have similar characteristics to the holder of the option (DSOP, paragraph 4.51).

In some cases, when a renewal option becomes valuable, its exercise results in a net cash outflow for the insurer, who has a current obligation to accept the renewal premium and pay the resulting claims. That is the case of fixed premiums and significant surrender charges which locks the insurer into a level of premiums that becomes uneconomic (e.g. because the insured becomes uninsurable or current market premiums increase).

In other cases, a policyholder decision to renew (i.e. not to lapse) will lead to a net cash inflow. That would reduce the insurance liability and could result in the recognition of an asset.

¹⁰It is the cost of using a resource, measured by the benefit from the next-best alternative available.

In substance, a renewal option written by the insurer can or cannot be an asset. Looking at the issue from the point of view of lapses: if it can, expected lapses will be reflected in measurement (with an adjustment for risk and uncertainty: see section 4.2); if it cannot, then one should assume that policyholders exercise lapse options in the way that results in the largest reported liability.

Some contracts include renewal and cancellation options held by the insurer. As the insurer cannot be required to exercise such options, they cannot be an insurer's liability. However, they may decrease the measurement of an insurance liability. The DSOP deems it unlikely that any such options will ever come into the money for contracts meant to be profitable. If there is a significant risk of that happening, however, they should be included in measurement.

4.1.3. *Cash Flows Excluded.* . The following cash flows should not be included in the expected present value computation:

Income tax payments and receipts. Their exclusion implies the use of a pre-tax discount rate in discounting pre-tax cash flows, in order to avoid double counting.

Cash flows arising from future insurance contracts, consistently with the closed book approach adopted in the DSOP (see section 3.2).

Payments to and from reinsurers. Insurers should recognise an insurance asset arising under reinsurance contracts rather than reduce the related direct insurance liability. This is particularly relevant for financial reinsurance treaties.

Investment return from current or future investments (except for some performance-linked contracts, see section 4.3). This is consistent with the independence of fair value on the assets actually held (see section 3.5). Note, however, that future inflows from explicit investment management charges that will be levied on policyholders must be included, together with the cash outflows incurred by the insurer to generate those investment charges.

Regulatory requirements and cost of capital, consistently with the neutrality purpose of financial statements (see section 3.6).

Transaction costs. Those affecting the settlement (sale) of an insurance liability (asset) should not be included in the fair value. The exception regarding entity-specific value has been pointed out in section 4.1.1.

An insurer's own credit standing should not be reflected by fair value, although it should, at least conceptually. This issue has not been completely

solved yet. There appear to be reasons of a more practical kind for the exclusion of this item (see IAA (2000), Effect of Insurer's Credit Standing on Insurance).

Provisions for catastrophes and equalisation. Their exclusion derives from the definition of insurance liability and from the closed book approach adopted.

4.1.4. ***Assumptions and Source of Assumptions.*** The DSOP favours an *explicit approach* to assumptions.

A net premium valuation is an example of implicit approach, a gross premium valuation of an explicit approach. In the former case, future premiums are considered after deductions for assumed level of expenses. In the latter, total premium inflows and separate deductions for estimated future expenses are considered.

When an implicit approach is used, assumptions are selected in combination, with main focus on the overall measurement result. As a consequence, sometimes individual assumptions may not be meaningful in isolation. Under an explicit approach instead, each significant assumption is meaningful in its own right. That does not preclude, of course, the possibility to use stochastic modelling or similar techniques, or to capture joint effects of different assumptions.

Two main classes of assumptions will be needed for the measurement of insurance liabilities:

- *market assumptions*, such as interest rates, for which transactions in the capital markets provide easier estimation.
- *non-market assumptions*, such as lapse rates and mortality, which are not readily estimated from capital markets.

Market assumptions should be consistent with current market prices and other market-derived data, unless there is reliable evidence that current trends will not continue. Under unit-linked or variable insurance contracts, for example, the present value of the units to which the policyholder's benefit are linked should be equal to their current fair value. To the same value should be calibrated any stochastic model used.

Non-market assumptions must be consistent with market assumptions and with the most recent financial budgets and forecasts that have been approved by management. Adjustments should be made to those assumptions that are not current anymore or appear to be biased estimates of future events.

Sources of non-market assumptions are:

- claims already reported by policyholders;
- historical data regarding the insurer's experience;
- industry average experience;
- if available, recent market prices for transfers of book of contracts.

Any assumptions should reflect not just up-to-date information about the current level of claims, but also expected or well-established historical trends (e.g. mortality improvements). Any data gathered should be adjusted to reflect the specific characteristics of the book of contracts concerned. Key factors to be considered are past underwriting, the insurer's marketing actions, mix of business and distribution channels. Assumptions, in other words, should make sense and reflect the current contract portfolio as well as the current operating environment.

The border between entity-specific and fair value does not seem to be clear when dealing with items such as future operating expenses. That would suggest the adoption of a *fair-value-in-use* approach (see IAA (2000), Market Expectations Regarding Experience Assumptions). The following paragraphs describe the DSOP's views on the issue.

In determining *entity-specific value*, each cash flow scenario used to determine expected present value should be based on assumptions reflecting:

- (a) all future events that may affect future cash flows from the closed book of existing contracts;
- (b) inflation;
- (c) all entity-specific future cash flows that would arise in that scenario for the insurer concerned.

The same principles should be used to estimate *fair value*, when it is not observable directly in the market. However, two main differences must be pointed out:

- all entity-specific cash flows mentioned at point (c) should be excluded;
- fair value should reflect market information when there is data indicating that market participants would not use the insurer's assumptions.

If not observable, fair value would need to be estimated using valuation techniques that reasonably mimic how the market could expect to price a book of insurance contracts. In doing so, assumptions will need to be representative of market expectations on the risk-return characteristics inherent in the book of contracts considered.

Future events (point (a) above) include changes in legislation (e.g. changes in tax rates and tax laws), technological change (refinement of new technology or development of completely new technology) and regulatory approvals (e.g. sale of new drugs).

Inflation (point (b)) should be reflected by using cash flows and discount rates either both in real terms (i.e. inflation-adjusted) or both in nominal terms. The DSOP allows both approaches, since in principle they should produce the same result. However, the second one is perhaps more viable, because in countries where the government does not issue inflation-linked securities the estimation of adjustments may be difficult or unreliable.

Note that the adoption of a closed book approach does not imply the use of *run-off assumptions*. These are used when an insurer stops writing some or all types of contracts and allows the existing book of insurance contracts to run off. In that situation, peculiar assumptions regarding expense levels, lapse rates and claims management procedures may be needed. The DSOP states that run-off assumptions can be used only if they represent a reasonable and supportable estimate of what will occur.

There are different kind of *markets* that can provide useful information. *Primary markets* for the issuance of direct insurance contracts can provide information about the current pricing of risk, but it is obvious that retail markets can only partially reflect prices paid in wholesale markets.

Secondary markets for the transfer of books of contracts are the ideal place where true exit values can be observed. Unfortunately, they are extremely thin and exchange prices are often not publicly available. When they are, they include an implicit margin for future benefits arising from: future renewals that are not in the closed book, cross-selling opportunities and customer lists. The margin is usually not easily quantifiable, but it must be excluded if it can be determined reliably, since the DSOP considers not recognisable the value of extra-contractual intangible items, such as customer relationships.¹¹

Reinsurance markets may provide an indication of prices that would prevail in secondary markets, but have some limitations. First, they are not generally true exit prices, because most reinsurance contracts do not extinguish the cedant's contractual obligations under the direct contract. Second,

¹¹This is consistent with IAS 38, Intangible Assets, under which recognition of those items would be unlikely.

they may include a margin for intangible items. Finally, reinsurance premiums can be used as a basis for measuring the entire liability only with care, since the cedant's liability is often only partially covered by reinsurance.

Capital markets are of course the main source of market assumptions. If a suitable replicating portfolio (an asset portfolio closely matching the characteristics of the insurer's obligations: see section 4.2.4) can be constructed, they can provide a proxy for the fair value of an insurance liability.

4.2. Adjustments for Risk and Uncertainty. Although some writers distinguish the terms 'risk' and 'uncertainty', the DSOP uses them interchangeably, referring to a two-tailed probability distribution in which the outcome may be either more favourable or less favourable than expected.

The DSOP classifies three kinds of risk affecting the estimation of future cash flows: *occurrence risk*, i.e. risk that the number of insured events will differ from expectations; *severity risk*, i.e. risk that the cost of events will differ from expectations; *development risk*, i.e. risk that the amount of an insurer's obligation may change after the end of a contract period. The last factor is more important for general insurance, where litigation and claims settlement may pose particular problems.

Under the DSOP's proposal, both entity-specific and fair value should always reflect risk and uncertainty (principle 5.1). That is justified on the ground that the pricing of all rational economic transactions takes risk into account.

Note that the expected present value computation described in section 4.1 does not already reflect risk. Indeed, it places the same weights on favourable and unfavourable outcomes, thus not reflecting *risk preferences*. Most individuals and most enterprises are risk-averse, i.e. they would rather avoid a loss of a given amount than make a gain of the same amount. That has an impact on the amount that a rational investor would pay for a set of uncertain cash flows: typically, an investor would place more weight on the unfavourable outcomes, less on the favourable ones.

Sections 4.2.3 and 4.2.4 try to answer the questions about where and how risk preferences should be reflected. Moreover, we will see the convergence of entity-specific and fair value when covering the issue of whose preferences should be reflected (section 4.2.6).

4.2.1. *Diversifiable and Undiversifiable Risks.* A usual classification proposed for risks is the following:

- *model risk*: it is the risk of choosing an incorrect model of future cash flows (e.g. choice of a normal instead of a gamma distribution) or overlooking a factor that will influence the future cash flows;
- *parameter risk*: it arises because information about the underlying probability distribution chosen must be estimated, and estimates may be incorrect (e.g. because of sampling errors or parameters changing over time);
- *process risk*: it is the risk of random accidental fluctuations, which are unavoidable even when model choice and parameter estimation are correct.

Moreover, a distinction can be made between risks that have a significant effect only on one or a few enterprises (or sectors) and risks that tend to affect all investments. The former are usually called *diversifiable* (or specific or idiosyncratic) risks, because holding a well-diversified portfolio would enable an investor to exploit the low correlation between enterprises to offset enterprise-specific losses with equally likely enterprise-specific gains. The latter are called *undiversifiable* (or systematic) risks, because a well-diversified portfolio would not help to protect against risks that affect all investments simultaneously, although to a different extent.

Conceptually, at least, process risk is a diversifiable risk from the perspective of a large institutional investor. It is more debated whether model and parameter risk can also be diversifiable. Some believe that they are, because they relate to information about individual investments. Others argue that at least model risk, but also parameter risk sometimes, are undiversifiable because the market as a whole can draw wrong conclusions or make errors in using information available to price assets and liabilities.

Some popular asset pricing models, such as the Capital Asset Pricing Model (CAPM), take the view that market prices reflect only undiversifiable risks. Their claim is justified by equilibrium arguments: if some assets could compensate investors for risks that could be eliminated by diversification, investors willing to hold a diversified portfolio would bid up the market prices of those assets until they no longer incorporated a return for diversifiable risks.

Unlike the CAPM, the DSOP states that entity-specific value and fair value should always reflect *both* diversifiable and undiversifiable risks (principle 5.4). That has mixed implications on liability measurement. In particular:

- diversifiable risks, if relevant, will always increase the measurement of the liability;
- undiversifiable risks, if relevant, will:
 - always increase the measurement of the liability
 - or
 - decrease (increase) the measurement of the liability if payments under the liability are positively (negatively) correlated with the return on the market portfolio.

The first view about undiversifiable risks is based on the argument that a liability with uncertain cash flows is always more onerous than a risk-free liability with the same expected timing and amount. The second view is based on portfolio theory. One way of implementing the second view is to use the replicating portfolio approach described in section 4.2.4.

Since the measurement of a liability includes both types of risk, the DSOP does not find it useful to distinguish them nor to require insurers to make such a distinction. Model, parameter and process risk should all be taken into account. Particular care should be put in determining adjustments for model and parameter risk: in order to avoid undue subjectivity, the DSOP states that they should be quantified by reference to observable market data (if they are available).

4.2.2. ***Unit of account.*** Since the impact of risk and uncertainty will depend on the size of the book, it becomes very important to define the *unit of the account*. The DSOP gives the following guidance (principle 5.5):

- measurement of insurance contracts should focus on books of contracts that are subject to substantially the *same risks*; and
- measurement should reflect all benefits of diversification and correlation *within* that book of contracts.

As a consequence, the entity-specific value or fair value of a book of contracts is likely to be lower than what would be obtained by considering first individual contracts and then aggregating. This could happen because of: reduced exposition to process risk; presence of economies of scale; greater statistical evidence for supporting decisions on model choice and parameters estimation; joint effects arising when contracts are combined.

As far as the last point is concerned, note that under the DSOP's proposal term assurances and annuities would not qualify for inclusion in the same

book of contracts, as opposed to current practice in some countries¹². Moreover, the unit of account must not (necessarily) correspond to the level of aggregation used in the internal reporting system (as under US GAAP). The internal reporting system may provide a useful starting point for determining the unit of account though.

Finally, it is clear from this and previous section that the DSOP does not distinguish between *pooling* and *diversification*. The first term refers to the aggregation of a large number of homogeneous contracts or risk exposures,¹³ while the second to the aggregation of risk exposures that are uncorrelated, or not perfectly correlated. Both principles relying on the “law of large numbers”, the DSOP deems that both terms can be used interchangeably for accounting purposes.

4.2.3. Reflecting risk and uncertainty. There are three main methods, all theoretically correct, for valuing a set of (risky) future cash flows. We go through them by using a simple two-state, one-period example (we follow Babbel *et al.* (2001)). Consider a market where: trading is allowed only at dates 0 and 1; r is the one-period risk-free rate; S_0 is the price of a security that will pay either S_1^u or S_1^d at time 1 (assume, without loss of generality, that $S_1^u > S_1^d$).¹⁴ Then, there are three ways of expressing S_0 :

(1) First method:

$$S_0 = \frac{p \cdot S_1^u + (1 - p) \cdot S_1^d}{1 + r + \lambda \cdot \sigma_S} \quad (1)$$

where p ($0 < p < 1$) and $(1 - p)$ are the ‘true’ probabilities¹⁵ of the payoff being respectively S_1^u or S_1^d , λ is the market price of risk associated with the uncertainty about the random payoff S_1 and σ_S is a volatility parameter associated with the same uncertainty.¹⁶

¹²In Canada, for example.

¹³It can be shown that the ‘risk ratio’ (the standard deviation to expected value ratio) of the aggregate claims arising from a pool of homogeneous risks decreases as the number of risks increases (see Daykin *et al.* (1994)). The risk of mortality random fluctuations is a pooling risk.

¹⁴Note that $S_1^u > S_0 \cdot (1 + r) > S_1^d$ must hold for arbitrage opportunities not to exist.

¹⁵i.e. realistic probabilities (based, for example, on ‘historical’ estimation).

¹⁶The market price of risk is the excess reward-to-risk ratio. Consider the random rate of return on the risky security, defined as $R_S = (S_1 - S_0)/S_0$. Then, set $\mu_S = \mathbb{E}_{\mathbb{P}}[R_S]$ and $\sigma_S^2 = \text{Var}_{\mathbb{P}}[R_S]$, where by $\mathbb{E}_{\mathbb{P}}$ we denote the expectation taken under the “true” probability measure. Clearly, $\mu_S = [p \cdot S_1^u + (1 - p) \cdot S_1^d]/S_0 - 1$ and $\sigma_S^2 = p \cdot (1 - p) \cdot [(S_1^u - S_1^d)/S_0]^2$. The market price of risk is thus defined as $\lambda = (\mu_S - r)/\sigma_S$ (with $\sigma_S > 0$). In equilibrium,

(2) Second method:

$$S_0 = \frac{q \cdot S_1^u + (1 - q) \cdot S_1^d}{1 + r} \quad (2)$$

where q ($0 < q < 1$) and $(1 - q)$ are the so-called “risk-neutral” probabilities of the payoff being respectively S_1^u or S_1^d . The link with the true probabilities is given by $q = p - \lambda \cdot \sqrt{p \cdot (1 - p)}$.¹⁷

(3) Third method:

$$S_0 = \frac{[p \cdot S_1^u + (1 - p) \cdot S_2^d] - Z}{1 + r} \quad (3)$$

where Z is a quantity that makes the numerator of equation (3) the certainty equivalent of the risky payoff S_1 .¹⁸

We see, therefore, that all three methods account for risk, but in different ways. When a risk-free security is considered, i.e. when $S_1^u = S_1^d$, all methods collapse to the same result $S_0 = S_1/(1 + r)$.¹⁹

In the first approach we can recognise the traditional *discounted cash flow* method. It uses true probabilities to determine the expected end-period payoff, which is then discounted by using the risk-free rate plus a risk premium incorporating both the security riskiness and the market preferences for risk.²⁰ The method is widely used in capital budgeting problems and in pricing non-traded or thinly traded securities.

Under the second approach, true probabilities are converted into risk-neutral probabilities and discounting is carried out by using the risk-free rate. Risk-neutral probabilities can be interpreted (see Hull (2002)) as the probabilities that investors would place on uncertain outcomes in a world

this ratio is constant for all securities. In a CAPM framework, for example, λ would be defined with the “beta” of the risky security in the denominator.

¹⁷The link is obtained by comparing formulae (1) and (2). Note that q can be rewritten as $q = (S_0 \cdot (1 + r) - S_1^d)/(S_1^u - S_1^d)$, and is thus a probability by no-arbitrage arguments (see footnote 14).

¹⁸The certainty equivalent of a random payoff is the certain amount of money which an individual would find indifferent to exchange with the random payoff concerned. If $u(\cdot)$ is a *utility function*, a non-decreasing function describing market participants’ preferences among different levels of wealth, then the certainty equivalent of S_1 is the amount M such that $u(M) = E_{\mathbb{P}}[u(S_1)]$. In a CAPM framework, Z is equal to $S_0 \cdot \lambda \cdot \sigma_S$.

¹⁹We have $\sigma_S = 0$ and the market price of risk is simply 0. Moreover, the adjustment Z becomes zero, since $u(E_{\mathbb{P}}[S_1]) - Z = u(S_1 - Z)$ must be equal to $E_{\mathbb{P}}[u(S_1)] = u(S_1)$.

²⁰Note that the risk adjustment is written explicitly in (1). The overall discount rate is actually equal to the expected return on the security S , since $r + \lambda \cdot \sigma_S = \mu_S$.

where the expected return on all securities is the risk-free rate. A risk-neutral world is a world where investors do not require a premium to take on risks, i.e. the market price of risk is zero for all securities. The existence of such world is irrelevant: we use it as a merely artificial valuation device. The method, called *risk-neutral valuation*, has become increasingly popular over last years, because of its application to the pricing of derivative instruments and interest-sensitive financial instruments.

The third approach, finally, adjusts the cash flows to their *certainty equivalent* levels and discounts their expected value (under true probabilities) by the risk-free rate. It is somewhat similar to the risk-neutral method, but adjustments are incorporated directly in the cash flows rather than in the probabilities. The need to specify a utility function is a drawback. Nevertheless, the method has been successfully used.

The DSOP takes the view that (principle 5.2):

Adjustments for risk and uncertainty should be reflected preferably in the cash flows, or alternatively in the discount rate(s), without any double counting.

Adjustments to discount rate(s), for example, may be preferable if an insurance contract has cash flows similar to cash flows from a financial instrument traded in an active market (e.g. a bond or a floating rate note).

4.2.4. ***Replicating portfolio approach.*** There are several factors influencing the uncertainty of an insurer's cash flows. They can be distinguished in:

- *non-tradable factors*, such as mortality, longevity or morbidity;
- *tradable factors*, such as equity returns, interest rates and real estate.

The first category may also include factors that are not *frequently* traded in the capital markets, such as inflation when there is not a liquid market for inflation-linked securities.

The distinction is important because when all factors driving the uncertainty in cash flows are tradable, then perfect matching of those cash flows can be achieved. In other words, it is possible to manufacture a portfolio of tradable instruments that match the cash flows concerned and is therefore called *replicating portfolio*. In this case, the market is said to be ***complete***, and valuations are simplified in many aspects. For example, risk-neutral probabilities can be showed to be unique, and the market value of assets and liabilities is uniquely determined by the cost of the replicating portfolio.

The risk-neutral valuation approach is ready to go, with all of the arsenal of derivative valuation techniques at its disposal.

Insurance markets are typically *incomplete*, because of the relevance of non-tradable factors. Incompleteness, however, characterises many financial markets as well, because of lack of actively traded assets or high transaction costs for example.

The International Association of Actuaries (IAA) has proposed some guidance regarding the possibility of hedging and pricing insurance liabilities, in response to the Issues Paper published by the IASC in 1999 (see IAA(2000), Valuation of Risk Adjusted Cash Flows and The Setting of Discount Rates). The focus is mainly on the practical issues, since it is acknowledged that valuation techniques for incomplete markets are very complex.

The IAA's proposal involves three steps in the valuation process:²¹

1. Remove incompleteness due to non-tradable factors, by including appropriate adjustments in the cash flows.
2. Remove any remaining financial incompleteness by adopting reasonable assumptions and extrapolating techniques; then determine fair value as if the market were complete.
3. Adjust the fair value computed in previous step by reflecting the actual assets availability in the market.

1. The first step involves the estimation of the so called *market value margins*, i.e. adjustments to cash flows consistent with market risk preferences about the (non-financial) riskiness of the cash flows.

The DSOP does not propose any specific benchmark or quantitative guidance about them, but states that margins should be inferred as far as possible from market data, and that their assessment should be based on a consistent methodology over time.

The estimation of these margins is a debated issue (see Bice (2002) and Wright (2002), for example). A number of examples of how margins could be set are reported in Vanderhoof and Altman (1998) and Vanderhoof and Altman (2000). Here, we present a brief overview of how the problem has been dealt with in some papers which have been published recently.

Perrott and Hines (2001), in considering a block of single premium deferred annuities, express margins as adjustments to the Treasury yield curve. A fixed spread to the spot rates is determined at inception, by calibrating the

²¹For details on some of the topics covered, see Pliska (1997).

discounted cash flows of liabilities to the present value of projected statutory profits (both computed under best-estimate assumptions).

Wallace (2001), in valuing a block of structured settlements policies,²² interprets the difference between the initial GAAP reserve and the value of a suitable replicating portfolio as a risk premium for insurance risk. This premium is then spread over the policy life term by adjusting liability cash flows by a constant factor.

Van Broekhoven (2002) focuses on mortality risk and proposes a model for taking into account departures from best estimate assumptions. He expresses market value margins in terms of a 90% confidence level in estimates.²³

Abbink and Saker (2002) link fair value to embedded value: they express margins as adjustments to best estimate assumptions by calibrating a risk neutral valuation model to an embedded value computation. The approach, which seems to be very consistent with the DSOP's proposal, is the one we adopt in section 5.

Some authors (e.g. Girard (2000)) are persuaded that margins for insurance risk should only rely on an expert's opinion and that market-implied assumptions, although useful in providing some information, could be misleading. In this regard, the expertise developed by actuaries in estimating provisions for adverse deviations (PADs) in US GAAP, and margins for adverse deviations (MADs) in Canadian GAAP,²⁴ would certainly be helpful in assessing market value margins.

2. The second step requires the identification of financial factors affecting the uncertainty of cash flows that are not tradable. If none of them can be identified, then cash flows can be replicated by a suitable mixture of assets available in the capital market and fair value is simply given by the cost of assembling that replicating portfolio or by the direct application of derivative valuation techniques.

If some factors are non tradable instead, the market is incomplete and we cannot hedge our cash flow stream. The IAA (2000) suggests to remove

²²These contracts provide a defined or structured set of future payouts in exchange for upfront payments.

²³The Australian Prudential Regulatory Authority has proposed a 75% confidence level in estimates made by general insurers (DSOP, paragraph 5.29). The same approach is followed in the example of section 4.2.6, although in that context the confidence level is assumed to be inferred from the market.

²⁴See Doyle (1999) for an overview of the main differences between US and Canadian GAAP (and, specifically, between PADs and MADs).

incompleteness by making reasonable assumptions about the ‘missing assets’. For example, if bonds of some key maturities are not available, then the current yield curve could be extrapolated or stochastically modelled so as to get a reasonable price²⁵ for the missing security.

The value of a replicating portfolio manufactured in such artificially completed market would be a proxy for the true fair value of the liabilities. An adjustment would be needed to reflect the actual assets availability (step 3). It is clear that the proxy fair value would be subjective to some extent, because of the assumptions and method used to complete the market. However, disclosure of assumptions and consistency with overall market views should prevent undue distortions.

In some cases reasonable assumptions or extrapolations would be enough to place a fair value on the liabilities, without having to adjust the proxy fair value. That is the case, for example, of interest-insensitive cash flows with maturity beyond 30 years, which is usually the maximum maturity of bonds. Following IAS 19, the DSOP argues that the expected present value of a defined benefit obligation is unlikely to be particularly sensitive to the discount rate applied to the portion of benefits that is payable beyond the final maturity of the available corporate or government bonds.

For a practical overview on methods for modelling the yield curve, see Martellini and Priaulet (2001). Techniques for determining the replicating portfolio in a discrete time setup can be found in Pliska (1997).

3. The third step involves the assessment of an adjustment to be put on the fair value calculated as if the market were complete. The IAA suggests to use a stochastic model in order to generate a set of possible future scenarios around an extrapolated mean (set in step 2). A margin could then be determined, for example, in terms of the standard deviation of the outcomes generated.

In more complex situations, especially when interest-sensitive cash flows are concerned, it is better to use a stochastic model to determine the hedging portfolio directly, without passing through margins. For this purpose, the IAA proposes two approaches.

The first one aims at determining a hedging strategy that matches the insurers cash flows irrespective of movement in interest rates or returns. It consists in the specification of several dynamic hedging strategies, each involving an initial investment on the assets available and a reinvestment

²⁵Consistent with a no-arbitrage setting.

strategy for returns, coupons and principals. A strategy is said to be hedging if it leads to a net change in fair value belonging to a pre-specified interval. Here, net fair value change is the difference between the change in fair value of assets and the change in fair value of liabilities at each point in time. A stochastic model incorporating assumptions about the evolution of the market can be run for each strategy. Then the lowest cost hedging strategy is chosen. The current cost of setting up the strategy represents the fair value of the insurer's assets or liabilities.

The second method has a limited scope and can therefore be more viable. The same stochastic model as before is run, but a hedging strategy is chosen so as to minimise the volatility of the net fair value change over a shorter time horizon. The strategy would need to be rebalanced at the end of each time period. The cost of setting up the strategy is the so called *fair value immunising portfolio*.

4.2.5. **Discount rates.** If an asset or a portfolio of assets can be found that closely matches the insurer's cash flows, then the proper discount rate entering the expected present value formula would be the yield on that asset or portfolio (see previous section).

In all other cases, estimation of the risk-free discount rate would be needed, irrespective of the approach adopted in valuing the stream of cash flows, since such rate actually enters all expected present value formulae described in section 4.2.3.

The DSOP defines the risk-free rate as the pre-tax market yield at the balance sheet date on risk-free assets, where these are assets with readily observable market prices and with lowest variability in cash flows for a given maturity and currency. The risk free rate can be used directly, to discount cash flows already adjusted for risk, or as a basis for discount rate adjustments.

No assets ensure certain cash flows, but a benchmark for the risk free component of discount rates is usually provided by securities issued by highly creditworthy governments. However, these may carry margins for risks other than default, such as inflation, interest rate and liquidity risk. Nevertheless, they are the closest assets to securities yielding what the JWG Draft calls 'basic interest', i.e. compensation for the time value of money.

In countries where there is no active market in government securities, yields on other securities with high credit ratings could be used, but only

after deductions of margins for estimated defaults and for risk of variability in defaults or in returns.

High quality corporate bonds may be taken as primary benchmark also in countries where an active market in central government bonds exists, but the government or the currency are not stable or when rigidity of supply and demand distorts bond prices.

The definition of risk-free asset makes explicit reference to the maturity of cash flows. It is clear that different risk-free assets exist for different maturities, usually implying different discount rates reflecting the time preferences of market participants. The DSOP suggests to use the whole yield curve as a set of discount rates, in order to reflect the estimated timing of the cash flows to be discounted. However, the use of a single rate is allowed when results are reasonable approximations of what would have been obtained by using all discount rates available.

As far as *foreign currency* cash flows are concerned, expected present values should first be computed by discounting at an appropriate discount rate for that currency, and then translated into the measurement currency using the spot exchange rate at the reporting date. The same procedure would apply for currencies of hyperinflationary economies. The DSOP does not allow the use of a hard currency as a proxy for the translation into the measurement currency.

4.2.6. *Whose risk preferences?* The DSOP states that both entity-specific value and fair value should reflect the *market's preferences* inferred, as far as possible, from observable market data (principle 5.3).

That is not surprising for fair value, since it refers to a hypothetical transaction in the marketplace and should be independent from factors that are specific to the particular insurer that holds the assets or liabilities concerned (see section 3.3).

For entity-specific value instead, the DSOP's proposals mean that the insurer is allowed to make its own estimates of the amount, timing and uncertainty of future cash flows, but must then price those cash flows in the same way that the market would price similar cash flows. As a consequence, the specific insurer's degree of risk-aversion is reflected by neither entity-specific value nor fair value.

This approach leads to greater comparability, because it allows to deal with overall risk preferences rather than having to cope with possible conflicts between the risk preferences of insurers and other users of financial statements.

As an example, suppose an insurer has a book of contracts that will generate cash flows whose random present value (X) follows a normal distribution. The market agrees on the use of that distribution for a book with similar characteristics. However, there is disagreement about the parameter estimates: the insurer estimates a mean of 100 and a standard deviation of 20, while the consensus view in the market is that the same book of contracts will generate cash flows with expected present value of 125 and standard deviation of 35. Now, suppose both the insurer and the market express the market value margins as a fraction of the standard deviation, but:

- the insurer requires a margin equal to 25% of the standard deviation;
- according to the market's risk preferences, the margin required should be 15% of the standard deviation.²⁶

Then, the insurer should report:

- a liability of $100 + 15\% \cdot 20 = 103.0$ under entity-specific measurement;
- a liability of $125 + 15\% \cdot 35 = 130.2$ under fair value measurement.

The difference in the two values would not be attributable to the insurer's risk-preferences, but only to the fact that the insurer expects lower cash flows than the overall market participants.

4.2.7. *Additional Remarks.* Some points presented in previous sections need additional information.

Market value margins. If no reliable estimate can be made of the market value margin at initial recognition of an insurance asset or liability, then an insurer should set the margin so as to have no net underwriting profit or loss from the contract (principle 5.7). This is, however, an exceptional case according to the DSOP, and circumstances should be reviewed carefully at later reporting dates.

Illiquidity and market imperfections. Conceptually, they should be included in both entity-specific and fair value. However, to promote comparability the DSOP limits the inclusion of adjustments for such factors to cases

²⁶Those figures correspond to confidence levels of 60% for the insurer, and 56% for the market. In the first case, for example, the probability that the liability X would exceed $100 + 25\% \cdot 20$ is not greater than 40%.

when observable market data enable to estimate them reliably (principle 5.7).

Options and guarantees. The DSOP does not require the insurer to account separately for options contained in insurance assets and liabilities. However, it does require the insurer to use option pricing models to measure cash flows that contains options or guarantees. In doing so, attention should be paid to the policyholder behaviour. For example, models used for financial options assume that an investor, unlike a policyholder, never exercises out-of-the-money options. Similarly, policyholders often do not exercise in-the-money-options, when it would be optimal. The importance of behavioural features of insurance options make them close to mortgage-backed securities. As a consequence, valuation techniques adopted in that field, could be successfully used (see Britt (2001)).

4.3. Performance-linked contracts. The IASB is still working on the section of the DSOP relating to performance-linked insurance contracts. However, some notes for observers attending the IASB's meetings have been released and can be downloaded from the IASB's website. Here, we give an overview of the issues that are being discussed.

Performance-linked insurance contracts are insurance contracts under which the payments to policyholders, or the premium payable from policyholders sometimes, depend (fully or partly) on: the performance of the contract or of a pool of contracts; realised and/or unrealised investment returns on a specified pool of assets held by the insurer; the net profit or loss of the company. Contracts of this type are participating (with-profit) and variable (unit-linked) policies, for example.

Such contracts are characterised by several features that make their valuation quite complex. For example:

- *smoothing*: their objective is often to smooth the impact of fluctuations of experience over time, by spreading profits and losses over different generations of policyholders;
- *discretion*: the insurer has often discretion in choosing the amount and timing of bonuses to policyholders;
- *guarantees*: they usually provide guarantees, and often with features difficult to price (e.g. ratchet guarantees):
- *distributable surplus*: the amount available for distribution to policyholders is sometimes determined on a contractual or statutory basis

and can be different from what recognised under a reporting framework.

The DSOP refers to the assets in which the policyholders have an effective interest as *participating net assets*. *Pre-allocation surplus* is the aggregate carrying amount of the participating net assets, after deduction of any expenses or guaranteed minimum benefits, and addition of future premium receipts (all these items at their expected present value). It is clear that pre-allocation surplus need not be positive, and that its size will influence the insurer's discretionary *allocations* (i.e. dividends, distributions, bonuses).

The fundamental issue concerning performance-linked contracts is whether pre-allocation surplus is a liability. Some argue that it should be classified as equity, some as liability and others as neither clearly a liability nor clearly equity, but as an intermediate category.

The IASB seems to favour the view that pre-allocation surplus should be classified as a liability to the extent that the insurer has a legal or constructive obligation at the balance sheet date to allocate part of the surplus to current (or future) policyholders. It should be classified as equity for the remaining part.

A crucial issue is represented by the measurement of *policyholders' interest in the pre-allocation surplus*, which is likely to be necessary even if some or all of that interest will be presented as equity. The IASB proposes a measurement approach that reduces the need to estimate future investment returns and the amount and timing of allocations. The approach is based on the argument that if assets are carried at fair value, then policyholders' interest in those assets is consistent with market prices. In particular, the present value of cash flows arising from existing assets and liabilities can be considered to be the carrying amount of those assets and liabilities. Of course, guarantees and options would require the use of stochastic models with a set of 'entity-specific' modelling assumptions, but their effect would be limited to some components of the policyholders' interest.

A simple example will outline the main features of the approach. Suppose that policyholders are collectively²⁷ entitled to a guaranteed benefit equal to the premiums paid, plus a guaranteed annual investment return of 3%, plus 90% of the amount by which the investment return (on a fair value basis)

²⁷That means we consider policyholders as a group, including both current and future policyholders.

to the date of maturity exceeds the guaranteed annual return of 3%. For simplicity, insurance risk arising from lapses and mortality is disregarded.

In a one-period setting, let: A_0 be the assets acquired at time 0 by policyholders' premiums; A_1 be the random fair value of those assets at time 1; G_1 be the guaranteed amount at time 1 (i.e. $G_1 = (1 + 3\%) \cdot A_0$). Then, the end-period payoff to policyholders is given by:

- G_1 , if the investment return on the assets over the period does not exceed the 3% threshold;
- $G_1 + 90\% \cdot (A_1 - G_1)$, if the return on A_0 is above 3%.

Some simple algebra enables us to rewrite the final payoff in an equivalent, more meaningful way:

- if the guarantee bites, then policyholders will receive:

$$10\% \cdot G_1 + 90\%A_1 + 90\% \cdot (G_1 - A_1); \quad (4)$$

- otherwise, they will receive:

$$10\% \cdot G_1 + 90\%A_1 \quad (5)$$

The payoff can therefore be summarised by the following expression:²⁸

$$10\% \cdot G_1 + 90\%A_1 + 90\% \cdot (G_1 - A_1)^+ \quad (6)$$

Thinking in terms of expected present values, therefore, we can say that at time 0 policyholders have an effective interest in the present value of a risk-free payment from shareholders of 10% of the guaranteed benefit at maturity, in 90% of the fair value of the assets held, and in the fair value of an option to put 90% of the assets to shareholders for a strike price equal to 90% of the guaranteed benefit.

The measurement of the first two components is straightforward and unambiguous. The present value of $10\% \cdot G_1$ can be obtained by discounting that known quantity at the risk-free rate. The present value of $90\%A_1$ is simply equal to 90% of the carrying amount of the assets held (i.e. to $90\%A_0$).

The pricing of the last component is entity-specific: the characteristics of the assets actually held will influence the risk of the put option being in-the-money at maturity.

Things become more complicated when one has to take into account mortality and lapses affecting future cash flows. Moreover, policyholders could have an effective interest in the expenses that will be borne by the assets

²⁸By $(a - b)^+$ we mean the greater between $(a - b)$ and 0.

held. When allocations are linked to underwriting performance, the present value of those future expenses reduces the pre-allocation surplus. As a consequence, policyholders will bear at least part of those expenses. On the other hand, shareholders may have an interest not only in part of the assets held, but also in estimates of future premium receipts²⁹ (net of policyholders' interest in assets to be acquired with those expected premiums) and in charges that will be levied on policyholders' interest by the insurer.

The approach proposed by the IASB could nevertheless be applied to complex situations, at least in order to isolate the components of the liabilities that are harder to measure, thus favouring a more explicit analysis of those components.³⁰

Note also that under this approach, one could consider *unit-linked policies* as a special case of with-profit policies, in which policyholders' effective interest is (typically) 100%. In the context of our simple example, the fair value of assets is divided between policyholders and shareholders in the ratio 90:10. With a unit-linked product we would possibly have to consider a ratio 100:0. Asset management charges, which generally constitute a significant part of the insurer's return, would actually reduce the policyholders' effective interest in the assets held, thus decreasing the fair value of the insurer's liability. This is a consequence of principle 1.6 (see section 3.7), which states that the investment component should not be unbundled.

An example of unit-linked product valuation is provided in the next section.

5. NUMERICAL EXAMPLE

In this section we apply the DSOP's principles to the valuation of an insurance product sold in the Italian market. We assess the value of the liabilities by using first an embedded value approach and then a direct prospective approach. We determine the market value margins by calibrating the fair

²⁹to the extent that they qualify as part of the closed book under principle 4.2 (see section 4.1.2 about renewals).

³⁰Recently, the actuarial community has been discussing stochastic models which are consistent with risk-neutral valuation and enable to value very complex cash flows. These models are based on the theory of the so called 'deflators', stochastic discount rates derived in a no-arbitrage framework. See Jarvis *et al.* (2001) for an overview of the theory and some examples.

value model to the results given by the embedded value approach at inception. We then compare the emergence of profits under prudential, embedded value and fair value reporting.

5.1. Characteristics of the insurance policy. The policy is a unit-linked deferred annuity. In particular, the contract is a unit-linked product during the deferment period and a non-profit annuity from the time of retirement onwards.

The policyholder can choose among different funds where to put his money. Premium payment is free all over the policy term. Only the first premium is imposed to be above a minimum level P^{\min} .

All premiums are levied a fixed percentage charge c ($0 < c < 1$) and a variable charge determined with respect to the age of the insured. If the insurer decides to retire later, no variable fee is charged.

Premiums after charges are used to buy units from the fund chosen. The insurer collects an asset management charge α ($0 < \alpha < 1$) from the unit fund at the end of each year.

Death and survival benefits are linked to the accumulated units in the following way:

- in case of death, a benefit equal to $(1 + \delta)$ (with $\delta > 0$) times the value of the units held is provided;
- at retirement, the value of the units held is converted into an immediate annuity through a conversion factor.

For example, if $\delta = 10\%$, in case of death a benefit equal to 110% of the value (after asset management charges) of the units held at the time of death is provided.

At inception, an annuity conversion factor is declared. During the deferment period, the conversion factor can be varied by the insurer, in response to significant mortality improvements or dramatic movements in interest rates, but only subject to approval of the regulatory authority. However, new conversion factors do not apply to the amount of units already bought. They only affect the units that will be acquired by the premiums payable thereafter.

At the time of conversion, a percentage charge a ($0 < a < 1$) is levied on the conversion factor to cope with initial and recurrent expenses related to the annuity.

Since the policy enjoys particular tax benefits, it has several features preventing the policyholder to cancel. Surrender is allowed only after 8 years are elapsed from inception and only in specific circumstances (development of dread diseases, need for costly therapies, purchase of first home for children, etc.). The termination benefit is equal to the value of the accumulated unit fund at the time of surrender.

The policyholder can transfer his funds to a similar pension regime run by another company only after 3 years are elapsed. The transfer is subject to a fixed charge on the units held.

At retirement, only 50% of the accumulated unit fund can be drawn down. The other 50% must be necessarily converted into an immediate non-profit annuity.

Assume that the retirement age is ξ and that an insured aged x years enters the contract at time 0. In what follows, we assume that all cash flows occur at year dates $t \geq 0$.

Let P_t be the premium paid at time t (with $P_0 > P^{\min}$) and u_t the price of units at that time (for simplicity, we assume that there is no bid-ask spread). If U_t indicates the number of units that the policyholder will be allocated, the premium can be expressed in the following way:

$$P_t = c \cdot P_t + \delta \cdot {}_{n-t}A_{x+t}^{(0\%, q^c)} \cdot U_t \cdot u_t + U_t \cdot u_t, \quad (7)$$

where $\{0\%, q^c\}$ ³¹ is the technical basis used in determining the term assurance factor (${}_{n-t}A_{x+t}$), which in standard actuarial notation is given by:

$${}_{n-t}A_{x+t}^{(0\%, q^c)} = \sum_{h=1}^{n-t} {}_{h-1/1}q_{x+t}^c \quad (8)$$

From (7), we see that the premium presents three components. The first component represents the charges levied for expenses and administration costs. The second component is a mortality charge that covers the cost of providing the additional $\delta\%$ benefit in case of death during the deferment period.³² The third component represents the amount of units that will be given back to the policyholder anyway: as part of the death benefit, in case of death; as part of the amount to be converted into an annuity, in case of survival at maturity.

³¹In what follows, we refer to death probabilities by writing q , to survival probabilities by writing p .

³²Note that the mortality charge covers the additional benefit with respect to the only units acquired at time t .

The last component, which we denote by S_t ($S_t = U_t \cdot u_t$), is the amount which is actually used to allocate the units. From (7), it can be expressed as:

$$S_t = (1 - c) \cdot P_t \cdot \frac{1}{1 + \delta \cdot {}_{n-t}A_{x+t}^{(0\%, q^c)}} \quad (9)$$

Let k_t be the conversion factor declared at time t for the units then acquired (it can be equal to k_{t-1} or not, depending on whether the insurer has changed conversion factor or not). It can be expressed as:

$$k_t = \frac{1}{\ddot{a}_\xi^{[t]}} \quad (10)$$

where $\ddot{a}_\xi^{[t]}$ is the single premium of an immediate annuity payable to an individual aged ξ and determined through the technical basis $\{i^{[t]}, q^{[t]}\}$ declared at time t . We have:

$$\ddot{a}_\xi^{[t]} = \sum_{h=0}^{\omega-\xi-1} (1 + i^{[t]})^{-h} \cdot {}_h p_\xi^{[t]}, \quad (11)$$

where ω is the ‘extreme age’ according to the demographic basis $q^{[t]}$.

Now, let UF_t be the unit fund available at time t , after deduction of unit charges. It is given by the aggregate amount of units (acquired between 0 and t and netted of charges) multiplied by the current value of each unit:

$$UF_t = \left(\sum_{h=0}^t U_h \cdot (1 - \alpha)^{t-h} \right) \cdot u_t, \quad (12)$$

Then, assuming the policyholder draws down a fraction γ ($0 \leq \gamma \leq 0.5$) of the total accumulated fund at maturity ($UF_{\xi-x}$), the annuity amount R that will be paid in the retirement period can be expressed as:

$$R = (1 - \gamma) \cdot \left(\sum_{t=0}^{\xi-x} U_t \cdot (1 - \alpha)^{\xi-x-t} \cdot k_t \cdot (1 - a) \right) \cdot u_{\xi-x} \quad (13)$$

Note that each annuity conversion factor k_t applies to the only units bought at each time t . In this way, we allow possible changes in the conversion factor to affect only the units acquired at the time such changes occur.

5.2. Example of product valuation. Assume a portfolio of homogeneous risks is considered, such as a group of 50-year old men that enter the contract at time 0. From now on, we will think of a single insured and we will reason in terms of expected values. It will be immediate to scale the results back to a book level.

Let $\xi = 65$, $c = 6\%$, $\delta = 1\%$, $\alpha = 1.5\%$ and $a = 0.9\%$. Assume the first premium paid is $P_0 = 1000$ (above the threshold $P^{\min} = 900$ fixed by the insurer). If the technical basis used to determine the mortality charge is $\{0\%, q^c = q^{SIM92}\}$,³³ from (8) we have:

$$1 + \delta \cdot {}_{15}A_{50}^{(0\%, q^{SIM92})} = 1.0015 \quad (14)$$

Thus, the part of premium used to allocate units is $S_0 = 938.60$ (see (9)).

For simplicity, and without loss of generality, assume that $u_0 = 1$. Then, the amount of units bought at inception is simply $U_0 = S_0/u_0 = 938.60$.

Assume further that the annuity conversion factor (k_0) declared at inception (and thus guaranteed for the units bought at that time in case any change is decided by the insurer at future dates) is based on the technical basis $\{i^{[0]} = 2.5\%, q^{[0]} = q^{RG48}\}$.³⁴ By applying (10), we get $k_0 = 0.62085$.

Looking at the book of contracts from the point of view of the DSOP's proposals, it is clear that the contract chosen qualify as insurance contract. Insurance risk is present and relevant because the insurer is committed to the provision of an annuity at the end of the deferment period. If the maturity benefit were not to involve an annuity, but simply a lump sum payment at maturity, then the additional 1% benefit provided in case of death would be insufficient for the contract to qualify as insurance contract.

Estimates of future premium payments should not be considered, because the insurer is free to change the annuity conversion factor (see section 4.1.2 and DSOP, paragraph 4.59). We do not consider new business either, consistently with principle 2.2 (see section 3.2).

We do not take into account: surrenders, transfers to other insurers' plans and switches between different fund. The exercise of any of those options by the policyholder would relieve the insurer from the burden of providing the annuity and would determine additional cash inflows to the insurer (transfer and switch charges), thus decreasing the liabilities.

As far as the annuity is concerned, note that the policyholder has the right to draw down at most 50% of the accumulated benefit. This means that the insurer guarantees the conversion of 50% of the final accumulated fund and has sold an option to put an additional 50% back to the insurer. The situation is actually that of a sequence of put options being written to

³³We mean that death probabilities are derived by mortality table SIM92. Table SIM92 is an italian mortality table used for term assurances and endowments.

³⁴Table RG48 is an italian projected mortality table widely used for annuity products. In what follows, we assume that $\omega = 112$, consistently with the use of this table.

the policyholder every time a premium is paid into the fund, thus locking the conversion rate declared at that time.

Valuing these ‘options to annuitise’ is a difficult task. We refer the reader to Milevsky and Promislow (2001), Ballotta and Haberman (2003) and Olivieri and Pitacco (2003) for some results on the topic.

In what follows, we value the insurer’s liability assuming the policyholder does not draw down anything at retirement (i.e. $\gamma = 0$). The results obtained will constitute an upper bound for the liability.

5.2.1. *Embedded Value Method.* We perform an embedded value analysis of the policy for three reasons:

- (1) to show how this indirect method works and to point out the main differences with the fair value framework;
- (2) to determine a benchmark to which calibrate our fair value model in order to quantify the market value margins required;
- (3) to show the differences in expected profit patterns between prudential, embedded value and fair value (with and without adjustments) reporting.

As far as the second point is concerned, we may think of it in two ways. First, many transactions in the secondary insurance market are based on embedded or appraisal value (i.e. embedded value plus goodwill) calculations. We could therefore think of the value put on the liability through the embedded value method as of a potential or hypothetical ‘observed price’.

Second, insurers have matured a great amount of expertise in allowing for risk in embedded value calculations. They generally adopt best estimate assumptions as far as deaths, lapses and expenses are concerned, while setting margins in the discount rates to allow for adverse departures from ‘true’ assumptions.³⁵ Calibrating the fair value model to an embedded value model would thus allow to translate those margins into corrections to best estimate assumptions (as done in Abbink and Saker (2002)). Moreover, we could achieve a better understanding of how adjustments to true assumptions compare to prudential assumptions.

Note that embedded value methods discount future statutory profits by an adjusted rate which usually reflects not only the risk being taken on by the insurer, but also items such as cost of supervisory capital requirements and

³⁵Note that by ‘true’ assumptions we mean ‘realistic’ assumptions based on best estimates. It is clear that no true assumptions exist, since we are dealing with random future mortality rates, lapse rates, inflation rates, etc..

future tax liabilities. It is generally not easy to unbundle the adjustments made for each item. Moreover, the discount rate used for book transfers of some class of business does not seem to move much with yield curve changes (Perrott and Hines (2001)). See Simpson and Wells (2000) for an overview about the use of embedded value methods in insurance mergers and acquisitions, and Sheard *et al.* (2001) for a survey of the assumptions used in life office valuations.

In Table 1, we provide true and prudential assumptions used in the valuation. In what follows, we use the superscript ‘*true*’ and ‘*prud*’ to refer to the one or the other type of assumptions.

	Prudential	True
Interest (i) on sterling reserves and cash flows	2.5%	4.5%
Mortality adjustment factor (MF)	80%	110%
Unit growth rate (g)	7%	7%
Commission	100	100
Expenses:		
inflation rate (f)	4%	3%
deferment period:		
(initial)	20	20
(regular)	5	5
annuity expenses:		
(initial)	20	20
(regular)	0.3%	0.3%

Table 1: Prudential and realistic assumptions.

For simplicity, we use a constant mortality adjustment factor MF ($0 < MF < 1$) to describe different basis with respect to the one used for pricing (table RG48). As an example, death probabilities reflecting true assumptions are given by:

$$q^{true} = MF^{true} \cdot q^{RG48} = 110\% \cdot q^{RG48} \quad (15)$$

The unit price is assumed to grow at a constant rate g and can therefore be expressed as:

$$u_t = (1 + g)^t \quad (16)$$

for a generic time $t = 0, \dots, 15$.

As far as expenses are concerned, we assume that renewal expenses are paid at the beginning of each year after inception. Note that annuity renewal expenses are expressed as a percentage of the annuity amount paid. Expense levels do not vary under the two basis, but the assumed expenses inflation rate (f) does. Let E_t indicate the expenses relating to year $[t-1, t)$ and paid at time $(t - 1)$. Under the true basis, for example, E_t can be expressed as follows:

$$E_t^{true} = \begin{cases} 100 + 20 & t = 1 \\ 5 \cdot (1 + f^{true})^{t-1} & t = 2, \dots, 15 \\ 20 & t = 16 \\ R \cdot 0.3\% \cdot (1 + f^{true})^{t-1} & t = 17, \dots, \omega - 50 - 1 \end{cases} \quad (17)$$

where $f^{true} = 3\%$ and R is given by (13) or (20). Note that we include commission as an expense incurred at time 0.

We first focus on the deferment period ($t = 0, \dots, 15$), and then move on to the annuity payment period ($t = 15, \dots, \omega - 50 - 1$). As usual with unit-linked policies, we have to deal with a unit fund and a sterling fund. We can focus on the latter, because 100% of the units credited will be returned back to the policyholder anyway (in case of death or in case of survival at maturity). The 1% additional benefit in case of death is provided by the sterling fund, which in turn receives the asset management charges levied at the end of each year on the unit fund.

Prudential annuity reserves are determined through the following expression (for $t = 16, \dots, \omega - 50 - 1$):

$$V_{t-1} = R \cdot \ddot{a}_{50+t-1}^{prud} = R \cdot \left(\sum_{h=0}^{\omega-t-1} (1 + i^{prud})^{-h} \cdot {}_h p_{50+t-1}^{prud} \right) \quad (18)$$

where \ddot{a}_{50+t-1}^{prud} is the single premium of an immediate annuity payable at age $(50 + t - 1)$. Table 4 provides some of their values.

We now check that the amount provided by the unit fund at maturity will not be enough to set up the prudential annuity reserve required in case of survival. Since we do not consider any future premium payments, we have that the unit fund at maturity is (see (12)):

$$UF_{15} = S_0 \cdot (1 - \alpha)^{15} \cdot (1 + g)^{15} = 2064.35 \quad (19)$$

Therefore, the annual amount provided by the immediate annuity will be (see (13)):

$$R = UF_{15} \cdot k_0 \cdot (1 - a) = 127.01, \quad (20)$$

and an amount equal to:

$$R \cdot V_{15} - UF_{15} = 99.49 \quad (21)$$

will be needed at the beginning of year 16.

That figure certainly depends on the unit growth assumption made, but not so much as it might at first appear. In fact, while a higher unit growth assumption generates a higher annuity amount R (and a greater prudential requirement), at the same time it leads to greater inflows from asset management charges to the sterling fund. It is indeed the sterling fund that has to provide for the building up of the amount required at maturity, as will become clear soon.

Table 2 shows the cash flow analysis for the sterling fund under prudential assumptions. Data reported are *per policy in force at the beginning of each year*. The quantity $(P_t - S_t)$ represents the premium charges collected at the beginning of year t . E_t^{prud} is a start year item as well and is obtained by applying (17), but with prudential assumptions.

The asset management charges collected from the unit fund at the end of year t (independently of the policyholder being alive or not at that time) are expressed as:

$$AMC_t = \alpha \cdot UF_{t-} \quad (22)$$

where UF_{t-} represents the accumulated unit fund at the end of year t ($t = 1, \dots, 15$), before benefits are paid.

Item D_t^{prud} represents the expected claim costs at the end of year t for a policy in force at the beginning of that year, and is given by:

$$D_t^{prud} = \delta \cdot UF_t \cdot q_{50+t-1}^{prud} \quad (23)$$

Note that UF_t is the unit fund at the end of year t after asset management charges are collected, i.e. $UF_t = (1 - \alpha) \cdot UF_{t-}$.

Item AR_t represents the expected additional amount to be provided at the end of year t , in case of survival, to set up the annuity reserve. It is zero in each year $t < 15$. At the end of year 15 instead, the amount given by (21) must be provided. We have, therefore:

$$AR_{15}^{prud} = 99.84 \cdot p_{64}^{prud} \quad (24)$$

The sterling cash flows computed on a prudential basis are defined as follows:

$$SCF_t^{prud} = (P_t - S_t - E_t^{prud}) \cdot (1 + i^{prud}) + AMC_t - D_t^{prud} - AR_t^{prud} \quad (25)$$

From the results reported in Table 2, we can see that a negative cash flow arises at the end of year 15. We need to use the profits available in previous years to set up a sterling reserve so as to zeroise any negative cash flows after year 1. This can be done through the classical *zeroisation algorithm* (see Hare and McCutcheon (1991), for example). As a result, we get the sterling reserves V_{t-1}^{ST} (with $V_{15}^{ST} = 0$) that must be set up at the beginning of each year t , per policy in force at that time.

We can then determine the sterling fund profits emerging at the end of each year as follows:

$$PRO_t = SCF_t^{prud} + V_{t-1}^{ST} \cdot (1 + i^{prud}) - p_{50+t-1}^{prud} \cdot V_t^{ST} \quad (26)$$

Table 2 shows that under the prudential basis used, the setting up of the sterling reserve requires the complete use of profits in the years preceding retirement.

t	$(P_t - S_t)$	E_t^{prud}	AMC_t	D_t^{prud}	AR_t^{prud}	SCF_t^{prud}	V_{t-1}^{ST}	PRO_t
1	61.40	120.00	15.06	0.015	0.00	-45.02	0.00	-45.02
2	0.00	5.20	15.88	0.016	0.00	10.53	0.00	10.53
3	0.00	5.41	16.73	0.018	0.00	11.17	0.00	11.17
4	0.00	5.62	17.64	0.020	0.00	11.85	0.00	11.85
5	0.00	5.85	18.59	0.023	0.00	12.57	0.00	12.57
6	0.00	6.08	19.59	0.026	0.00	13.33	0.00	13.33
7	0.00	6.33	20.65	0.030	0.00	14.13	0.00	14.13
8	0.00	6.58	21.76	0.035	0.00	14.98	0.00	14.98
9	0.00	6.84	22.94	0.040	0.00	15.88	0.00	15.88
10	0.00	7.12	24.17	0.046	0.00	16.83	0.00	16.83
11	0.00	7.40	25.48	0.052	0.00	17.84	0.00	6.26
12	0.00	7.70	26.85	0.059	0.00	18.90	11.61	0.00
13	0.00	8.01	28.30	0.069	0.00	20.03	30.91	0.00
14	0.00	8.33	29.83	0.080	0.00	21.21	51.89	0.00
15	0.00	8.66	31.44	0.093	99.02	-76.55	74.68	0.00

Table 2: Sterling Fund: prudential assumptions.

Under true assumptions instead, the sterling fund generates the cash flows reported in Table 3. By PRO_t^{STAT} we indicate the statutory profit emerging over year t per policy in force at the beginning of year t . Profit emerges as true assumptions are borne out in practice and the insurer is released from

risk. We have:

$$PRO_t^{STAT} = SCF_t^{true} + V_{t-1}^{ST} \cdot (1 + i^{true}) - p_{50+t-1}^{true} \cdot V_t^{ST}, \quad (27)$$

where $t = 1, \dots, 15$ and SCF_t^{true} is determined as in (25), but with true assumptions.

Statutory profits *per policy issued* (EP_t^{STAT}) can be determined by taking into account the probability of a policyholder being alive at the beginning of each year, i.e.:

$$EP_t^{STAT} = {}_{t-1}p_{50}^{true} \cdot PRO_t^{STAT} \quad (28)$$

The sequence $\{EP_t^{STAT}; t = 1, \dots, 15\}$ represents what is usually called the *profit signature*.

t	$(P_t - S_t)$	E_t^{true}	AMC_t	D_t^{true}	AR_t^{true}	V_{t-1}^{ST}	PRO_t^{STAT}	EP_t^{STAT}
1	61.40	120.00	15.06	0.020	0.00	0.00	-46.20	-46.20
2	0.00	5.15	15.88	0.022	0.00	0.00	10.47	10.45
3	0.00	5.30	16.73	0.025	0.00	0.00	11.17	11.12
4	0.00	5.46	17.64	0.028	0.00	0.00	11.90	11.82
5	0.00	5.63	18.59	0.031	0.00	0.00	12.68	12.56
6	0.00	5.80	19.59	0.036	0.00	0.00	13.50	13.34
7	0.00	5.97	20.65	0.041	0.00	0.00	14.37	14.16
8	0.00	6.15	21.76	0.048	0.00	0.00	15.29	15.02
9	0.00	6.33	22.94	0.055	0.00	0.00	16.26	15.93
10	0.00	6.52	24.17	0.063	0.00	0.00	17.29	16.87
11	0.00	6.72	25.48	0.071	0.00	0.00	6.82	6.63
12	0.00	6.92	26.85	0.082	0.00	11.61	0.90	0.87
13	0.00	7.13	28.30	0.094	0.00	30.91	1.42	1.36
14	0.00	7.34	29.83	0.109	0.00	51.89	1.98	1.89
15	0.00	7.56	31.44	0.127	98.84	74.68	2.61	2.48

Table 3: Sterling Fund: true assumptions.

Looking at the retirement period, we can compute the statutory profits emerging from the deferred annuity in a similar way. Profits emerging at the end of year t ($t = 16, \dots, \omega - x - 1$), per policy in force at the beginning of that year, are given by:

$$PRO_t^{STAT} = (V_{t-1} + CC_t - R - E_t^{true}) \cdot (1 + i^{true}) - p_{50+t-1}^{true} \cdot V_t \quad (29)$$

where R is the annuity amount defined by (13) or (20), E_t^{true} are annuity expenses computed according to (17) and CC_t represents the conversion charges levied on the final accumulated fund. CC_t is zero for every $t > 16$, while in year 16 it is given by:

$$CC_{16} = UF_{15} \cdot a = 18.58 \quad (30)$$

Profits per policy issued can be computed through (28). Results for $t = 16, \dots, 30$ are provided in Table 4.

t	\ddot{a}_{50+t-1}^{prud}	$R \cdot V_{t-1}$	E_t^{true}	CC_t	PRO_t^{STAT}	EP_t^{STAT}
16	17.04	2163.84	20.00	18.58	43.38	41.04
17	16.52	2098.76	0.61	0.00	43.26	40.63
18	16.01	2032.95	0.63	0.00	42.29	39.40
19	15.48	1966.47	0.65	0.00	41.35	38.17
20	14.95	1899.41	0.67	0.00	40.44	36.96
21	14.42	1831.88	0.69	0.00	39.57	35.75
22	13.89	1764.01	0.71	0.00	38.75	34.56
23	13.35	1695.92	0.73	0.00	37.98	33.38
24	12.82	1627.78	0.75	0.00	37.28	32.22
25	12.28	1559.76	0.77	0.00	36.65	31.08
26	11.75	1492.04	0.80	0.00	36.10	29.95
27	11.22	1424.84	0.82	0.00	35.66	28.85
28	10.70	1358.44	0.85	0.00	35.31	27.75
29	10.18	1293.05	0.87	0.00	35.02	26.62
30	9.67	1228.81	0.90	0.00	34.77	25.43
...

Table 4: Annuity payment period: true assumptions.

We then assume that no additional shareholders' assets assist the business and that all statutory profits (or losses) are distributed (capital is contributed), so that statutory surplus is zero at the end of each year. The embedded value at inception (EV_0) is thus simply given by the value of in force business at time 0, i.e. by discounting the profits per policy issued by a risk discount rate (RDR) suitably chosen. We have, therefore:

$$EV_0 = \sum_{h=1}^{\xi-x} (1 + RDR)^{-h} \cdot EP_h^{STAT} \quad (31)$$

With a discount rate of 6.15%,³⁶ we get an embedded value figure of 209.07. We can then indirectly place a value MVL_0 on the liabilities, by subtracting EV_0 from the market value of assets ($MVA_0 = 1000$):

$$MVL_0 = MVA_0 - EV_0 = 790.93 \quad (32)$$

5.2.2. Fair Value Method. We now show how a direct prospective method can be applied to the policy concerned. We can rely almost entirely on the current fair value of the units underlying the policyholder's conversion capital (see the approach described in section 4.3).

³⁶It incorporates a 100 basis point spread above the return on 30-year zero coupon bonds (see section 5.2.2).

We first need to distinguish non-unit related from unit related items. The former are given by expenses and premium charges. The latter by death and survival benefits, and by the annuity expenses expressed in percentage of annuity amount.

We adopt the replicating portfolio approach described in section 4.2.4. We first use the true assumptions reported in Table 1 to specify the expected cash flows that need to be replicated. Later, we will focus on the adjustments for risk to those cash flows.

We match yearly expected cash flows by using riskless zero coupon bonds. Let $B(0, T)$ indicate the price at time 0 of a riskless zero coupon bond with maturity T and face value 1. Under annual compounding, we have:

$$B(0, t) = (1 + r(0, t))^{-t} \quad (33)$$

where $r(0, t)$ is the spot annual zero rate for maturity t . Values of $B(0, T)$ (for $T = 0, \dots, 30$) are showed in Table 5. They are derived from the zero rates available on the Reuters information system on 17 December 2002 at 4pm (see Figure 1).³⁷ We assume that the yield curve remains flat for maturities longer than 30 years, at a rate of 5.15% consistent with the longest zero coupon bond available (see section 4.2.4).

T	$B(0, T)$	T	$B(0, T)$
1	0.9724994616	16	0.4558231224
2	0.9416192153	17	0.4316530681
3	0.9063795664	18	0.4082589776
4	0.8678668150	19	0.3859290073
5	0.8284894308	20	0.3644469005
6	0.7884208943	21	0.3462300859
7	0.7479983281	22	0.3288236145
8	0.7094520304	23	0.3121930118
9	0.6719879701	24	0.2962210306
10	0.6358393117	25	0.2810882698
11	0.6019689835	26	0.2679536161
12	0.5687168170	27	0.2554165666
13	0.5384418641	28	0.2434830219
14	0.5093350929	29	0.2321237789
15	0.4809168275	30	0.2212824596

Table 5: Zero coupon bond prices.

³⁷Data were kindly provided by Valeria Mazzone.

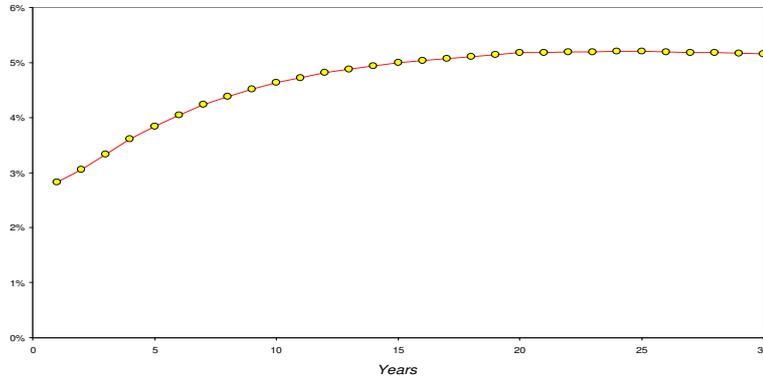


Figure 1: Zero rates for the Euro market (17/12/2002, 4pm).

The *non-unit related* component of the insurer's liability ($FVL_0^{(1)}$) can be valued by discounting the future expenses cash flows (under true assumptions) at the risk-free zero rates. Put it another way, we can replicate the cash flow stream by using a portfolio of zero coupon bonds:

$$FVL_0^{(1)} = \sum_{h=0}^{14} B(0, h) \cdot \left(E_{h+1}^{true} + P_h - S_h \right) + B(0, 15) \cdot E_{16}^{true} \quad (34)$$

Note that we included the annuity initial expenses.

For the *unit-related* items, we can use the current fair value of units and the risk-free yield curve in an effective way. We give a simple example of that by considering three basic cases which will be useful later.

Suppose we want to compute the expected present value V_0 at time 0 of a future cash flow C_T dependent on the unit fund growth. Let g be the random unit growth rate and $f(0, t, T)$ the deterministic annual forward rate implied by the current risk-free yield curve for the period $[t, T]$ (for $0 < t < T$). By no arbitrage arguments (see Hull (2002)), the latter can be expressed as follows:

$$f(0, t, T) = \left(\frac{B(0, T)}{B(0, t)} \right)^{-\frac{1}{T-t}} - 1 \quad (35)$$

We could be presented with the following situations:

- C_T is a cash flow at time T based on the investment in the unit fund at time 0 of a known quantity c_0 . We must obviously have:

$$V_0 = c_0 \cdot (1 + g)^t \cdot (1 + g)^{-t} = c_0 \quad (36)$$

and V_0 is simply the current fair value of the units bought with c_0 .

- C_T is a cash flow at time T deriving from the investment in the unit fund at time $t > 0$ of a known quantity c_t , i.e.:

$$C_T = c_t \cdot (1 + g)^{T-t} \quad (37)$$

In this case, we have:

$$\begin{aligned} V_0 &= (1 + r(0, t))^{-t} \cdot (1 + g)^{-(T-t)} \cdot C_T \\ &= B(0, t) \cdot c_t \end{aligned} \quad (38)$$

so that the expected present value reduces to the valuation of the risk-free cash flow c_t .

- C_T is a cash flow at time T whose amount is given by the value at time t of a known quantity c_0 invested in the unit fund between time 0 and time t . That is,

$$C_T = c_0 \cdot (1 + g)^t \quad (39)$$

The expected present value is therefore:

$$\begin{aligned} V_0 &= (1 + g)^{-t} \cdot (1 + f(0, t, T))^{-(T-t)} \cdot C_T \\ &= \frac{B(0, T)}{B(0, t)} \cdot c_0 \end{aligned} \quad (40)$$

From (40) we understand that V_0 is just given by the number of units we can currently buy with an amount of money equal to $c_0 \cdot B(0, T)/B(0, t)$.

In the first example, we are projecting forward the amount of units bought at time 0, and discounting them back again, always using the unit growth rate. This prevents the insurer from recognising spurious profits and losses.

In the context of our policy, the first example refers to the case of the expected death benefits at the end of each year and to the asset management charges levied on the policyholder's funds.

The third example clearly refers to the situation of the annuity payments, which depend upon the fund performance over the deferment period only, and not beyond the conversion date. Since we do not consider future premiums, the carrying amount of units after charges will be converted at maturity at a known annuity rate.

Were we to consider future premiums, we could use the second example to take them into account. The amounts of units bought in the future would

depend on the fund performance only from the time of premium payment onwards.

Theoretically, we could achieve the same results within the embedded value method by choosing a set of discount rates differentiated by maturity and items concerned. The procedure would become extremely cumbersome even for a simple product (Abbink and Saker (2002)).

We apply (36) to determine the liabilities relating to the expected death benefits under true assumptions:

$$FVL_0^{(2)} = \sum_{h=1}^{15} (1 + \delta) \cdot S_0 \cdot (1 - \alpha)^h \cdot {}_{h-1/1}q_{50}^{true} \quad (41)$$

As far as the cash flows relating to the annuity payment are concerned, we can apply jointly (36) and (40) to get:

$$FVL_0^{(3)} = \left[\left(S_0 \cdot (1 - \alpha)^{15} \right) \cdot (1 - a) \cdot k_0 \right] \cdot {}_{15}p_{50}^{true} \cdot (\ddot{a}_{65}^* - E_{16}^{true}) \quad (42)$$

where \ddot{a}_{65}^* is the expected present value of the cash flows of an immediate annuity paying 1 unit a year from year 15 onwards and of the annuity expenses stream. We have:

$$\ddot{a}_{65}^* = \sum_{h=15}^{\omega-65-1} (1 + E_{h+1}^{true}) \cdot \frac{B(0, h)}{B(0, 15)} \cdot {}_{h-15}p_{65}^{true} \quad (43)$$

Note that the annuity initial expenses are excluded from (42), since already included in (34).

The fair value of the insurer's liabilities at time 0 is then given by the sum of all liability components:

$$FVL_0 = FVL_0^{(1)} + FVL_0^{(2)} + FVL_0^{(3)} \quad (44)$$

Computations result in data reported in Table 6.

$FVL_0^{(1)}$	190.96
$FVL_0^{(2)}$	44.42
$FVL_0^{(3)}$	528.15
FVL_0	763.53

Table 6: Fair value of liabilities.

It is interesting to note that the RDR which would make the market value of liabilities equal under both embedded value and fair value method is 5.53%.

We then move on to calculating the adjustments for risk to cash flows. We set some reasonable margins on expenses and expenses inflation rate (see Sheard *et al.* (2001)) and then calibrate the fair value model to the embedded value method result to get an estimate of the margin MF required for mortality.

Risk-adjusted assumptions are given in Table 7. The mortality reduction factor reported is the one that matches (32) and (44) at inception, once (34), (41) and (42) are computed under risk-adjusted assumptions. Figures obtained for the latter three items are given in Table 8. We can see that the adjustments increase expenses and annuity liabilities, while the death benefits component is reduced, since we chose a mortality basis which is on the safe side with respect to longevity risk.

	True	Risk-adjusted
Mortality adjustment factor (MF)	110%	89.45%
Expenses:		
inflation rate (f)	3%	3.50%
annuity expenses:		
(initial)	20	25
(regular)	0.3%	0.5%

Table 7: True and risk-adjusted assumptions.

$adj FVL_0^{(1)}$	195.77
$adj FVL_0^{(2)}$	36.28
$adj FVL_0^{(3)}$	558.88
$adj FVL_0$	790.93

Table 8: Risk-adjusted fair value of liabilities.

5.2.3. Profit Emergence. In this section we compare the emergence of profits for prudential, embedded value and fair value reporting.

Immediately after the policy is taken out, we have no excess assets under prudential reporting. We saw that units must be bought out of the premium and that charges must flow to the sterling reserve, so as to provide for expenses, claims costs and setting up of sterling reserves. Under both embedded value and fair value reporting instead, we are holding excess assets,

of $1000 - 790.93 = 209.07$ in the first case, of $1000 - 763.53 = 236.47$ in the latter.

In order to determine future profits, we assume that assets are kept equal to statutory liabilities and that all statutory profits are distributed at the end of the year in which they emerge. Moreover, we assume that all true assumptions are borne out in practice and that the yield curve stays the same at future reporting dates.

As far as prudential reporting is concerned, statutory profits are the ones derived in section 5.2.1 for the purpose of calculating the value of the business in force (see (28)). Statutory liabilities, equal to the carrying amount of assets held (MVA_t) in our setup, are simply given by: the unit fund plus possible sterling reserves, during the deferment period; the annuity reserves, during the retirement period.

Moving to the embedded value case, we can calculate successive embedded value figures by using the following expression,³⁸ easily derived from (31):

$$p_{50+t-1}^{true} \cdot EV_t = EV_{t-1} \cdot (1 + RDR) - PRO_t^{STAT}, \quad (45)$$

where EV_t indicates the embedded value per policy in force at time t (start of year $(t + 1)$).

In terms of embedded values per policy issued (i.e. $\overline{EV}_t = {}_t p_{50}^{true} \cdot EV_t$), we have:

$$\overline{EV}_t = \overline{EV}_{t-1} \cdot (1 + RDR) - EP_t^{STAT}, \quad (46)$$

Embedded value profits per policy issued (EP_t^{EV}) are then given by the unwinding of the risk discount rate from year to year, since we are assuming that the embedded value assumptions are borne out in practice:

$$EP_t^{EV} = \overline{EV}_{t-1} \cdot RDR \quad (47)$$

We compute the fair value of liabilities at future dates by using formula (44). Fair value profits (PRO_t^{FV}) are equal to the change in fair value surplus plus statutory profits (see Perrott and Hines (2001) for alternative equivalent definitions). Formally, we have:

$$PRO_t^{FV} = FVS_t \cdot p_{50+t-1}^{true} - FVS_{t-1} + PRO_t^{STAT} \quad (48)$$

where FVS_t is the fair value of surplus at time t , expressed as the difference between the market value of assets and the fair value of liabilities:

$$FVS_t = MVA_t - FVL_t \quad (49)$$

³⁸A generalised recurrent formula, suitable for more complex situations, is given in Collins and Keeler (1993).

Fair value profits per policy issued are thus given by:

$$EP_t^{EV} = {}_{t-1}p_{50}^{true} \cdot PRO_t^{FV} \quad (50)$$

Risk-adjusted profits per policy issued (EP_t^{adj}) can be computed in the same way, but using risk-adjusted values. These are derived assuming that the market value margins determined at inception are locked in over the policy term.

Results for the value of assets and liabilities are reported in Table 9. Table 10 shows the expected profits for policy issued under different accounting standards. In the row relating to time 40, profits are aggregated for the subsequent years. It can be checked that under all accounting methods the sum of the initial liability and the profits released is the same. If experience unfolds according to true assumptions, the same total profit is recognised, although it is differently released over time. We can see that by figure 2, which depicts the profit patterns implied by the reporting methods considered from year 1 onwards.

t	MVA_t	EV_t	FVL_t^{true}	FVL_t^{adj}	t	MVA_t	EV_t	FVL_t^{true}	FVL_t^{adj}
0	1000.00	209.07	763.53	790.93	21	1764.01	297.01	1382.94	1469.45
1	989.24	268.12	683.75	712.95	22	1695.92	280.71	1334.53	1422.49
2	1042.61	274.16	722.31	753.48	23	1627.78	264.59	1285.30	1374.58
3	1098.86	279.90	763.92	797.21	24	1559.76	248.65	1235.40	1325.84
4	1158.14	285.30	806.92	842.38	25	1492.04	232.86	1185.02	1276.45
5	1220.63	290.28	854.16	892.02	26	1424.84	217.24	1134.36	1226.57
6	1286.48	294.79	904.10	944.48	27	1358.44	201.75	1083.70	1176.48
7	1355.88	298.75	958.10	1001.24	28	1293.05	186.41	1033.29	1126.37
8	1429.03	302.10	1016.72	1062.87	29	1228.81	171.25	983.25	1076.39
9	1506.13	304.75	1079.38	1128.80	30	1165.79	156.35	933.68	1026.63
10	1587.39	306.62	1149.09	1202.17	31	1104.03	141.78	884.61	977.13
11	1684.64	318.85	1226.66	1283.87	32	1043.70	127.60	836.21	928.07
12	1794.19	337.59	1312.75	1374.60	33	985.07	113.84	788.79	879.77
13	1910.30	356.99	1411.08	1478.29	34	928.34	100.58	742.57	832.43
14	2033.36	377.05	1524.03	1597.49	35	873.71	87.88	697.76	786.30
15	2163.84	397.75	1651.41	1732.04	36	821.40	75.83	654.61	741.65
16	2098.76	381.18	1609.03	1686.98	37	771.75	64.48	613.50	698.86
17	2032.95	363.99	1566.20	1645.98	38	725.10	53.92	574.84	658.36
18	1966.47	346.97	1522.12	1603.69	39	681.68	44.24	538.90	620.43
19	1899.41	330.13	1476.83	1560.13	40	641.50	35.55	505.74	585.16
20	1831.88	313.48	1430.41	1515.36

Table 9: Assets and liabilities values (per policy in-force at the beginning of each year).

t	EP_t^{STAT}	EP_t^{EV}	EP_t^{true}	EP_t^{adj}	t	EP_t^{STAT}	EP_t^{EV}	EP_t^{true}	EP_t^{adj}
1	-46.20	12.86	22.20	20.46	21	35.75	19.28	12.88	12.48
2	10.45	16.49	24.54	22.65	22	34.56	18.27	12.31	12.16
3	11.12	16.86	24.94	22.90	23	33.38	17.26	11.77	11.91
4	11.82	17.21	27.16	25.08	24	32.22	16.27	11.25	11.72
5	12.56	17.55	26.72	24.45	25	31.08	15.29	10.74	11.59
6	13.34	17.85	28.02	25.63	26	29.95	14.32	10.25	11.50
7	14.16	18.13	28.15	25.56	27	28.85	13.36	9.77	11.46
8	15.02	18.37	27.92	25.11	28	27.75	12.41	9.29	11.44
9	15.93	18.58	28.49	25.48	29	26.62	11.46	8.80	11.42
10	16.87	18.74	26.39	23.03	30	25.43	10.53	8.30	11.36
11	6.63	18.86	23.80	20.04	31	24.18	9.62	7.79	11.25
12	0.87	19.61	21.34	17.14	32	22.90	8.72	7.26	11.10
13	1.36	20.76	15.95	11.13	33	21.60	7.85	6.72	10.91
14	1.89	21.95	8.73	3.15	34	20.27	7.00	6.17	10.65
15	2.48	23.19	2.27	-4.06	35	18.89	6.19	5.60	10.31
16	41.04	24.46	16.23	19.29	36	17.46	5.40	5.02	9.87
17	40.63	23.44	15.52	14.40	37	16.01	4.66	4.44	9.35
18	39.40	22.39	14.80	13.82	38	14.52	3.97	3.86	8.73
19	38.17	21.34	14.13	13.31	39	13.00	3.32	3.29	8.02
20	36.96	20.30	13.49	12.86	40+	55.37	11.12	11.54	36.57

Table 10: Expected profits per policy issued under different reporting standards.

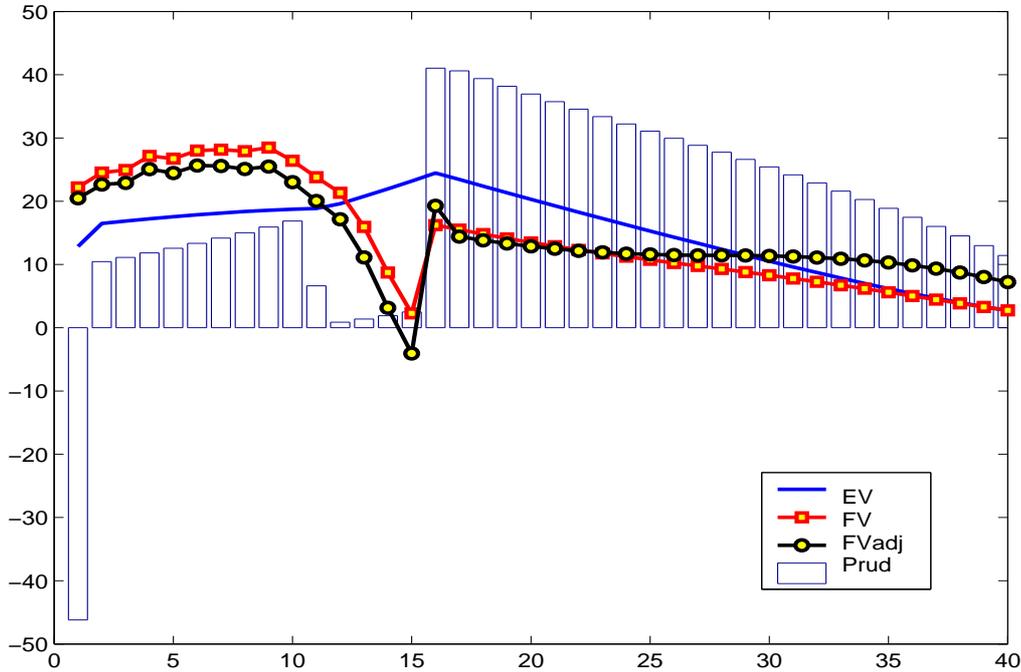


Figure 2: Expected profits under prudential (Prud), embedded value (EV), unadjusted fair value (FV) and risk-adjusted fair value (FVadj).

6. CONCLUSION

In this work, we have outlined the principles underlying the IASB's proposal for the measurement of insurance assets and liabilities at market value. We have focused on life insurance liabilities, but the scope of the considerations presented is much more general.

We have seen that the framework proposed by the IASB relies on an asset and liability approach, which defines income and expenses in terms of changes in measurement of insurance assets and liabilities, as opposed to the deferral and matching approach, which associates claim costs with premium revenue so as to recognise revenue and expenses progressively over time.

Two models of measurement are proposed, depending on whether a successor standard of IAS 39 will introduce fair value measurement for the substantial majority of assets and liabilities or not. They are respectively Fair Value and Entity-specific Value. While the former is defined as the value in exchange of a book of contracts in a hypothetical arm's length transaction between knowledgeable parties, the latter is the value of the book to the enterprise that holds it. We have seen that the two measures are not fundamentally different and that they should lead to not too different results.

Both fair and entity-specific valuation rely on the computation of the expected present value of the insurer's cash flows. The approach adopted is thus a direct prospective method, which differs from the embedded value methodology in that the value of the liabilities is obtained by discounting directly the insurer's cash flows, rather than subtracting the value of in-force business from the market value of assets.

A number of issues have been presented regarding the estimation of timing and amount of the insurer's cash flows and to the way in which risk and uncertainty should be included in those estimates. With regard to the last point, the IASB favours the approach of reflecting risk in the cash flows, rather than in the discount rates. That leads to the important and still debated issue of the assessment of market value margins, i.e. adjustments to cash flows consistent with market risk preferences.

The IASB has not finished discussing many issues regarding performance-linked contracts, but some indications have been provided. We implemented some of them in valuing a unit-linked product sold in the Italian market.

The numerical example presented in the last part of the dissertation enabled us to describe the main differences between prudential, embedded value and fair value reporting methods. We saw in what respect these approaches

differ and quantified the impact that they have in the emergence of profit over time. Market value margins were determined by using the embedded value computation as a benchmark to which calibrate our fair value model. As a result, we could achieve a better understanding of how adjustments to realistic assumptions compare to prudential assumptions in a fair value accounting system.

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Contact information:

Enrico Biffis

Dipartimento di Matematica Applicata “B. de Finetti”

Facoltà di Economia, Università degli Studi di Trieste

Piazzale Europa, 1 – 30147 Trieste (Italy)

E-mail: enricob@econ.units.it