An Integrated Notional Defined Contribution (NDC) Pension Scheme with Retirement and Permanent Disability[♥]

Manuel Ventura-Marco[♣] and Carlos Vidal-Meliá[♠]

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This paper develops a multistate overlapping generations model (MOLG) that integrates old-age and permanent disability into a generic NDC framework. In the model, the account balances of participants who do not survive are distributed as inheritance capital to the accounts of the (non-disabled) active survivors on a birth cohort basis. The model includes realistic demography insofar as it takes into account an age schedule of mortality and the uncertainty concerning the timing of disability, and allows for changes in the economically active population and for a large number of generations of contributors and pensioners to coexist at each moment in time. The results achieved in the numerical example we present endorse the fact that the model really works and show an optimal integration of both contingencies into the NDC framework. The model can easily be linked to real practices in social security policies because, to mention just a few positive features, it could be implemented without much difficulty, it would help to improve actuarial fairness, it would uncover the real cost of disability and minimize the political risk of disability insurance being used as a vote-buying mechanism.

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[♣] Department of Financial Economics and Actuarial Science, University of Valencia, Avenida de los Naranjos s.n., 46022 Valencia. (Spain). (e-mail: manuel.ventura@uv.es).

[♠] Department of Financial Economics and Actuarial Science, University of Valencia, Avenida de los Naranjos s.n., 46022 Valencia. (Spain). (e-mail: carlos.vidal@uv.es). (Corresponding author).

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1.-Introduction

A notional defined contribution scheme is a pay-as-you-go system (PAYG) that deliberately mimics a financial defined contribution scheme (FDC) by paying an income stream whose present value over a person's expected remaining lifetime equals the amount accumulated at retirement. It therefore has many of the features of an FDC scheme, but not all of them. This type of pension scheme is based on a notional account, which is a virtual account reflecting each contributor's individual contributions and the fictitious returns that these contributions generate over the course of the participant's working life. In principle the contribution rate is fixed.

The NDC system has many well-known positive features[1], although here we only highlight two of them that have to do with the aim of this paper:

✓It has stronger immunity against political risk than more traditional defined benefit (DB) PAYG systems because NDC increases the financial stability of the pension system by making it very difficult for politicians to make promises about future retirement benefits.

✓It encourages actuarial fairness and stimulates contributors' interest in the pension system as it brings to light any improper or hidden redistribution of benefits to privileged groups and reveals who really benefits from the legislation.

For Holzmann & Palmer (2006) and Chłoń-Domińczak et al (2012), compared to an FDC scheme, the three most important differences are:

✓The internal rate of return (G) in a generic NDC account is a function of productivity growth, labour force growth and factors linked to contribution and benefit payment streams as opposed to the financial market rate of return.

✓The only financial saving that can arise under the NDC scheme is in the form of a buffer fund as opposed to the funded character of the FDC scheme.

✓The way the pension balances of deceased persons are used (i.e. inheritance gain, also known as the survivor dividend). In FDC schemes the survivor dividend is usually inherited by the late contributor's survivors. It can be used to enhance the survivor's retirement savings or be paid out as a lump sum or as a phased withdrawal survivor benefit. However, among the countries in which NDC systems are in place, only Sweden applies what is called "inheritance gains". These will be technically defined later.

Another important difference, not often mentioned in the literature, is the way disability benefits are integrated into the scheme. In most Latin American countries with mandatory private pension systems based on individual capitalization accounts, disability and survivor benefits are linked to the funded individual account. However, Wiese (2006), people who become disabled at a young age might lack sufficient capital in their individual accounts to finance an adequate disability pension. The standard solution to this shortfall problem has been to allow disability benefit to remain a defined benefit, and to adopt various measures to stitch together the defined benefit (DB) and defined contribution (DC) components of the system into a coherent whole. Nevertheless, as described by Reyes (2010) and Kritzer et al (2011), big differences can be found in the way the benefits are integrated into the scheme.

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[1] See the papers by Holzmann & Palmer (2006), Barr & Diamond (2009), Auerbach & Lee (2009), Auerbach & Lee (2011), Chłoń-Domińczak et al (2012) and Holzmann et al (2012) to name just a few.

In Chile, James et al (2009), disabled workers who qualify are guaranteed a DB for the balance of their lives: 70% of their average wage (if totally disabled) and 50% (if partially disabled), i.e. disability insurance tops up the capital accumulated in the individual account if the balance is less than the minimum required to finance a permanent disability pension. The difference between the capital necessary to pay the pension and the balance available at the time of the disability or death event is called the 'additional contribution' and is one of the main cost components of disability and survivor insurance[2].

In NDCs, retirement and permanent disability are not fully integrated. In Sweden, for example, the current regulations on disability pension are closely linked to the old-age pension system but not integrated into it. According to Palmer (2006) and Chloń-Domińczak et al (2012), the Swedish model for retirement pension rights for persons receiving disability benefits involves imputing contributions for insured periods of disability and paying them into the retirement contingency. These payments, which are made annually from general tax revenues, are entered as a cost for the disability system in the country's accounts and are part of the transfer from state revenues to the NDC pension fund. Permanent disability benefits are converted into retirement benefits at age 65[3].

In Italy, disability pensions are based on the notional capital at the time of disability, and this is integrated taking into account the gap between the individual's age when the pension is granted and the reference age of 60 years. However, as Gronchi & Nistico (2006) have pointed out, the formula used to calculate the disability pension provides only a weak link between benefits and contributions.

Working-age disability policy today is one of the biggest social and labour market challenges for policy makers and currently occupies an important place on the economic policy agenda in many developed countries. According to the OECD (2010) and Autor & Duggan (2006), disability benefit in a number of countries has become the benefit of last resort for people unable to remain in, or enter, the labour market. Encouraged by the economic crisis, most of these countries, Burkhauser et al (2013), are now considering how best to reform their disability pension schemes.

Many social security systems, De Jong et al. (2010) and OECD (2010), face ever higher disability costs. Spending on disability pensions has become a significant problem for public finances in most OECD countries. Apparent public spending on disability benefits totals 2% of GDP on average across the OECD, rising to as much as 4-5% in countries such as Norway, the Netherlands and Sweden[4]. On average around 6% of the working-age population relies on disability benefits, with this figure reaching 10-12% in some countries in the north and east of Europe.

The US Social Security Disability Insurance (USDI) programme, BOT (2013), is suffering serious financial problems. Since 2009, it has been paying out more in annual benefits than it receives in contributions and interest from its trust fund. Based on current growth, it is projected to be insolvent by 2016. Burkhauser et al (2013) point out that the factors driving unsustainable USDI programme growth are similar to those that led to unsustainable growth in four other OECD countries (Australia, the Netherlands, Sweden and the United Kingdom).

According to Koning & Van Vuuren (2007), employers and trade unions have cooperated in the past on the use of disability benefits as a substitute for unemployment and early retirement programmes, notably in the Netherlands and Sweden. In Norway, Rege et al (2009) find that

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[2] Chilean law stipulates that members who die or become disabled before legal retirement age generate a survivor or disability pension respectively.

[3] As Wiese (2006) pointed out, a disabled individual's replacement rate may increase or decrease upon reaching the retirement age because disability benefits are defined by a formula, while old age benefits are a function of interest rates, mortality rates and the accumulated notional capital.

[4] The word "apparent" has been used because public spending is generally underestimated due to the practice known as conversion, Zayatz (2011), or pension reclassification, Ventura-Marco & Vidal-Meliá (2014).

downsizing substantially increases the disability entry rate of workers in the plants affected. Milligan & Wise (2012) point out that disability insurance programmes still play a big role in the departure of older persons from the labour force, as many pass through disability insurance on their path from employment to retirement.

Benítez-Silva et al (2010) find international evidence that the business cycle has much to do with explaining both the stock of disability benefit claimants and inflows to and outflows from that stock. They conclude that the rise in unemployment due to the current global economic crisis is expected to increase the number of disability insurance claimants. Laun & Wallenius (2013) find that generous early retirement benefits create strong incentives for early retirement, in large part through disability insurance, in France, Spain, Sweden and to a lesser extent Germany.

Political risk also seems to play an important role in disability insurance. According to Marin (2006), it enables short-term political popularity to be achieved at the cost of long-term sustainability. Easier access to early retirement, broader coverage, more generous replacement income and a more relaxed screening of eligibility and assessment of claims buy the immediate satisfaction of interest groups and voters. In the US, Iyengar & Mastrobuoni (2010) provide fairly strong evidence that some governors are using the USDI as a vote-buying mechanism. Similarly in the case of Spain, Jiménez-Martin et al (2007) have shown that there are significant regional differences in the probabilities of receiving a benefit without deserving it, which seems to suggest, although the authors do not actually put it into words, that permanent disability benefits have been used as an electoral tool, especially in the less developed regions of Spain.

To sum up, in most developed countries and in a similar way to DB PAYG retirement systems, disability insurance (DI) has many complex problems that need to be addressed, and as Marin (2006) pointed out, disability pensions seem to have become what might be considered the 'garbage can' of the social security system.

The aim of this paper is to develop a multistate overlapping generations model (MOLG) that integrates old-age and permanent disability into a generic NDC framework. In the model, the account balances of participants who do not survive are distributed as inheritance capital to the accounts of surviving contributors on a birth cohort basis. The model includes realistic demography insofar as it takes into account an age schedule of mortality and the uncertainty concerning the timing of disability, and it allows for changes in the economically active population and for a large number of generations of contributors and pensioners to coexist at each moment in time. The results achieved in the numerical example we present endorse the fact that the model really works and show an optimal integration of both contingencies into the NDC framework.

As far as we know, the model proposed is an innovation in this field and we have been unable to find similar models in the economic literature. The model can easily be linked to real practices in social security policies because, to mention just a few positive features, it could be implemented without much difficulty, it would help to improve actuarial fairness, it would uncover the real cost of disability and minimize the political risk of disability insurance being used as a vote-buying mechanism.

The structure of the paper is as follows. After this introduction, Section 2 presents an actuarial OLG model that integrates retirement and permanent disability into a generic NDC system. For the sake of clarity, this section is separated into three subsections dealing with the determination of the year in which the system reaches a mature state, the definition and determination of the survivor dividend, and the effect of the survivor dividend on the system's financial equilibrium. Section 3 shows a numerical illustration representing a generic NDC pension system with two contingencies. This section is divided into two different parts according to the assumptions made about the growth of the economically active population. Section 4 shows our conclusions and

discusses some issues that would have to be taken into account when putting the model into practice. The paper ends with the bibliographical references.

2. The Model

This section develops a multistate overlapping generations model (MOLG) that integrates retirement and permanent disability into a generic NDC system taking into account the survivor dividend. To a great extent the model includes realistic demography, Bommier & Lee (2003), insofar as it takes into account an age schedule of mortality and the uncertainty concerning the timing of disability and allows for changes in population.

We build on the models developed by Boado-Penas & Vidal-Meliá (2014) and Ventura-Marco & Vidal-Melia (2014), based on those first put forward by Settergren & Mikula (2005), Boado-Penas et al. (2008) and Vidal-Meliá & Boado-Penas (2013). Boado-Penas & Vidal-Meliá (2014) develop a model to show whether it would be justified to include the survivor dividend when calculating affiliate pension balances in a generic NDC framework. They conclude that the survivor dividend has a strong financial basis which enables the macro contribution rate applied to be the same as the individual rate credited. The model by Ventura-Marco & Vidal-Melia (2014) presents a theoretical base for applying a Swedish type actuarial balance sheet (ABS) to both retirement and disability contingencies in a DB PAYG system, thereby taking a step towards filling the large gap in the literature in this area. They indicate that their model has many other practical implications which could be of interest not only to DB systems but also to NDCs.

These papers were to some extent inspired by the accounting framework for organizing, summarizing and interpreting data on transfer systems and the life cycle developed in Lee (1994), Willis (1988) and Arthur & McNicoll (1978).

The main starting assumptions are:

- ✓ The affiliates contribute for retirement and disability contingencies.

- ✓ There is a defined contribution rate (fixed over time) to cope with both contingencies.

- ✓ The initial disability and retirement pensions depend on the value of the accumulated notional account, the expected mortality of the cohort in the year the contributor becomes disabled and a notional imputed future indexation rate λ, i.e. pensions in payment increase or decrease at an annual rate of λ.

- ✓ The capital accumulated in the notional account reflects each participant's individual contributions and the fictitious returns these contributions generate over the course of the participant's working life, plus the inheritance capital.

- ✓ The account balances of participants who do not survive to retirement are distributed as inheritance capital to the accounts of the (non-disabled) active survivors on a birth cohort basis.

- ✓ The accumulated notional capital is not split into contingencies because permanent disability is considered a type of compulsory early retirement for health reasons.

- ✓ The system does not provide a minimum pension.

- ✓ It is assumed that contributions and benefits are payable yearly in advance.

- ✓ Participants' lives last $(w\text{-}1\text{-}x_e)$ periods, where $(w\text{-}1)$ is the highest age to which it is possible to survive and x_e is the earliest age of entry into the system.

✓ The age giving entitlement to retirement pension, x_e+A, is fixed. This assumption does not imply loss of generality because, as we will see later for the disability contingency, the ages that give entitlement could be defined as an interval.

✓ As regards disability pension, it is supposed that initially the ages that give entitlement are to be found in age interval $[x_e+1, x_e+A]$[5]. The age interval is later widened to $[x_e+A+1, w-1]$.

✓ The only reason for a disabled worker's benefit to terminate is through the death of the pensioner.

✓ We do not take conversions or recoveries into account, i.e. conversion and recovery rates are null in our model.

✓ The contribution base (coinciding with earnings) grows at an annual rate of g.

✓ The economically active population increases or decreases over time at an annual accumulative rate of γ, affecting all groups of contributors equally.

✓ The system's income from contributions (wage bill growth) also grows (decreases) at rate $G = (1+g)(1+\gamma)-1$.

✓ When the system reaches the mature state $t = w-x_e-1$ years from inception, A generations of contributors and $(w-(x_e+1))$ generations of pensioners coexist at each moment in time.

Once the main assumptions have been detailed, for the sake of clarity this section will be divided into three subsections dealing with the determination of the year in which the system reaches a mature state, the definition and determination of the survivor dividend, and the effect of the survivor dividend on the system's financial equilibrium.

2.1.-Description of the system and determination of the year in which it reaches a mature state.

Diagram 1 shows the relationships (transitions) between the various collectives (states) that will be separated in the model. The difference between this model and the one found in Ventura-Marco & Vidal-Melia (2014) is that the pension system is NDC instead of DB PAYG and the survivor dividend is explicitly taken into account. With regard to the model developed by Vidal-Meliá et al (2013), a new state - disability - is introduced, along with the new relationships shown by broken lines in the diagram.

We work with a simplified type of "multiple state transition model", Haberman & Pitacco (1999), which is a probability model that describes a subject's movements among various states: active (a), disabled (i), retired (r) and dead (d)

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[5] Indeed a person of x_e years may become disabled after having paid contributions and therefore starts to receive disability pension at age x_e+1 years. Similarly, a person of x_e+A-1 years may become disabled at that age after contributing and will therefore receive benefit for being disabled at age x_e+A years.

Diagram 1
NDC scheme with permanent disability
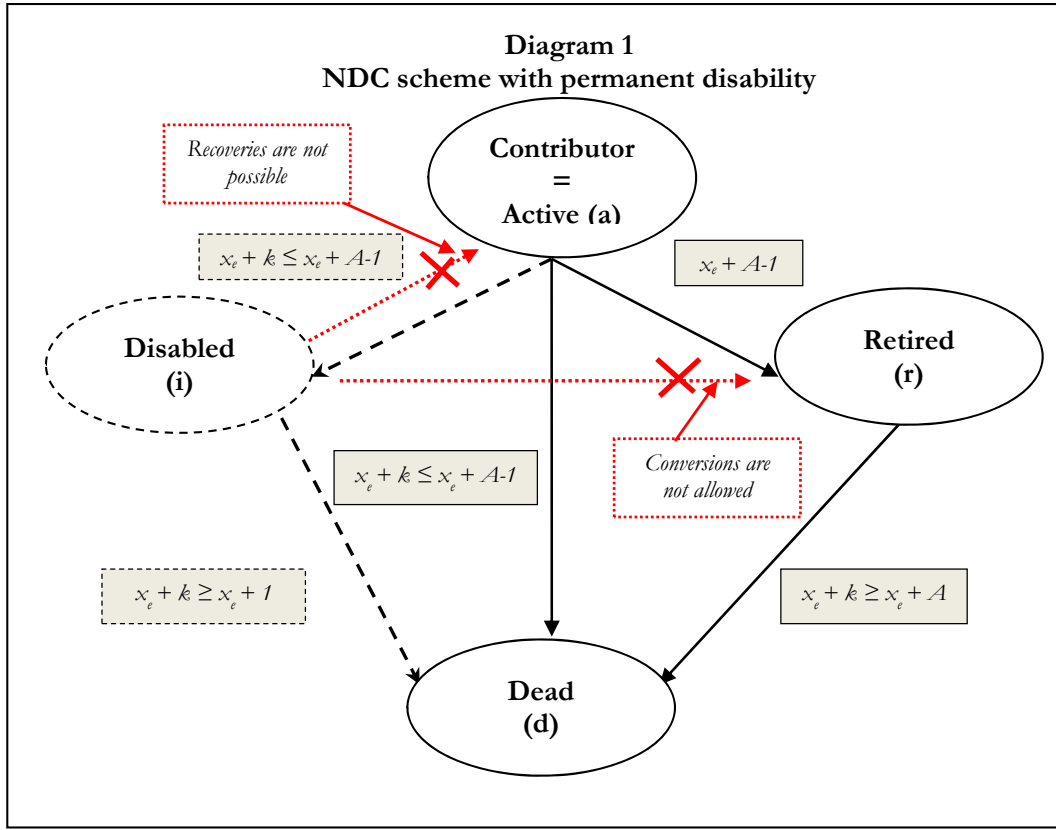
1.-Transition probabilities:

The discrete model could be expressed as a four-state non-homogeneous Markov chain with the following transition probabilities, in which no more than one transition within a year is assumed:

$p_{x_e+k}^{aa}$, the probability that an active person aged x_e+k will reach age x_e+k+1 being active,

$p_{x_e+k}^{ai}$, the probability that an active person aged x_e+k will become disabled during the year,

$p_{x_e+k}^{ar}$, the probability that an active person aged x_e+k will be retired one year later,

$p_{x_e+k}^{ad}$, the probability that an active person aged x_e+k will die during the year,

$p_{x_e+k}^{ii}$, the probability that a disabled person aged x_e+k will reach age x_e+k+1 in the same state,

$p_{x_e+k}^{ir}$, the probability that a disabled person aged x_e+k will be retired one year later,

$p_{x_e+k}^{id}$, the probability that a disabled person aged x_e+k will die during the year,

$p_{x_e+k}^{rr}$, the probability that a retired person aged x_e+k will reach age x_e+k+1 in the same state,

$p_{x_e+k}^{rd}$, the probability that a retired person aged x_e+k will die during the year.

2.-Age:

$$\begin{cases} \overbrace{x_e, \ x_e+1, \ x_e+2, \ ...., \ x_e+A-1,}^{\text{Contributors' ages}} \underbrace{x_e+A, \ x_e+A+1, ...\ w\text{-}1}_{\text{Pensioners' ages (retirement)}} \\ \underbrace{x_e+1, \ x_e+2, \ ...., \ x_e+A-1, x_e+A, \ x_e+A+1, ...\ w\text{-}1}_{\text{Pensioners' ages (disability)}} \end{cases} \quad \textbf{[1.]}$$

We assume that the affiliate cannot contribute and receive pension in the same year. If an individual becomes disabled at contribution age $x_e+k \in [x_e, x_e+A\text{-}1]$, the corresponding disability pension payable will be at age $x_e+k+1 \in [x_e+1, x_e+A]$.

3.-Number of contributors by age at time t:

$$\left\{ N_{(x_e, t)}, \ N_{(x_e+1, t)}, \ ...., \ N_{(x_e+A-1, t)} \right\}$$
$$=$$
$$\left\{ N_{(x_e, 0)}(1+\gamma)^t, \ N_{(x_e+1, 0)}(1+\gamma)^t, \ ...., \ N_{(x_e+A-1, 0)}(1+\gamma)^t \right\} \quad \textbf{[2.]}$$

where $N_{(x_e+k, t)} = N_{(x_e, t)} \cdot {_k}p_{x_e}^{aa}$ and $_k p_{x_e}^{aa}$ is the stable-in-time ratio between the numbers of individuals aged x_e and x_e+k years. Stable ratios or probabilities include the decrements due to death and disability associated with each age, with the possibility of a return to active life not being considered. It is a different matter when it comes to considering decrements or new entries due to migratory movements; these are included in parameter γ.

4.-Average wage (average contribution base) by age at time t:

$$\left\{ y_{(x_e, t)}, \ y_{(x_e+1, t)}, \ ...., \ y_{(x_e+A-1, t)} \right\}$$
$$=$$
$$\left\{ y_{(x_e, 0)}(1+g)^t, \ y_{(x_e+1, 0)}(1+g)^t, \ ...., \ y_{(x_e+A-1, 0)}(1+g)^t \right\} \quad \textbf{[3.]}$$

The demographic framework above implies that the age-wage structure only undergoes proportional changes. The slope of the age-wage structure is constant.

5.-Number of disabled people:

In age interval $[x_e+1, x_e+A]$ at $t = 1$

$$I_{(x_e+k, 1)} = N_{(x_e+k-1, 0)} \cdot p_{xe+k-1}^{ai} = N_{(x_e, 0)} \cdot {_{k-1}}p_{xe}^{aa} \cdot p_{xe+k-1}^{ai} \quad \textbf{[4.]}$$

where:

p_{xe+k-1}^{ai} is the probability that an active person aged $x_e+k\text{-}1$ will become disabled during the year.

$_{k-1}p_{xe}^{aa}$ is the probability that an active person aged x_e will reach age $x_e+k\text{-}1$ being active.

$I_{(x_e+k, 1)}$ is the number of people who become disabled in year t of age x_e+k, becoming disabled as far as the system is concerned because their disability really began in the previous period [0, 1).

For $t \geq 2$ and age interval $[x_e+1, x_e+A]$ we need to consider two types of disabled people: those aged x_e+k years who became disabled in the current year, $I_{(x_e+k, t)}^{N}$, and those whose disability began

earlier or survivors aged x_e+k years who continue from previous years, $I^S_{(x_e+k,\,t)}$. The structure for the number of people who became disabled during the year is always given by:

$$I^N_{(x_e+k,\,t)} = N_{(x_e+k\text{-}1\,,\,t\text{-}1)} \cdot p^{ai}_{xe+k+1} = N_{(x_e+k\text{-}1\,,\,0)} \cdot (1+\gamma)^{t\text{-}1} \cdot p^{ai}_{xe+k+1} \qquad [5.]$$

After age x_e+A+1 years, all the disabled in the system are by definition considered survivor disabled because, once the state of activity disappears, nobody can become disabled for the purposes of the system. Therefore, and always for t ≥ 2, as far as the continuing disabled are concerned a distinction has to be made between two age intervals, $[x_e+2,\ x_e+A]^6$ and from age x_e+A+1 years onwards.

The structure of the survivor disabled in $[x_e+2,\ x_e+A]$, whose evolution will depend on survival probabilities $p^{ii}_{x_e+k\text{-}1}$, which are different from those for the active population[7], $p^{aa}_{x_e+k\text{-}1}$, incorporates all those who became disabled in successive earlier periods and have survived.

In general, when all the disabled people who began in $t=1$ have died, this means that $t \geq w\text{-}x_e$, and therefore from here on for all this disability band we get $k < t$,

$$I^S_{(x_e+k,\,t)} = \sum_{s=1}^{k\text{-}1} I^N_{(x_e+s,\,t\text{-}k+s)} \cdot {}_{k\text{-}s}p^{ii}_{x_e+s} \qquad [6.]$$

where ${}_{k\text{-}s}p^{ii}_{x_e+s}$ is the probability that a disabled person aged x_e+s will reach age x_e+k in the same state.

The total number of disabled for each age in t can be calculated by:

$$I_{(x_e+k,\,t)} = I^S_{(x_e+k,\,t)} + I^N_{(x_e+k,\,t)} = \sum_{s=M\acute{a}x\{1,k\text{-}t+1\}}^{k} I_{(x_e+s,\,1)} \cdot (1+\gamma)^{t\text{-}1\text{-}k+s} \cdot {}_{k\text{-}s}p^{ii}_{x_e+s} = \sum_{s=1}^{k} I^N_{(x_e+s,\,t\text{-}k+s)} \cdot {}_{k\text{-}s}p^{ii}_{x_e+s} \quad [7.]$$

From age x_e+A+1 years onwards, no more new disabled people are taken into account, and so for age interval $[x_e+A+1,\ w\text{-}1]$, i.e. $k \in \{1,\ w\text{-}1\text{-}(x_e+A)\}$, we get:

$$I_{(x_e+A+k,\,t)} = \left(I^S_{(x_e+A,\,t\text{-}k)} + I^N_{(x_e+A,\,t\text{-}k)}\right) \cdot {}_kp^{ii}_{x_e+A} = \left(\sum_{s=1}^{A} I^N_{(x_e+s,\,t\text{-}k+s\text{-}A)} \cdot {}_{A\text{-}s}p^{ii}_{x_e+s}\right) \cdot {}_kp^{ii}_{x_e+A} \qquad [8.]$$

According to the starting assumptions, the amount of initial pension for disability paid at age x_e+k with $k \in \{1, \ldots, A\}$ and $c \in \{1, \ldots, k\}$ is:

$$P^I_{(x_e+k,\,c,t)} = \frac{\left(K^{ac}_{(x_e+k\text{-}1,c\text{-}1,t\text{-}1)} + \theta_a \cdot y_{(x_e+k\text{-}1,t\text{-}1)}\right) \cdot (1+G)}{{}^I\ddot{a}^\lambda_{x_e+k}} \qquad [9.]$$

where $K^{ac}_{(x_e+k\text{-}1,c\text{-}1,t\text{-}1)}$: Accumulated notional capital at time $t\text{-}1$ for one individual aged $x_e+k\text{-}1$ who has been contributing for the last $c\text{-}1$ years,

where for k = 1

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[6] In $k=1$ the disabled are always newly disabled as they come from age x_e in $t\text{-}1$, and therefore $I(x_e+1, t) = I^N(x_e+1, t)$.

[7] According to Pittaco (2012), the mortality of disabled people contains an "extra-mortality" term and can be represented either as a specific mortality (via the appropriate numerical tables or parametric mortality laws) or via adjustments to the standard age pattern of mortality. Plamondon et al (2002) show that considering specific mortality for permanently disabled people is standard practice in social security.

$$P^I_{(x_e+1,\,1,\,t\text{-}k+1)} = \frac{\theta_a \cdot y_{(x_e,\,t-k)} \cdot (1+G)}{^I a^\lambda_{x_e+1}}$$ [10.]

and for k = A

$$P^I_{(x_e+A,\,c,\,t)} = \frac{\left(K^{ac}_{(x_e+A-1,\,c-1,\,t-1)} + \theta_a \cdot y_{(x_e+A-1,\,t-1)}\right)\cdot(1+G)}{^I a^\lambda_{x_e+A}}$$ [11.]

Whereas for the retirement contingency:

$$P^R_{(x_e+A,\,c,\,t)} = \frac{K^{ac}_{(x_e+A,c,\,t)}}{^R a^\lambda_{x_e+A}} = \frac{\left(K^{ac}_{(x_e+A-1,c-1,\,t-1)} + \theta_a \cdot y_{(x_e+A-1,c-1,\,t-1)}\right)\cdot(1+G)\cdot\left(1+\frac{p^{ad}_{x_e+A-1}}{p^{aa}_{x_e+A-1}}\right)}{^R a^\lambda_{x_e+A}}$$ [12.]

with $^I a^\lambda_{x_e+k}$ and $^R a^\lambda_{x_e+A}$ respectively being annuity factors (Af^D, Af^R), i.e. the present value of a lifetime annuity for the disabled or retired of 1 monetary unit per year payable in advance and growing at real rate λ, valued at age x_e+k years and age x_e+A years, with a technical interest rate equal to $d=G$. In formula [12] the term $\left(1+\frac{p^{ad}_{x_e+A-1}}{p^{aa}_{x_e+A-1}}\right)$ is the annual inheritance gain factor that will be developed in detail in Section 2.2.

For $c=k$, these pensions would be for people with a full contribution history, i.e. those who enter the labour market at the earliest age x_e and exit aged $x_e + k\text{-}1$ and $x_e + A$ years for disability and retirement respectively.

$\overline{P}^I_{(x_e+k,\,t)}$ is the average pension for disabled individuals at age x_e+k, with $k \in \{1, 2, \ldots, A\}$, while $\overline{P}^R_{(x_e+A,\,t)}$ is the average pension for individuals who retire at the ordinary retirement age. The former is a weighted pension of the k different disability pensions once settled, while the latter is a weighted pension of the k different retirement pensions once settled. Therefore, for $k \in \{1, \ldots, A\}$ and $c \in \{1, \ldots, k\}$

$$\overline{P}^I_{(x_e+k,\,t)} = \frac{\sum_{c=1}^{k} P^I_{(x_e+k,\,c,\,t)} \cdot I^N_{(x_e+k,\,c,\,t)}}{I^N_{(x_e+k,\,t)}} = \frac{\left(\overline{K}^{ac}_{(x_e+k-1,\,t-1)} + \theta_a \cdot y_{(x_e+k-1,\,t-1)}\right)\cdot(1+G)}{^I a^\lambda_{x_e+k}}$$ [13.]

and

$$\overline{P}^R_{(x_e+A,\,t)} = \frac{\sum_{c=1}^{A} P^R_{(x_e+A,\,c,\,t)} \cdot N_{(x_e+A,\,c,\,t)}}{N_{(x_e+A,\,t)}} = \frac{\overline{K}^{ac}_{(x_e+A,\,t)}}{^R a^\lambda_{x_e+A}}$$ [14.]

The total accumulated notional capital in year t for the generation aged x_e+k, $K^{Tac}_{(x_e+k,\,t)}$ includes contributions made by all contributors in $t-1$ plus the credited account balances of contributors in year $t-1$ corresponding to those dying in the period $[t-1, t)$ and active contributors in year t, both capitalized for a period. However, we have to remove the credited account balances of active participants in year $t-1$ who become disabled during the year because they receive the disability pension in year t, i.e. contributions allocated to the disabled have to be deducted from total contributions:

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$$K^{Tac}_{(x_e+k,\ t)} = \theta_a \cdot \left[\sum_{s=0}^{k-1} y_{(x_e+s,\ t)} \cdot N_{(x_e+s,t)} \cdot \left[\prod_{h=s}^{k-1} \left(p^{aa}_{xe+h} + p^{ad}_{xe+h} \right) \right] \right] \qquad \textbf{[15.]}$$

If we include contributions made in year t for the generation aged x_e+k years and take into account formula **[15]**, we get $K^{Tac\ [+]}_{(x_e+k,\ t)} = K^{Tac}_{(x_e+k,\ t)} + \theta_a \cdot y_{(x_e+k,\ t)} \cdot N_{(x_e+k,\ t)}$. Hence, with the total accumulated notional capital in year t for the generation aged x_e+k, with contributions for time t being included at age "x_e+k" $\in \{x_e+1,\dots,\ x_e+A\text{-}1\}$ for all contributors who reach that age, the spending on disability pensions in year $t \geq w - x_e - 1$ for beneficiaries aged x_e+k years is:

$$\underbrace{\underbrace{\frac{K^{Tac\ [+]}_{(x_e+k-1,t-1)} \cdot (1+G)}{N_{(x_e+k-1,t-1)}}}_{K^{ac\ [+]}_{(x_e+k-1,\ t-1)} \cdot (1+G)} \cdot \frac{1}{Af} \cdot I^N_{(x_e+k,\ t)}}_{\overline{P}^I_{(x_e+k,\ t)}} = I^N_{(x_e+k,\ 1)} \cdot (1+\gamma)^{t-1} \cdot \overline{P}^I_{(x_e+k,\ 1)} \cdot (1+g)^{t-1} \qquad \textbf{[16.]}$$

The spending on new retirement pensions awarded in year t and the amount of the annual average pension paid at retirement age are:

$$\frac{K^{Tac}_{(x_e+A,\ t)}}{N_{(x_e+A,\ t)}} \cdot \frac{1}{Af^R} \cdot N_{(x_e+A,\ t)} = \frac{K^{ac}_{(x_e+A,\ t)}}{Af^R} \cdot N_{(x_e+A,\ t)} = \overline{P}_{(x_e+A,\ t)} \cdot N_{(x_e+A,\ t)} \qquad \textbf{[17.]}$$

where $K^{Tac}_{(x_e+A,\ t)}$ is the total accumulated notional capital at age x_e+A of all the contributors who reach that age:

$$K^{Tac}_{(x_e+A,\ t)} = \left(\underbrace{K^{Tac}_{(x_e+A-1,\ t-1)} + \theta_a \cdot y_{(x_e+A-1,\ t-1)} \cdot N_{(x_e+A-1,\ t-1)}}_{K^{Tac\ [+]}_{(x_e+A-1,\ t-1)}} \right) \cdot (1+G) \cdot \left(1 - p^{ai}_{xe+A-1} \right)$$

$$= \qquad \textbf{[18.]}$$

$$\theta_a \cdot \left[\sum_{s=0}^{A-1} y_{(x_e+s,\ t)} \cdot N_{(x_e+s,\ t)} \cdot \left[\prod_{h=s}^{A-1} \left(p^{aa}_{xe+h} + p^{ad}_{xe+h} \right) \right] \right]$$

In the financially sustainable NDC framework, the spending on pensions has to be equal to the aggregate income from contributions according to balanced rate θ_t, and therefore:

$$\theta_t \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ t)} \cdot N_{(x_e+k,\ t)} = \sum_{k=1}^{A} \left(\sum_{s=1}^{k} \overline{P}^I_{(x_e+s,\ t)} \cdot I_{(x_e+s,\ t)} \cdot F^{k-s} \cdot {}_{k-s}p^{ii}_{xe+s} \right)$$

$$+ \qquad \textbf{[19.]}$$

$$\sum_{k=1}^{t-1} \left(\sum_{s=1}^{A} P^I_{(x_e+s,\ t)} \cdot I_{(x_e+s,t)} \cdot F^{A+k-s} \cdot {}_{A+k-s}p^{ii}_{xe+s} \right) + \overline{P}^R_{(x_e+A,\ t)} \cdot \sum_{k=0}^{t-1} N_{(x_e+A+k,\ t)} \cdot F^k$$

where $F = \left[\dfrac{1+\lambda}{1+G} \right]$ is an indexation factor which depends on λ (indexation of pensions in payment) and G.

This leads to:

$$\theta_t \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ t)} \cdot N_{(x_e+k,\ t)} = \sum_{k=1}^{A} \left(K^{ac}_{(x_e+k-1,\ t-1)} + \theta_a \cdot y_{(x_e+k-1,\ t-1)} \right) \cdot (1+G) \cdot I^N_{(x_e+k,\ t)} + K^{Tac}_{(x_e+A,\ t)} \quad \textbf{[20.]}$$

It can be said that the system's income from contributions is equivalent to the present actuarial value of the pensions awarded in that year (commitments the system takes on with pensioners who have just retired or become disabled), i.e. the accumulated notional capital belonging to new beneficiaries in year t.

From year w-x_e-1=t counting from the system's inception, the ratio between the number of pensioners (D, R) and the number of contributors (C) – (dr_t) – stabilizes and the average pension-average contribution base quotient ($\dfrac{\overline{P^I}}{\overline{W}}$, $\dfrac{\overline{P^R}}{\overline{W}}$) – ($fr_t$) – is already constant due to the fact that the numerator and denominator evolve equally (at the rate of variation in wages). Therefore the contribution rate, also called the macro contribution rate, is the product of the demographic dependency ratio and the financial ratio (the system's average replacement rate):

$$\theta^S = fr \cdot dr = \left(fr^I, fr^R\right) \cdot \begin{pmatrix} dr^I \\ dr^R \end{pmatrix} = \theta^I + \theta^R = \frac{\overline{P^I}}{\overline{W}}\frac{D}{C} + \frac{\overline{P^R}}{\overline{W}}\frac{R}{C} = \frac{\overline{P^I} \cdot D + \overline{P^R} \cdot R}{\overline{W} \cdot C} \qquad \text{[21.]}$$

2.2.-Definition and determination of the survivor dividend.

Like with the Swedish NDC model, we follow the principle that each monetary unit contributed is paid out in the form of retirement benefit but not necessarily to the individual who made the contributions. The main difference between the Swedish NDC model and ours is that we consider two integrated contingencies. Therefore, for the individual who becomes disabled or reaches retirement age, there is an accumulated survivor dividend. The account balances of participants who do not survive to retirement are distributed as inheritance capital on a birth cohort basis to the accounts of surviving contributors.

For population growth, $\gamma > 0$, for age x_e+k there are k different contribution trajectories as contributors might be working for 1 year, 2 years…, k years. The only exits considered are death and disability. Therefore:

$$N_{(x_e+k,\, t)} = \sum_{c=0}^{k} N_{(x_e+k,\, c,\, t)}. \qquad \text{[22.]}$$

For a given credited contribution rate, θ_a, the accumulated survivor dividend (or accumulated inheritance gain) at age $x_e + k$ in t for one contributor who belongs to the initial group and has contributed since entering the system, $D^{ac}_{(x_e+k,\, k,\, t)}$, is the difference between the accumulated notional capital, $K^{ac}_{(x_e+k,\, k,\, t)}$, including contributions and indexation on contributions from members of the same cohort who died while active (not disabled), and the individual accumulated notional capital, $K^{i}_{(x_e+k,\, k,\, t)}$.

The accumulated survivor dividend, at a specific age, is the portion of the credited account balances of participants resulting from the distribution, on a birth cohort basis, of the account balances of participants who do not survive to retirement while active. In this case for $k \in \{1, \dots, A\text{-}1\}$:

$$D^{ac}_{(x_e+k,\, k,\, t)} = K^{ac}_{(x_e+k,\, k,\, t)} - \underbrace{\theta_a \cdot \sum_{s=0}^{k-1} y_{(x_e+s,\, t-k+s)} \cdot (1+G)^{k-s}}_{K^{i}_{(x_e+k,\, k,\, t)}}$$

$$= \qquad \text{[23.]}$$

$$\sum_{s=1}^{k} D_{(x_e+s,\, t-k+s)} \cdot (1+G)^{k-s} = \theta_a \cdot \sum_{s=0}^{k-1} y_{(x_e+s,\, -k+s+t)} \cdot (1+G)^{k-s} \cdot \left[If^{ac}_{(x_e+k,\, k,\, t)} - 1\right]$$

Because the accumulated notional capital, $K^{ac}_{(x_e+k, k, t)}$, can be expressed as:

$$K^{ac}_{(x_e+k, k, t)} = \theta_a \cdot \left[\sum_{s=0}^{k-1} y_{(x_e+s, t-k+s)} \cdot (1+G)^{k-s} \cdot \overbrace{\prod_{h=s}^{k-1} \left(1 + \frac{p^{ad}_{xe+h}}{p^{aa}_{xe+h}}\right)}^{If^{ac}_{(x_e+k, k, t)}} \right]$$ [24.]

where $If^{ac}_{(x_e+k, k, t)} = \prod_{h=s}^{k-1} \left(1 + \frac{p^{ad}_{xe+h}}{p^{aa}_{xe+h}}\right)$ is the cumulative inheritance gain factor, and the result is a

formula that is very similar in structure to the formula used by the Swedish authorities for the NDC system, which only includes the retirement contingency[8].

Similarly for *k = A*:

$$D^{ac}_{(x_e+A, A, t)} = K^{ac}_{(x_e+A, A, t)} - \underbrace{\theta_a \sum_{s=0}^{A-1} y_{(x_e+s, t-A+s)} \cdot (1+G)^{A-s}}_{K^i_{(x_e+A, A, t)}} = \sum_{s=1}^{A} D_{(x_e+s, t-A+s)} \cdot (1+G)^{A-s}$$

$$=$$ [25.]

$$\theta_a \cdot \sum_{s=0}^{A-1} y_{(x_e+s, -A+s+t)} \cdot (1+G)^{A-s} \cdot \left[If^{ac}_{(x_e+k, A, t)} - 1 \right]$$

$$K^{ac}_{(x_e+A, A, t)} = \left(K^{ac}_{(x_e+A-1, A-1, t-1)} + \theta_a \cdot y_{(x_e+A-1, t-1)} \right) \cdot (1+G) + D_{(x_e+A, t)}$$

$$=$$ [26.]

$$\theta_a \cdot \sum_{s=0}^{A-1} y_{(x_e+s, -A+s+t)} \cdot (1+G)^{A-s} \cdot If^{ac}_{(x_e+k, A, t)}$$

and because at age x_e+A years there are no more contributions, the accumulated notional capital in year *t* is determined from the accumulated notional capital in year *t-1*, capitalized for one period using notional rate *G,* plus the inheritance gains generated over the period *[t-1, t).*

2.2.1.-The survivor dividend when the economically active population increases.

Assuming that the population changes at rate $\gamma > 0$, i.e. contributors are joining the system at all ages, if one contributor enters the system at age x_e+s, they will contribute for *(k-s)* years, $k \in \{s+1, ..., A-1\}$, until the age they become disabled when their notional capital will be:

$$K^{ac}_{(x_e+k, k-s, t)} = \theta_a \cdot \left[\sum_{h=s}^{k-1} y_{(x_e+s, t-k+s)} \cdot (1+G)^{k-h} \cdot If^{ac}_{(x_e+k, k-s, t)} \right]$$ [27.]

where $If^{ac}_{(x_e+k, k-s, t)} = \prod_{r=h}^{k-1} \left(1 + \frac{p^{ad}_{xe+r}}{p^{aa}_{xe+r}}\right)$ is the cumulative inheritance gain factor, or until retirement

age, *A,* when their notional capital will be:

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[8] See Pensionsmyndigheten (2013), Appendix A. Inheritance gain factors for the Inkomstpension.

$$K^{ac}_{(x_e+A,\ A-s,\ t)} = \left(K^{ac}_{(x_e+A-1,\ A-s-1,\ t-1)} + \theta_a \cdot y_{(x_e+A-1,\ t-1)}\right) \cdot (1+G) + D_{(x_e+A,\ t)} \tag{28.}$$

Finally, for $k \in \{1, \ldots, A\}$, the average accumulated dividend can be expressed according to the inheritance gain factor:

$$\overline{D}^{ac}_{(x_e+k,\ t)} = \overline{K}^{ac}_{(x_e+k,\ t)} - \overline{K}^{i}_{(x_e+k,\ t)} = \theta_a \cdot \sum_{s=0}^{k-1} \frac{y_{(x_e+s,\ t-k+s)} \cdot (1+G)^{k-s}}{(1+\gamma)^{k-s}} \cdot \left[\mathit{If}^{ac}_{(x_e+k,\ k,\ t)} - 1 \right] \tag{29.}$$

given that

$$\overline{K}^{ac}_{(x_e+k,\ t)} = \frac{K^{Tac}_{(x_e+k,t)}}{N_{(x_e+k,\ t)}} = \theta_a \cdot \left[\sum_{s=0}^{k-1} \frac{y_{(x_e+s,t-k+s)} \cdot (1+G)^{k-s}}{{}_{k-s}p^{aa}_{xe+s} \cdot (1+\gamma)^{k-s}} \cdot \prod_{h=s}^{k-1} \left(1 - p^{ai}_{xe+h}\right) \right] \tag{30.}$$

and

$$\overline{K}^{i}_{(x_e+k,t)} = \frac{K^{Ti}_{(x_e+k,t)}}{N_{(x_e+k,\ t)}} = \theta_a \cdot \left[\sum_{s=0}^{k-1} \frac{y_{(x_e+s,t-k+s)} \cdot (1+G)^{k-s}}{(1+\gamma)^{k-s}} \right] \tag{31.}$$

2.3.-The effect of the survivor dividend on the system's financial equilibrium.

The relationship between the credited contribution rate and the balanced rate according to equation **[18.]** is:

$$\theta_t \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ t)} \cdot N_{(x_e+k,\ t)} = \sum_{k=1}^{A} K^{Tac\,[+]}_{(x_e+k-1,\ t-1)} \cdot p^{ai}_{xe+k-1} \cdot (1+G) + K^{Tac}_{(x_e+A,\ t)} \tag{32.}$$

Equation **[32.]** expresses that in the mature state reached, the system's income from contributions (retirement and disability) at t are equivalent to the present actuarial value of the pensions awarded in that year (commitments that the system takes on with pensioners who have just retired and with those who become disabled), i.e. the accumulated notional capital belonging to the new beneficiaries in year t for both contingencies. This means that liabilities to pensioners and contributors remain constant over time.

It can be demonstrated that the second member of equation **[32.]**, the amount of accrued notional capital belonging to new beneficiaries in year t, matches the first member, the system's income from contributions. Consequently,

$$\theta_t \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\ t)}\, N_{(x_e+k,\ t)} = \overbrace{\theta_a \cdot \sum_{s=0}^{A-1} y_{(x_e+s,\ t)} \cdot N_{(x_e+s,\ t)}}^{\sum_{k=1}^{A} K^{Tac[+]}_{(x_e+k-1,\ t-1)} \cdot p^{ai}_{xe+k-1} \cdot (1+G) + K^{Tac}_{(x_e+A,\ t)}}$$
$$\Downarrow \tag{33.}$$
$$\theta_t = \theta_a$$

So what does this result imply for the system's financial equilibrium?

If the amount of the pension is determined using the individual notional capital without considering the survivor dividend, then the balanced contribution rate and credited rate are different since the benefits are strictly lower than they could be (as the survivor dividend is not distributed among the survivors).

The amount of the pension ignoring the survivor dividend is calculated for disability and retirement respectively as follows:

$$\overline{P}^{i\_I}_{(x_e+k,\,t)} = \frac{\sum_{c=1}^{k} P^{i\_I}_{(x_e+k,\,c,\,t)} \cdot I^{N}_{(x_e+k,\,c,\,t)}}{I^{N}_{(x_e+k,\,t)}} = \frac{\left(\overline{K}^{i}_{(x_e+k-1,\,t-1)} + \theta_a \cdot y_{(x_e+k-1,\,t-1)}\right)\cdot(1+G)}{{}^{I}\ddot{a}^{\lambda}_{x_e+k}} \qquad \text{[34.]}$$

and

$$\overline{P}^{i\_R}_{(x_e+A,\,t)} = \frac{\sum_{c=1}^{A} P^{i\_R}_{(x_e+A,\,c,\,t)} \cdot N_{(x_e+A,\,c,\,t)}}{N_{(x_e+A,\,t)}} = \frac{\overline{K}^{i}_{(x_e+A,\,t)}}{{}^{R}\ddot{a}^{\lambda}_{x_e+A}} = \frac{\left(\overline{K}^{i}_{(x_e+A-1,\,t-1)} + \theta_a \cdot y_{(x_e+A-1,\,t-1)}\right)\cdot(1+G)}{{}^{R}\ddot{a}^{\lambda}_{x_e+A}} \qquad \text{[35.]}$$

Therefore the spending on pensions is:

$$\sum_{k=1}^{A}\sum_{c=1}^{k} P^{i\_I}_{(x_e+k,\,c,\,t)} \cdot I^{N}_{(x_e+k,\,c,\,t)} \cdot {}^{I}a^{\lambda}_{x_e+k} + \sum_{c=1}^{A} P^{i\_R}_{(x_e+A,\,c,\,t)} \cdot N_{(x_e+A,\,c,\,t)} \cdot {}^{R}a^{\lambda}_{x_e+k}$$

$$= \qquad \text{[36.]}$$

$$\sum_{k=1}^{A}\left(\overline{K}^{i}_{(x_e+k-1,\,t-1)} + \theta_a \cdot y_{(x_e+k-1,\,t-1)}\right)\cdot(1+G)\cdot I^{N}_{(x_e+k,\,t)} + K^{Ti}_{(x_e+A,\,t)}$$

and this has to be equal to the aggregate income from contributions according to the new balanced rate θ^{*}_{t}:

$$\theta^{*}_{t} \cdot \sum_{k=0}^{A-1} y_{(x_e+k,\,t)} N_{(x_e+k,\,t)} = \overbrace{\sum_{k=1}^{A} K^{Ti\,[+]}_{(x_e+k-1,\,t-1)} \cdot p^{ai}_{xe+k-1} \cdot (1+G)}^{\text{Disability}} + \overbrace{K^{Ti}_{(x_e+A,\,t)}}^{\text{Retirement}} \qquad \text{[37.]}$$

The previous expression, after some algebra, can be rewritten as:

$$\theta^{*}_{t} \cdot \overbrace{\left[1 + \underbrace{\frac{\sum_{k=1}^{A} D^{Tac}_{(x_e+k-1,\,t-1)} \cdot p^{ai}_{xe+k-1} \cdot (1+G) + D^{Tac}_{(x_e+A,\,t)}}{\sum_{k=1}^{A} K^{Ti\,[+]}_{(x_e+k-1,\,t-1)} \cdot p^{ai}_{xe+k-1} \cdot (1+G) + K^{Ti}_{(x_e+A,\,t)}}}_{\text{Dividend effect}=De_t} \right]}^{\text{Dividend factor}=Df_t} \qquad \text{[38.]}$$

Consequently $Df_t > 1$ results from not including the survivor dividend in the calculation of the contribution rate. Therefore $\theta_a > \theta^{*}_{t}$ due to the system's "savings" after the non-inclusion of the survivor dividend. De_t is a weighted average of the two dividend effects for both contingencies, the weighting being the balanced contribution rates by contingency as part of the total balanced contribution rate for the system, θ^{*}_{t}.

If $\theta_t = \theta_a$ were contributed instead of θ^{*}_{t}, the system would continuously accumulate financial reserves because ignoring the survivor dividend produces savings. In practice these reserves could finance the increase in spending on pensions resulting from increases in longevity. They could even be used as a source of finance for other social security commitments with no specified source, e.g. legacy costs from old pension systems.

3.-Numerical illustration.

This section shows the results obtained for a numerical example representative of the model developed in the previous section. More specifically, for the three generic NDC schemes analysed, we present the main values that make up the system's equilibrium including the contribution rates assigned to each contingency, the dependency ratio, the financial ratio and the dividend effect. We pay special attention to the assumptions made about the mortality rate for disabled people and the disability incidence rate, which largely determine the contribution rate assigned to disability. The effects of population changes on the dividend effect by cohort is also analysed in detail.

This section is divided into two different parts according to population growth. Part a) assumes that the active population will remain constant. This is in line with the assumption made by the Swedish authorities when valuing the system's assets and liabilities, Pensionsmyndigheten (2013). Meanwhile Part b) incorporates population changes over time.

a) Baseline case: the active population will remain constant

Our starting point is the numerical example developed by Boado-Penas & Vidal-Meliá (2014) for an NDCr (retirement) scheme after the inclusion of the survivor dividend. It is assumed that individuals can join the labour market from age 16 upwards, that the credited contribution rate is constant and equal to 16% and that the fixed retirement age for all individuals is 65, i.e. the highest age that individuals can join the labour market is 64.

This initial system, NDCr, is extended from the start by adding a disability contingency, so the resulting scheme is now called NDCdr. However, with the aim of emphasizing the important role of the assumptions made about the mortality rate for disabled people and the disability incidence rate, in the numerical example we work with two integrated schemes: NDCdr1 and NDCdr2.

With regard to the disability contingency, a contributor who becomes disabled in year *t-1* receives an initial disability pension based on formulas **[10.]** and **[11.]**, i.e. the accumulated notional capital at time *t-1* divided by the disability annuity factor corresponding to the age of the disabled person. It is important to remember that a contributor who becomes disabled at age 64, the last age at which it is possible to contribute, despite having made exactly the same contributions, would receive an initial pension that was different from (higher than) the initial retirement pension because the annuity divisor is not the same for both contingencies.

In the case of zero population growth, it is supposed that all individuals enter the labour market at age 16 and work continuously until retiring at age 65 unless they die or become disabled.

The mortality table[9] used for the active population (contributors and retirement beneficiaries) is the same in all three schemes (NDCr, NDCdr1 and NDCdr2). Figure 1 shows the mortality rates (in black, first vertical axis) for active contributors and retirement pensioners by age.

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[9] Observed mortality rates for Poland in 2009, obtained from the Human Mortality Database (http://www.mortality.org/).
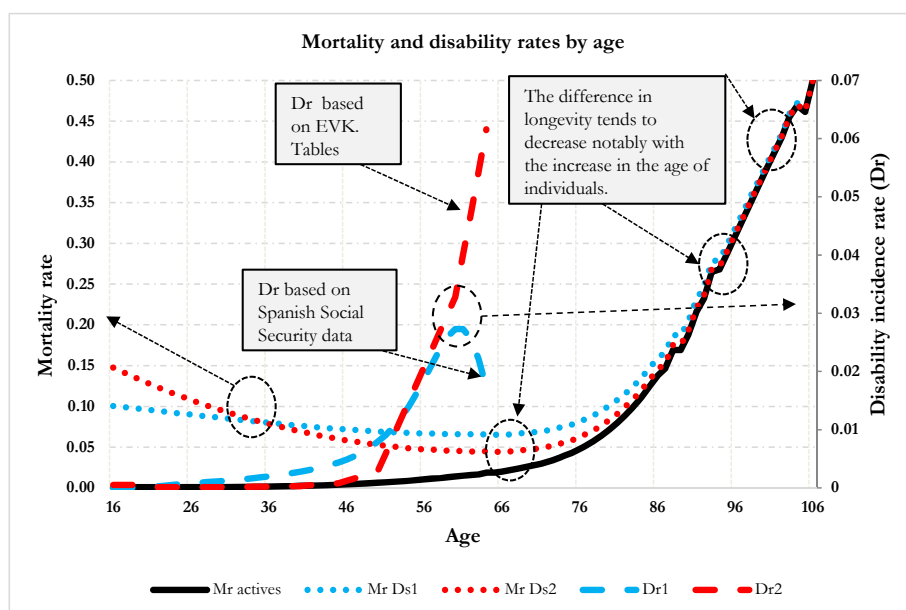
Figure 1: Mortality and disability incidence rates by grouped age structure.

NDCdr1 (blue in Figure 1) and NDCdr2 (red in Figure 1) show the differences in mortality rates for disabled people (Ds) (MR Ds1 and MR Ds2 in Figure 1, first vertical axis) and disability incidence rates (Dr) by age (Dr1 and Dr2 in Figure 1, second vertical axis). The disability incidence rate can be defined as the ratio between the new beneficiaries awarded benefits each year and the disability-exposed population[10]. The disability incidence rates are based on Spanish Social Security experience (Dr1) and EVK tables (Dr2) which rely on the Swiss federal government plan (no longer in existence).

As mentioned in Section 2 and shown in Figure 1, disabled people have a lower life expectancy than active people, but the difference in longevity tends to decrease notably with the increase in the age of the individuals [11]. The mortality rate for disabled people has been derived from that for the active population by adding an extra-mortality rate which decreases with the age of the individuals[12]. However, as Pitacco (2012) points out, the picture is much more complex given that the mortality of disabled people basically depends on the cause and severity of their disability.

The evolution of the pensioner and contributor collectives is shown in Figure 2 as a percentage of the initial group (contributors aged 16).

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[10] The disability incidence rate should not be confused with the disability prevalence rate, the latter being the ratio between the number of disabled pensioners in current-payment status each year and the insured-worker population (contributors).

[11] The RP-2000 Mortality Tables graduated by the US Society of Actuaries, SOA (2000), show the same mortality rate for healthier annuitants and disabled male pensioners from age 90 onwards.

[12] According to OSFI (2011), the mortality rates for male and female Canadian disability beneficiaries aged 55 to 59 are on average five to six times higher than the mortality rates for the general population for that age group and for each sex. Similar relationships are observed for other age groups.
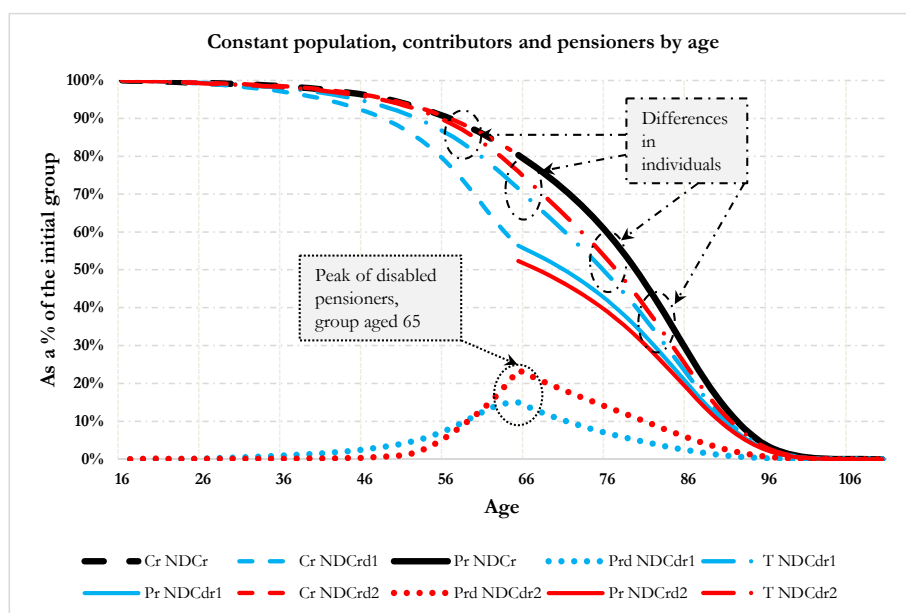
Figure 2: Evolution of the collectives by grouped age structure.

Figure 2 shows the evolution of contributors and pensioners for the three schemes: NDCr (contributors (Cr NDCr) and retirement pensioners (Pr NDCr)) in black; NDCdr1 (contributors (Cr NDCrd1), disability pensioners (Prd NDCrd1), retirement pensioners (Pr NDCrd1) and total (T NDCrd1)) in blue; and, NDCdr2 (contributors (Cr NDCrd2), disability pensioners (Prd NDCrd2), retirement pensioners (Pr NDCrd2) and total (T NDCrd2)) in red.

It can be seen that in the new model (NDCrd1 and NDCrd2) there are two types of beneficiary, disability pensioners and retirement pensioners, and that the collectives as a whole are smaller than the base system because the disabled have a lower life expectancy. Differences by age are shown in the graph by ellipses and reach their maximum at age 65, after which they are decreasing. The three collectives would only coincide under the additional assumption of equal longevity for both disabled and non-disabled (active or retired). If population growth had a positive value, then given the way in which disability is determined, the growth rate for the disabled would be lower than that for the contributing population. The differences between the NDCrd1 and NDCrd2 collectives basically arise due to the differences in the mortality rates for disabled people, as shown in Figure 1.

Figure 3 shows the evolution of average pensions and initial pensions by age and average pensions by contingency for the three schemes. The average disability pension (APd1 and APd2) by age is growing given that a higher pension is awarded when more contributions have been made. The maximum value is reached at age 64, from which time no more disability pensions can be awarded, and therefore for the retirement ages the amount is decreasing because once the pension is awarded it remains constant in real terms. The initial disability pension (IPd1 and IPd2) by age is also growing, and the differences between both can be explained by the annuity divisors used to calculate them that take into account different longevity for the disabled.

So what about retirement pensions? As can be seen in Figure 3, the average retirement pension (Pr) represents this value for the three schemes because, despite the very different assumptions for the disability contingency and although NDCr does not cover disability, the average retirement pension remains virtually the same in all three schemes. The average total pension by retirement (APTr) also remains virtually the same for all three schemes. This can be considered a sound result for our model and indicates an optimal integration of both contingencies into the NDC framework.
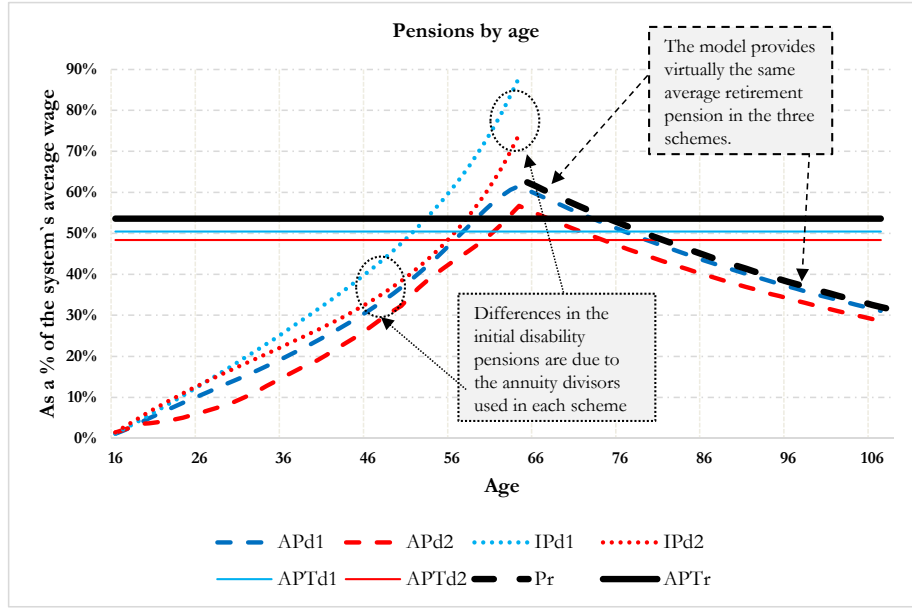
Figure 3: Average and initial pensions by grouped age structure.

The main values making up the system's equilibrium under the three generic NDC schemes (NDCr, NDCdr1 and NDCdr2) are shown in Table 1. We adopt the assumption that the contribution rate is the same for all schemes, but when disability is integrated into them (NDCdr1 and NDCdr2), the contribution rate is assigned to each contingency as a proportion of the spending on pensions per contingency as part of total spending. The contribution rate assigned to disability (3.97% in NDCdr1 versus 5.08% in NDCdr2) largely depends on the disability incidence rates and mortality rates of disabled people by age, which determine the stable prevalence disability rate of each scheme (7.88% in NDCdr1 versus 10.50% in NDCdr2), though the average disability pension also matters.

Table 1: NDCdr system with two contingencies: some selected values. Comparison with NDCr							
Items	**NDCr**	**NDCdr1**			**NDCdr2**		
		Disability	**Retirement**	**System**	**Disability**	**Retirement**	**System**
$\theta_a = \theta_t$	0.1600	0.0397	0.1203	0.1600	0.0508	0.1092	0.1600
dr_t	0.3020	0.0788	0.2246	0.3034	0.1050	0.2042	0.3092
fr_t	0.5298	0.5041	0.5355	0.5273	0.4835	0.5349	0.5175
De_t	0.1832	0.0880	0.1869	0.1607	0.1117	0.1893	0.1635
θ_t^*	0.1352	0.0365	0.1013	0.1378	0.0457	0.0918	0.1375
TD_t **(years)**	33.0800	25.5474	34.0325	31.9247	29.2994	33.7903	32.3650
A_r **(years)**	74.4500	65.9693	74.4544	72.3466	69.9635	74.4544	73.0291
A_c **(years)**	41.3800	40.4219	40.4219	40.4219	40.6641	40.6641	40.6641
$\bar{x}_t$ **(years)**	64.0000	56.1786	64.0000	62.0571	59.3199	64.0000	62.5146
pt_c **(years)**	18.2760	15.7567	23.5781	21.6352	18.6558	23.3359	21.8505
pt_r **(years)**	9.3160	9.7907	10.4544	10.2895	10.6436	10.4544	10.5145
Base scenario with $G=(1.016)(1.00)-1=0.016$							

The three schemes are in financial equilibrium because the contribution rate (see formula **[23.]**) is the product of the financial ratio (fr_t) and the dependency ratio (dr_t), and these ratios present slight

variations across the schemes. In the integrated schemes each contingency taken individually is also in financial equilibrium.

As shown by Boado-Penas & Vidal-Melia (2014), the effect of including the survivor dividend (De_t) on the initial pension is by no means irrelevant, and the pension rises by 18.32% in the NDCr scheme. The integration of disability into the NDC framework (NDC) keeps the dividend effect high. However, as the new contingency decreases the weighted average age at which contributions to the system cease[13] - 64 years in the NDCr plan against 62.05 years in the NDCdr1 scheme - the dividend effect is smaller.

The dividend effect for the integrated system, NDCdr, is a weighted average of the dividend effects for both contingencies, the weighting being the spending on pensions by contingency (without including the survivor dividend) as part of total spending. As shown in the previous section, the dividend effect can be calculated as that part of the total accumulated notional capital originating from contributions made by deceased contributors which belongs to new beneficiaries in the same year, divided by the yearly total spending on pensions (without including the survivor dividend). Although we do include the survivor dividend when calculating the amount of the retirement and disability pensions, if it were not included, then a discrepancy would arise between the credited contribution rate equal to 16% and the rate necessary to finance the benefits, θ_t^*, 13.78% and 13.75% for the NDCdr1 and NDCdr2 schemes respectively. Therefore, as already shown in the previous section, this numerical example illustrates the equivalence between the macro balanced contribution rate and the credited individual contribution rate in the new NDCdr framework introduced in this paper, and the fundamental role played by the survivor dividend in achieving the system's financial equilibrium.

Table 1 also shows the values for the turnover duration (TD), a well-known concept used for compiling the ABS of NDC systems[14]. Ventura-Marco & Vidal-Meliá (2014) developed the system's expected average TD for a DB PAYG scheme with retirement and disability benefits. Its application to NDCdr schemes is almost immediate. The system's TD is interpreted as the number of years expected to elapse before the committed liabilities with contributors and pensioners for retirement and disability are completely renewed at the current contribution level. Each monetary unit enters the system as if it were paid by a contributor of A_c years and remains within the contribution liability until retirement age is reached (pay-in). It is then received by the pensioner of A_r years after remaining within the liability to pensioners during the pay-out.

A system's TD can be calculated either as a weighted average of the TDs for both contingencies, the weighting being the spending on pensions by contingency as part of total spending, or as the difference between the weighted average of the average ages of disability $(A_r^D - A_c^D)$ and retirement $(A_r^R - A_c^R)$, the weightings here being spending on pensions per contingency as a part of total spending and the average age of the contributors.

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[13] For details on how to calculate the weighted average age at which contributions cease, $\overline{x}_t$, interested readers can consult Ventura-Marco & Vidal-Meliá (2014), formula **[50.]**.

[14] The legal definition and specific formulas used in the Swedish NDC system can be found in Pensionsmyndigheten (2013). This concept initially appears in connection with the contribution asset (CA) for NDCs, the general outline of which can be found in Settergren (2001) and (2003), while in Settergren & Mikula (2005) both concepts are modelled in continuous time, giving theoretical support. The search for valid expressions to apply to DB PAYG systems began with Boado-Penas et al (2008), continuing with Vidal-Meliá et al (2009), which in addition links to the concept of automatic balance mechanisms (ABMs). Vidal-Meliá & Boado-Penas (2013) obtain the analytical properties of the CA and confirm its soundness as a measure of a PAYG scheme's assets.

A system's TD is also the sum of the weighted pay-in, pt_c^S, and pay-out, pt_r^S, durations of one monetary unit in the system for the year's contributions and is based on population data obtained from a cross-section, not from an explicit projection.

The TD for retirement in the integrated schemes (34.03 and 33.79 years for NDCdr1 and NDCTdr2 respectively) is slightly different to the base system's TD (33.08 years). This comes about due to the slight change in the average age of the contributors after considering decrements through disability. The systems' TDs do change more noticeably (31.9 and 32.3 years for NDCdr1 and NDCTdr2 respectively) due to the introduction of disability, which makes the weighted average age at which the last contribution is made between 5 and 8 years earlier than for the retirement contingency.

To end these comments regarding Table 1, it is worth mentioning that our example is quite close to reality, not only because the OLG model developed works simultaneously with 49 and 85 generations of contributors and pensioners respectively, but also because the resulting values for the turnover duration - between 31.9 and 32.4 years for the integrated system - differ very little from those calculated by Settegren & Mikula (2007) for a large group of countries (32.7 years).

b) Population changes: the active population will not remain constant

The NDCdr1 scheme is taken as a reference when analysing the effect of active population changes, whether increases or decreases. Two additional assumptions are explored in this section: 1) the number of contributors of all ages grows at an annual rate of $\gamma = 0.01$ over time (henceforth NDCdr1+), and 2) the number of contributors of all ages decreases by an annual rate of $\gamma = -0.01$ over time (henceforth NDCdr1-).

Table 2 shows that although the ratio between the numbers of contributors and pensioners (dr_t) and the ratio between the average salary and pension (fr_t) change due to variations in the active population, the effect of the survivor dividend (De_t) remains unchanged for both contingencies.

The system's sustainable return (G) derives from an adjustment to the average initial pensions that are awarded in each case, directly linked to the annuity factors $^D\bar{\bar{a}}_{x_e+k}^{\lambda}$ [15] and $^R a_{x_e+A}^{\lambda}$ and the accumulated notional capital reached at retirement or disability age.

Despite the growth in population, the average initial pensions (retirement and disability) for NDCdr1+, expressed in Table 3 through the average replacement rate for each contingency ($\bar{\beta}_{(x_e+k,\ t)}^{D}$ [16] and $\bar{\beta}_{(x_e+A,\ t)}^{R}$), are higher than in the other two cases. The growth of the economically active population modifies the average years of contribution (AYC). As can be seen in Table 2, the average contributor, awarded a pension in year t, has been contributing for 36.74 years as opposed to 45.69 years for NDCdr1 and NDCdr1-. All contributors who reach retirement age are considered to have started working at the entry age of 16, i.e. A years ago. Likewise all contributors who become disabled at age x_e+k years started working k years ago. If the population grows over time, the retirees' generation and the generation of disabled people can be split into A and k different cohorts respectively, whose common factor is the number of years contributed since joining the labour market.

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[15] This is a weighted average calculated from the disability pensions awarded in year t.

[16] Like $^D\bar{\bar{a}}_{x_e+k}^{\lambda}$, the average replacement rate for the disability contingency is a weighted average calculated from disability pensions awarded in year t.

Items	NDCdr1			NDCdr1+			NDCdr1-		
	D	**R**	**S**	**D**	**R**	**S**	**D**	**R**	**S**
$\theta_a = \theta_t$	0.0397	0.1203	0.1600	0.0397	0.1203	0.1600	0.0397	0.1203	0.1600
dr_t	0.0788	0.2246	0.3034	0.0708	0.2012	0.2700	0.0886	0.2524	0.3435
fr_t	0.5041	0.5355	0.5273	0.5613	0.5976	0.5925	0.4486	0.4765	0.4658
Dq_t	0.0880	0.1869	0.1607	0.0880	0.1869	0.1607	0.0880	0.1869	0.1607
θ_t^*	0.0365	0.1013	0.1378	0.0365	0.1013	0.1378	0.0365	0.1013	0.1378
$^{D}\ddot{a}_{\overline{x_e+k}}^{\lambda}$	11.2443	n.a.	13.8284	10.3129	n.a.	12.6356	12.3544	n.a.	15.2336
$^{R}\ddot{a}_{x_e+A}^{\lambda}$	n.a.	15.0483		n.a.	13.7320		n.a.	16.5928	
AYC	38.6779	49.0000	45.6899	31.9944	38.9740	36.7358	38.6779	49.0000	45.6899
$\overline{\beta}_{(x_e+k,\, t)}^{D}$	0.8019	n.a.	0.8484	0.8812	n.a.	0.9369	0.7247	n.a.	0.7632
$\overline{\beta}_{(x_e+A,\, t)}^{R}$	n.a.	0.8703		n.a.	0.9632		n.a.	0.7814	
G	0.0160			0.0262			0.0058		

Table 2: NDCdr system with survivor dividend when the active population will not remain constant: some selected values.

In our example, generation members who retire at age 65 could come from 49 different cohorts depending on the number of years contributed. This determines 49 (*A*) different amounts of pension that set the average initial pension of the generation, linked to the average number of years contributed by those who reach retirement age. Similarly, generation members who become disabled at age 46 could come from 30 different cohorts depending on the number of years contributed.

Figure 4 shows initial retirement and disability pensions awarded at the ordinary retirement age broken down by years contributed as a percentage of the system's average wage. The pension amount for retirement (R NDCdr1+) and disability (D NDCdr1+) with an equal number of contribution years when the economically active population grows over time is much higher than for NDCdr1 (ARP NDCdr1 and ADP NDCdr1, in blue), and higher for NDCdr1 than for NDCdr1- (ARP NDCdr1- and ADP NDCdr1-, in red). This is only to be expected given that the sustainable return of the system with population growth (decline) is higher (lower) and, if the scheme is well designed, automatically increases (decreases) the amount of benefits awarded to retirement and disability pensioners.
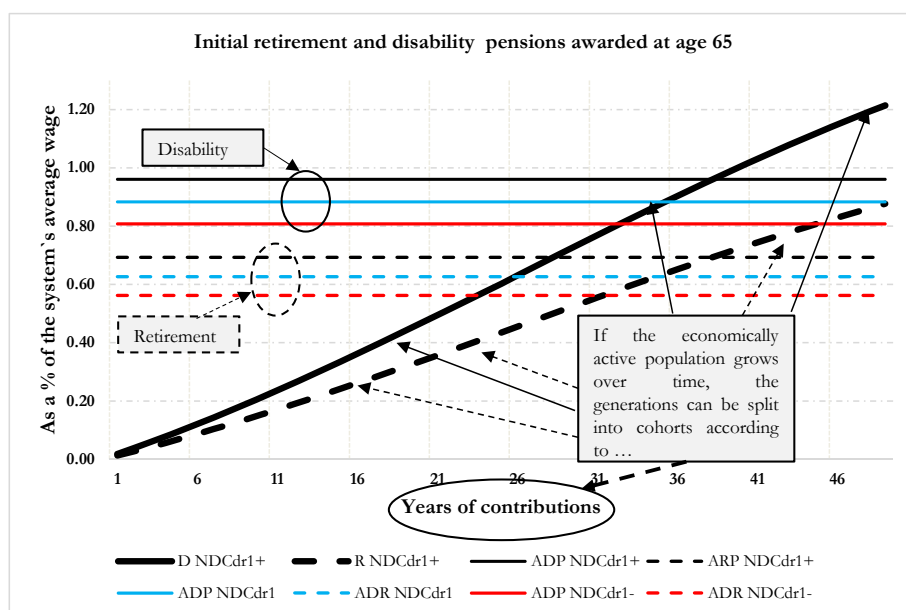
Figure 4: Breakdown of initial retirement and disability pensions by years contributed.

In line with the paper by Vidal-Meliá et al (2013), another essential concern is whether or not the variation in population has an influence on the dividend effect. For the NDCr model they find that, despite the fact that the dividend effect remains constant for any value of γ, growth in the economically active population enables cohorts with more years of contributions to benefit to a greater extent from the dividend effect, i.e. the more contributors there are, the larger the retirement pension for those cohorts with more years of contributions compared to what it would have been without including the survivor dividend.

Can the same cohort effect be seen in the integrated NDC model with disability? The answer can be found in Figures 5 and 6, which show the effect of the survivor dividend for each of the cohorts that make up the pensioner generation under the assumption that the economically active population grows at a constant rate of 1%, 2% or 4% per year. The value assigned to γ has an inverse influence on the average number of years contributed for the pensioner generation that retires at time t. For a value of $\gamma = 0.01$, as Table 3 shows, the average number of years contributed (AYC) is 31.99 (disability), 38.97 (retirement) and 36.64 (system). The AYCs for $\gamma = 0.02$ and $\gamma = 0.04$ are (26.90, 31.67, 30.14) and (19.89, 22.20, 21.46) respectively.

Figure 5 shows that the growth of the economically active population enables cohorts with more years of contributions to benefit to a greater extent from the dividend effect. Indeed some cohorts get a higher dividend effect than the average dividend achieved by the system. Also, retirement pensioners benefit from a higher effect than disability pensioners because, although both types of pension are awarded in the same year, the contributors who become disabled that year do not benefit from a distribution of the survivor dividend that year.
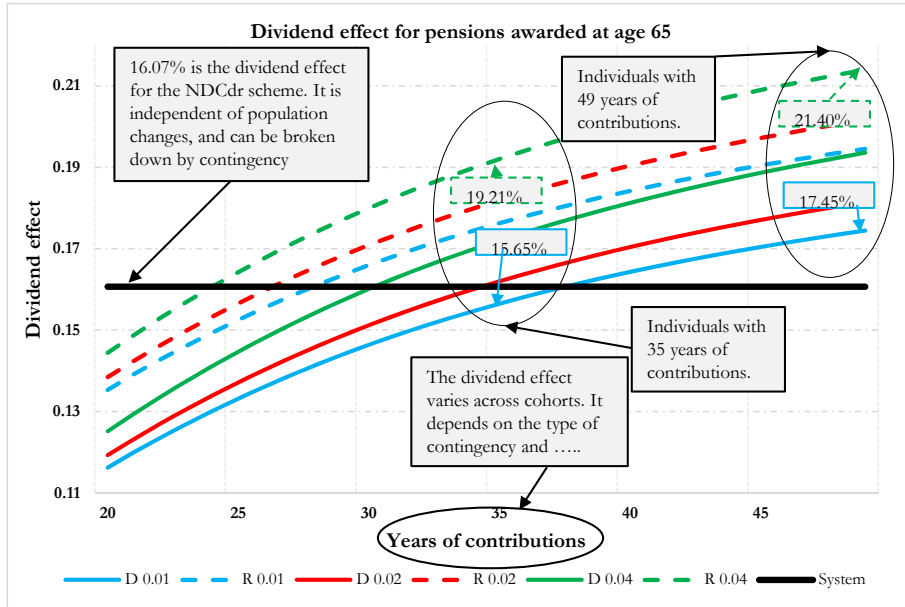
Figure 5: Dividend effect with population growth by years of contributions

Figure 6 shows the (relatively small) impact of the dividend effect on the amount of the disability pensions awarded to contributors who become disabled in earlier years. Even with a high rate of growth in the active population (0.04), the survivor dividend effect is small, much lower than the system's average dividend (0.1607), and lower even than the average survivor dividend effect for disability (0.0880).
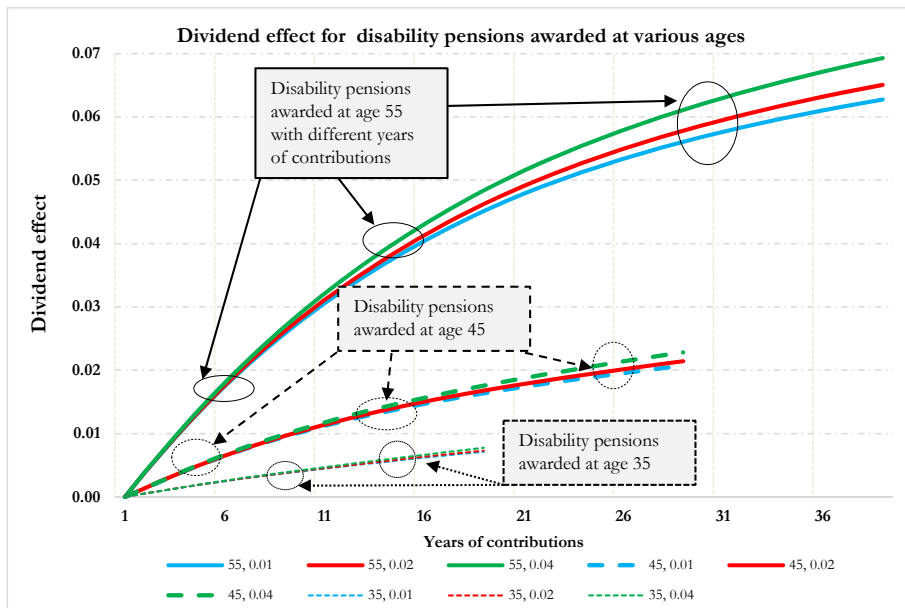

Figure 6: Dividend effect for disability pensions

In short, the system's average dividend remains constant for any value of γ, but the effect of the growth in active population is an increase in the amount of retirement and disability pensions, mainly for those people who become disabled at the last age at which it is possible to contribute - or at least close to that age - and who have long contribution records.

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4.-Conclusion, discussion and future research.

An NDC scheme is widely defined as a PAYG system that deliberately mimics a financial defined contribution scheme (FDC). However, this is not strictly true since the way disability benefits are integrated into the scheme varies greatly. In most countries with mandatory individual capitalization accounts, disability insurance (DI) is fully integrated into the FDC scheme.

Like (badly designed and managed) DB retirement PAYG systems, DB DI is today a big challenge for policy makers mainly because:

- ✓ it faces high and growing costs, and in most countries the real cost is underestimated because of the phenomenon identified as "pension reclassification",
- ✓ it creates strong incentives for early retirement,
- ✓ it hampers economic growth and reduces the effective labour supply,
- ✓ it hides the redistribution of benefits, and
- ✓ it faces significant political risk.

Hence, given that NDC pension schemes have positive features that could help to improve the efficiency of DI, it is not unreasonable to develop a theoretical model that fully integrates the disability contingency into an NDC framework.

For the above reasons, in this paper we have developed a multistate OLG model that integrates old-age and permanent disability into a generic NDC framework. Inspired by the Swedish NDC model currently in force, we have followed the principle that each monetary unit contributed is paid out in the form of benefit. However, this benefit is not necessarily paid to the individual who made the contributions, given that the account balances of contributors who do not survive are distributed as inheritance capital to the accounts of the (non-disabled) active survivors on a birth cohort basis.

To develop the model, DI has been considered as a contingency close to retirement because permanent work disability insurance enables people to get lifetime benefits before the age for early retirement if they are unable to work. Nevertheless, the authors are fully aware that disability policies have multiple implications for society that go beyond the scope of this paper.

With the aim of linking the survivor dividend and the disability contingency in the model, the so-called cumulative inheritance gain factor has been defined on the basis of a transition probabilities matrix. Unsurprisingly, the formula for this is very similar in structure to the formula used by the Swedish authorities for the NDC system, which only includes the retirement contingency.

The model shows that the survivor dividend has a sound financial basis that enables the balanced macro contribution rate applied to be the same as the individual credited rate. The main implication of this result is that, if the amount of the initial retirement and disability pensions were determined by the individual notional capital without considering the survivor dividend, the balanced contribution rate and the credited rate would be different because the system's benefits would be lower than they could be. On average for the base scenario shown in the numerical example, the effect of including the dividend on the initial pension is by no means irrelevant, and the initial benefit rises by 16.07%. Similarly, if the survivor dividend had not been included when calculating the amount of the benefit, a discrepancy would have arisen between the credited contribution used in the example, 16%, and the rate necessary to finance the benefits, 13.78%.

Another result that can be highlighted is the fact that the system's average dividend remains constant for any value of γ, but the effect of any growth in the economically active population is a proportionally higher increase in the amount of retirement and disability pensions, mainly for those people who become disabled at the last age at which it is possible to contribute - or at least close to that age - and who have long contribution records.

Our model can be said to be quite realistic insofar as it takes into account an age schedule of mortality and the uncertainty concerning the timing of disability, and allows for changes in the economically active population and for a large number of generations of contributors and pensioners to coexist at each moment in time.

On the practical side, the numerical example presented in the paper can also be considered as quite close to reality, not only because the OLG model developed works simultaneously with 49 and 85 generations of contributors and pensioners respectively, but also because the resulting values for the turnover duration – around 32.2 years for the integrated system - differ very little from those calculated in the literature for a large group of countries (32.7 years).

The results achieved in the numerical example, in the case of zero population growth and for when the economically active population changes, confirm that the model really works and show an optimal integration of both contingencies into the NDC framework. In spite of the very different assumptions for the disability contingency and even though NDCr does not cover disability, the average retirement pension remains virtually the same in the three schemes analysed.

This model can easily be linked to real practices in social security policies because, to mention just a few positive features, it could be implemented without too much difficulty, it would help to improve actuarial fairness, it would uncover the real cost of disability and minimize the risk of disability insurance being used as a vote-buying mechanism.

The question of putting the model into practice is by no means a minor topic. It would call for a new paper because at the very least it would need to thoroughly address the following issues:

- ✓ The transition rules from the old system to the NDC framework.
- ✓ The advisability of introducing a minimum pension.
- ✓ The transition from temporary disability to permanent disability.
- ✓ The updating of the annuity divisors.
- ✓ Communication to the public.
- ✓ The actuarial balance sheet (ABS) and the automatic balance mechanism (ABM).

Finally, based on the model presented in this paper, at least three important directions for future research can be identified:

✓To adapt the actuarial balance sheet (ABS) specifically designed for NDC systems to the new model with disability and to evaluate the impact of introducing a minimum pension on the system's financial equilibrium.

✓To extend the model to take into account different degrees of disability and/or the possibility of a return to active life. In practice there are usually various degrees of disability recognized and these have a direct effect on the amount of benefit paid and the likelihood of returning to active life. The papers by Aro et al (2013) and Zadeh et al (2013) could be useful for this purpose.

✓To incorporate insurance innovation into the model, as proposed by Murtaugh et al (2001) and Brown & Warshawsky (2013) for funded systems, with the integration of retirement and long-term care (LTC) annuities. The NDC framework could be useful for this purpose. This is not an unreasonable idea because LTC as a contributory contingency has been provided in the German contributory pension system, Rothgang (2010), since the mid-1990s. Barr (2010) also gives sound reasons for extending social security to provide mandatory cover for LTC.

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