2007 P&C SCOR Actuarial Prize

Modelling Utility under Uncertainty:

Further along the alternative path

By

George Aristides

October 2006

A dissertation submitted for the award of the degree of Master of Science in Actuarial Science

Supervisor: Dr Iqbal Owadally

Department of Actuarial Science and Statistics

Cass Business School

City University, London

Abstract

In this paper we analyse the theoretical treatment, modelling and applications of utility in the presence of uncertainty. We start by examining the limitations of the predominant *Expected Utility theory*, addressing the observed evidence for consistent violations of its assumptions. We then explore alternative methods for modelling utility, discussing their main features and predictions.

We proceed to introduce a new model of utility under uncertainty. The *Combined Risk Attitude* model aims to provide another way of representing economic agents' decision-making, using intuitively appealing assumptions. We discuss the applications of our theory, exploring the results predicted by the model in certain situations. Our model is finally compared to other alternative models of utility under uncertainty, highlighting its relative strengths and limitations, along with any scope for expansion and development.

The *Combined Risk Attitude* model does not aim to replace existing models of utility; it builds upon their foundations, offering a view of the topic from a slightly different angle. It is appropriate for complementing the existing models while arguably being more suitable for specific types of analysis – especially where the variable of interest is the magnitude of risk-aversion or risk-seeking associated with a simple prospect.

Aknowledgments

Firstly, I would like to thank my family, for providing me with their invaluable emotional and financial support and their endless encouragement.

I would like to dedicate this paper to my closest and dearest friends, and especially Christo, Dimitris and Philippos; thanks for always being there for me.

Last, but not least, I would like to thank my supervisor, Dr. Iqbal Owadally, for his contributions, feedback and constructive criticism, which went a long way towards improving the quality and clarity of this paper.

Table of Contents

1. Introduction

The concept of utility is a fairly controversial topic in economic theory. Its highly abstract nature, as well as the inherent inability to measure it directly, have led to disagreement as to whether it is a useful subject for analysis, and – if so – to what extent. Nevertheless, a large amount of literature revolves around the use of utility to explore, justify and predict human behaviour – a branch of economics called "utility theory". Contemporary applications of utility theory can be found in many areas that are related to both economics and psychology, such as finance, insurance, management and marketing.

In its most basic definition, utility is a measure of "pleasure" or "satisfaction" gained by economic agents from the consumption of goods or services. It is also often connected with the monetary amounts used by the agents to obtain those goods or services; thus, we can talk about the utility of money. Given that almost all contingencies – perhaps excluding those involving human mortality or disability – can be directly translated in terms of monetary value, utility of money can generally subsume any other type of utility in a given state of the world. In this paper we focus our analysis on utility of monetary outcomes, keeping in mind that – in reality – utility is derived from the ultimate use of the monetary value in question.

In situations of risk and uncertainty, where individuals are faced with uncertain contingencies of differing outcomes, the predominant utility theory analysing human behaviour and choice is the "Expected Utility" theory. First used by Bernoulli (1738), this theory has seen widespread use since the important contributions by the economists von Neumann and Morgenstern (1947), whose names often accompany the corresponding model of utility. Although based on a set of seemingly self-evident axioms, the *von Neumann – Morgenstern Expected Utility Model* has bred a multitude of paradoxes that contradict its assumptions and predictions. As a result of this, there have been several attempts to present alternative theories of utility under uncertainty, such as *Prospect Theory* by Kahneman and Tversky (1979) and models of *Regret* (Bell, 1982; Loomes and Sugden, 1982) and *Disappointment* (Bell, 1985; Loomes and Sugden, 1986).

In this paper we will analyse the theoretical treatment, modelling and applications of utility in the presence of uncertainty, focusing on the phenomenon of risk attitude reversal – from *Risk Averse* to *Risk Seeking* and vice versa – in certain situations. We will start by critically examining the limitations of the currently predominant *von Neumann* - *Morgenstern Expected Utility function* method, by showing how its assumptions are consistently violated in actual observed human behaviour under uncertainty. We will then analyse the alternative methods of modelling utility mentioned above, exploring their assumptions and discussing their main features and results.

We will further proceed to introduce a new alternative method of modelling utility under uncertainty, which features the cumulative effects of different factors on risk attitudes. The *Combined Risk Attitude* (henceforth referred to by the acronym *CRA*) model aims to provide another way of accurately representing economic agents' decision-making process, using intuitively appealing assumptions. We will discuss and analyse the applications of our theory in several areas, exploring the results predicted by the model in certain real world situations. The *CRA* model will finally be discussed in relation to other alternative models of utility under uncertainty; its strengths and limitations, along with any potential for further development, will be highlighted.

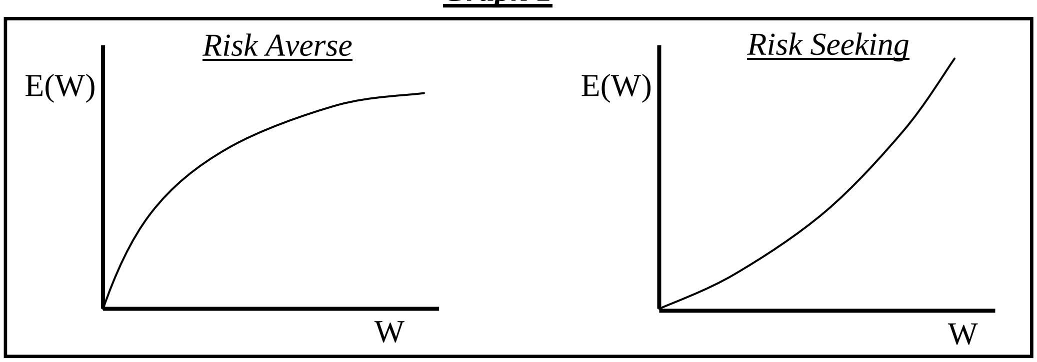
2. Why *not* Expected Utility?

In analysis of individual choice under uncertainty, where the concept of utility is often considered central to optimal decision making, the standard methodology is to consider the utility associated with each possible contingency. According to the Expected Utility theory, each utility value is multiplied by the ex-ante probability that the corresponding contingency takes place, and summed up. This gives us the von Neumann-Morgenstern Expected Utility function shown below. For each contingency $\boldsymbol{i}$, $\boldsymbol{p_i}$ refers to probabilities, $\boldsymbol{w_i}$ refers to levels of wealth and $\boldsymbol{u(w_i)}$ is the utility function of wealth:

$$EU = \sum_{i=1}^{n} p_i u(w_i)$$

The main attractiveness of Expected Utility theory lies in its simplicity. Besides knowledge of the probabilities of the different contingencies or states of the world, it relies only on the utility function of wealth. This function is commonly assumed to be globally concave, for most individuals, leading to risk averse attitudes for any level of wealth. In essence, this means that people would be willing to pay a premium for certainty, explaining the demand for insurance. Other individuals are considered to be risk seeking, preferring to take gambles such as investing in high risk/high reward projects and participating in lotteries at actuarially unfair odds. An example set of utility-of-wealth functions is displayed in Graph 1:

Graph 1



In their influential axiomatisation of Expected Utility, von Neumann and Morgenstern (1947) justified their theory by proving that a utility function over lotteries constructed on the basis of a set of mathematical axioms, would have the Expected Utility structure and properties. These four ***axioms of preference*** are: *completeness*, *transitivity*, *continuity* (often referred to as the *Archimedean* axiom) and *independence*; these are comparable with similar arguments formulated on the basis of choice under certainty.

"Ever since the formulation by John von Neumann and Oskar Morgenstern of a set of axioms of rational choice under uncertainty, a number of situations have been identified in which there are significant and repeated violations of one or more of those basic axioms" (Loomes and Sugden, 1983: pp.428). These situations – or *"Paradoxes"* – have since been a common subject of analysis. The St. Petersburg Paradox motivated Daniel Bernoulli (1738) to first introduce the Expected Utility function, featuring the property of diminishing utility of wealth. This was later on shown to be an unsatisfactory solution; for any unbounded utility function, there can exist a properly constructed game of chance offering the infinite expected utility predicted by the Paradox. Apparently, an upward bound in the utility function of wealth – not the most realistic of assumptions – would be necessary to explain the St. Petersburg Paradox under an Expected Utility framework.

Another violation of the Expected Utility axioms was analysed by Coombs (1975). The Archimedean axiom – in combination with transitivity of preferences – implies that, given two lotteries A and B, any compound lottery providing a weighted mixture of A and B would give a level of utility somewhere in between the two original lotteries. This "*in-betweenness"* property is shown to be violated by almost half the test subjects in an experiment where lottery A gave a 50-50 chance at 3 dollars and B gave a 50-50 chance at 5 dollars. In this situation, the violation consisted of a third lottery C - which was a 40-60 mixture of A and B - being ranked first or last in preference in comparison to the original lotteries. Although the very small amount of money at stake may lead someone to question the robustness of these findings, this experiment nevertheless contributed evidence against the Expected Utility theory.

Ellsberg (1961) with his commonly quoted paradox is an example of the effects of certainty versus uncertainty in individual decision-making. His experiment consisted of an urn A filled with a 50-50 proportion of red and black balls, and a second urn B filled with red and black balls, in a proportion undisclosed to the test subjects. According to Expected Utility theory, an individual would form a subjective evaluation of the odds for red or black balls in the “uncertain” urn B. When offered a monetary prize upon picking a red ball from a selected urn, a large majority of test subjects preferred the known 50-50 odds of urn A. When they were subsequently offered a similar prize upon picking a ***black*** ball from a selected urn, they still persisted in selecting urn A. This evidence suggests that some kind of psychological aversion to uncertainty is affecting individuals’ choices, something not modelled by Expected Utility.

Probably the most well-known Paradox of choice under uncertainty is the Allais Paradox. Formulated by the French economist Maurice Allais (1953), this experiment – and others of similar content that followed it in subsequent studies – provided strong evidence against the validity of the independence axiom of von Neumann and Morgenstern. When comparing a certain outcome A with a lottery B in one situation, and two lotteries C and D in another, the preferences in each situation varied among test subjects. The paradox was, however, that the lotteries C and D were constructed in a way that they were a combination between A and

B respectively, with another lottery E. Hence, the independence axiom would predict that a preference of A over B would necessarily lead to a preference of C over D, and vice versa. The experiment uncovered a *"so-called certainty effect, according to which outcomes obtained with certainty loom disproportionately larger than those that are uncertain"* (Schoemaker, 1982: pp.542).

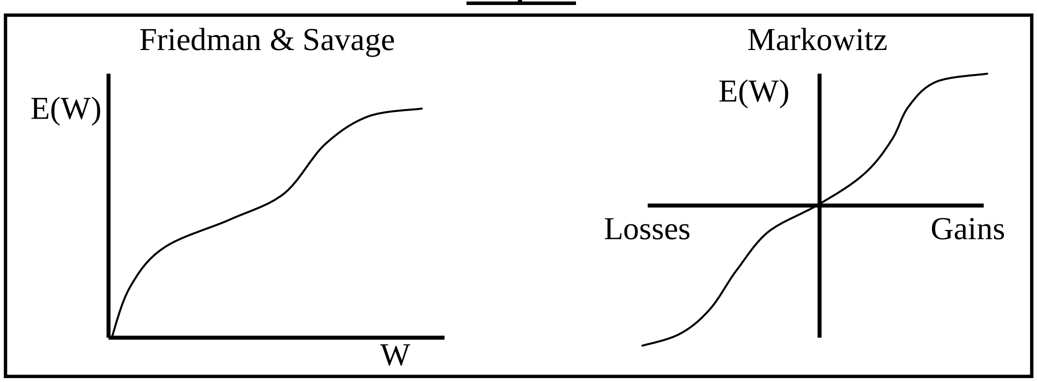
The evidence discussed above encouraged, in the last few decades, a considerable amount of economic literature challenging the Expected Utility model. According to Schoemaker (1982: pp.530) *"whereas the simplicity of EU theory, especially its mathematical tractability, may make it a very attractive model for purposes of social aggregation, its structural validity at the individual level is questionable."* The critique of Kahneman and Tversky (1979: pp.265) supports that *"if people are reasonably accurate in predicting their choices, the presence of common and systematic violations of expected utility theory in hypothetical problems provides presumptive evidence against that theory."* Although the Expected Utility theory will always remain a simple and useful model, giving reasonably accurate predictions in certain situations, its limitations produce a need for alternative ways of modelling utility under uncertainty.

3. The Alternative Path

3.1. A Review of Prospect Theory

In the previous section, we have seen the arguments that provide evidence against the validity and accuracy of the Expected Utility model as a representation of human choice under uncertainty. A number of papers have presented variants of the Expected Utility model, in a way that can address some of the observed behaviour that contradicts the original model. Friedman and Savage (1948) present a model where the utility function of wealth shifts from concave to convex, and again concave, as wealth increases. Markowitz (1952) presents a utility function with three points of inflection, one of which lies at the "status quo" level of wealth – thus separating risk attitudes between losses and gains. Illustrations of these two models are given in Graph 2:

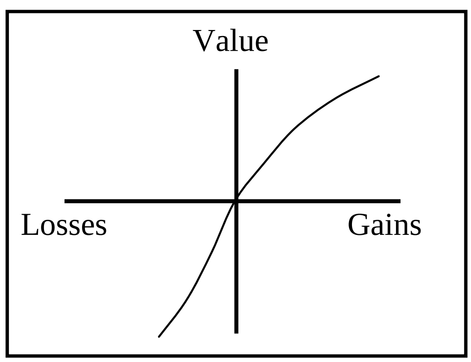
Graph 2



Karmarkar (1978) introduces subjective weighting of probabilities in risky choice, formulating the Subjective Weighted Utility model. Kahneman and Tversky (1979) use a similar concept in their formulation of the most influential departure from Expected Utility. Their model, called Prospect Theory, introduces several new concepts in utility analysis, and has many properties that successfully describe preferences in cases where the Expected Utility model fails to do so.

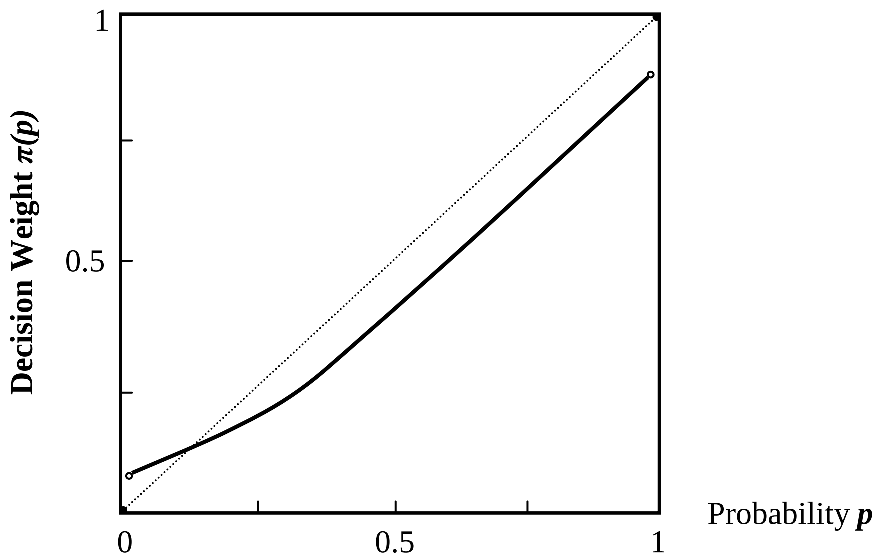
In most previous models of utility, the carrier of value is the wealth of each individual. Some models incorporated shifting reference points of wealth - this enables a differentiation in utility of a given level of wealth, according to its position in relation to the reference point. In Prospect Theory, this concept is taken one step further – the use of wealth as the carrier of value is rendered obsolete and, instead, we talk about the utility of ***gains and losses***. Therefore, a value function in the domain of gains and losses from a given reference point – usually the status quo level of wealth – replaces what used to be the utility function in the Expected Utility model. This value function is assumed to be concave for gains and convex for losses. The concept of loss aversion suggests that a loss causes disutility that is greater than the utility obtained by a gain of the same magnitude – thus the value function is steeper for losses than for gains. Graph 3 shows an example value function, as given in Prospect Theory.

Graph 3



A second major feature of Prospect Theory is the use of ***decision weights*** in choice under uncertainty. In a manner similar to the Subjective Weighting of utility used in Karmarkar (1978), the probabilities of each outcome in a prospect are weighted by a non-linear decision function, which is taken into consideration along with the value function for the corresponding outcome. The decision weight is typically lower than the probability associated with it, except in the case of very low probabilities, where it is higher. The decision function ***π(p)*** is discontinuous at the boundaries, having $\pi(0) = 0$ and $\pi(1) = 1$. An example of a decision weight function, found in Kahneman and Tversky (1979), is given in Graph 4:

Graph 4



Additionally, the concept of an ***editing phase*** is introduced. Prospect Theory suggests that individuals first edit the prospects under consideration, and then evaluate the edited prospects to make their decisions. One operation of the editing phase is to ignore extremely low probabilities, and treat extremely high probabilities as certainty. paraphrasing from Kahneman and Tversky (1979: pp.283), *"indivuduals are limited in their ability to evaluate extreme probabilities; thus, highly unlikely events are either ignored or overweighted, while the difference between highly likely events and certain events is either neglected or exaggerated."* Additionally, some prospects can be separated in riskless and risky components during the editing phase: for example, the prospect (100, 0.4; 500, 0.6) can be seen as a certain gain of 100 pounds and a 60% chance for a 400 pounds gain. The reference point separating losses and gains can be readjusted to differ from the status quo in some situations, according to the framing of the uncertain scenario and the expectations of the individual.

In the Prospect Theory formulation, the utility function is equal to the weighted average of the value functions – the weighting being done by the decision weight function on the basis of the probability of each contingency. Thus, for a prospect ***A (X, p; Y, 1-p)*** we have: ***U(A) = d(p)v(X) + d(1-p)v(Y)***. Given the shape of the value functions, in most situations Prospect Theory predicts risk aversion in the domain of gains and risk seeking in the domain of losses. This means that both insurance and gambling require the assumption of overweighting of low probabilities. The concept of loss aversion gives us the realistic prediction that in the case of a mixed prospect (offering both losses and gains with some probability) the individual will put greater weight on losses – which explains the observed unwillingness to take 50-50 win-lose gambles under most circumstances.

As a descriptive model of choice, Prospect Theory is highly successful in that its assumptions permit most types of observed deviations from the expected utility predictions. Its recent extension by the name of ***Cumulative Prospect Theory*** (Tversky and Kahneman, 1992) works with cumulative rather than individual probabilities of contingencies to give a formulation that is better suited to modelling complicated – even continuous – prospects. However, Prospect Theory suffers from the fact that its predictions rely on the arbitrary and highly abstract concept of overweighting/underweighting of probabilities. To deal with this fact, several other theories have attempted to model the properties of Prospect Theory by using a more intuitive and realistic explanation for departures from expected utility. The use of effects such as overconfidence and skill signalling has seen some use in contemporary theories, but the most widely known models make use of Regret and Disappointment as additional sources of utility beyond the monetary gains and losses given by a prospect.

3.2. Regret and Disappointment Models

"Expected utility appears to fail because the single outcome descriptor – money – is not sufficient. After making a decision under uncertainty, a person may discover, on learning the relevant outcomes, that another alternative would have been preferable. This knowledge may impart a sense of loss, or regret" (Bell, 1982: pp.961). ***Decision Regret*** is seen as the disutility caused by learning that one's decision under uncertainty was sub-optimal compared to alternative options, after the uncertainty is resolved. A corresponding positive utility effect, called ***"Rejoicing"***, occurs upon learning that your decision was optimal given the alternatives. Loomes and Sugden (1982) formulated Regret Theory, a descriptive theory of choice which models the ex-ante anticipation of Regret/Rejoicing in the utility function. A model of Regret with slightly different formulation but comparable results was independently created by Bell (1982).

In Regret Theory, we have a separation between ***Choiceless Utility*** – which occurs when a change in wealth did not result from a choice taken by the individual – and ***Modified Utility*** which includes regret or rejoicing effects. This makes use of a ***regret-rejoice function R(.)***, that gives the regret or rejoice associated with the action taken in comparison with alternative actions. An increasing, convex function ***Q(.)*** is defined such that ***Q(x) = x + R(x) – R(-x)*** to give the net utility after regret and rejoice effects are added in the equation. For example, Regret Theory predicts that an individual would weakly prefer action A_i to action A_k if and only if: $\sum_{j=1}^{n} p_j \left[Q\left(c_{ij} - c_{kj} \right) \right] \geq 0$, where ***j = 1..n*** refers to different possible states of the world occurring with probability p_j, and c_{ij} is the choiceless utility corresponding to the outcome of action A_i in state ***j***.

Regret Theory has some interesting properties and results. It is consistent with all the experiments presented in Kahneman and Tversky (1979), but in comparison it is simpler and uses a smaller number of assumptions. Loomes and Sugden (1982: pp.817) support that *"against the complex and somewhat ad hoc array of assumptions required by prospect theory, the principle of Occam's Razor strongly favours the straightforwardness of regret theory."* Regret Theory can explain the Allais Paradox, as well as violations of transitivity in preferences such as in Coombs (1975). Assuming a linear choiceless utility function, it predicts the reflection effect when we move from the domain of gains to the domain of losses, as well as the mixed risk attitudes phenomenon where we can observe simultaneous gambling and insurance by the same individual.

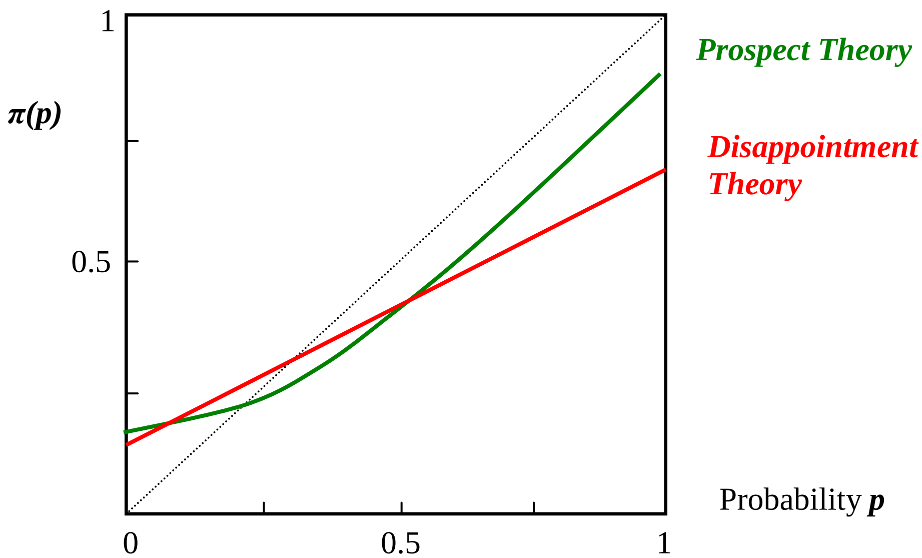
Regret Theory is not, of course, without its limitations. Loomes and Sugden (1986) admit that *"because regret theory makes comparisons across actions but within states of the world, it can predict violations of the transitivity axiom but not violations of the sure-thing principle"*. Lacking the sort of editing phase used in Prospect Theory, it cannot model "framing" and "context" effects (Tversky and Kahneman, 1981; Hershey and Schoemaker, 1980) that can affect behaviour according to how the prospect is presented. Some of its assumptions are also rather strong for several scenarios of choice under uncertainty: it requires that the uncertainty of foregone actions is resolved and their result, ex-post, known. Thus, it is unsuitable for gambling scenarios unless you get to know whether you would have won or lost - even if you don't actually participate.

As a follow-up to their respective models of Regret, Loomes and Sugden (1986) and Bell (1985) introduced models that revolve around the utility effect of Disappointment. Unlike Regret, which occurs by comparison of the result you would have gotten if you had taken another action, in the case of Disappointment the comparison is done across states. The basic assumption of these models is that individuals form expectations about the results of their choices, and then experience ***Disappointment*** – or its positive equivalent, ***Elation*** – according to whether the actual result is better or worse than expected. As in the case of Regret, the ex-ante anticipation by individuals of the psychological effects of disappointment/elation gives rise to observed behaviour that is predicted by this model, but is inconsistent with Expected Utility.

Bell's model of Disappointment adopts a gain-loss framework with constant marginal utility of money. Thus, risk attitudes are dependent on the effects of disappointment or elation associated with the prospect. For example, consider the simple prospect ***(1000, 5000, p)***, where you get 1000 with probability p and 5000 with probability 1-p. The disappointment associated with the low result of 1000 increases when p decreases; similarly, elation associated with gaining 5000 increases when 1-p decreases. This model can be extended to cases where no intrinsic money gains are involved. For example, you can be watching a football match where you don't have any intrinsic reason to support either team. In this scenario, Bell (1985: pp.16-17) suggests that *"uncommitted people tend to support the underdog, the team thought less likely to win… Supporting a cause with little chance of success offers only the possibility of elation and eliminates the possibility of sizable disappointment"*.

In Bell (1985) he also presents an extended model, where disappointment and elation are proportional to the ***odds*** against the state occurring, instead of the corresponding probabilities. Thus, winning an amount X at odds 6 to 1 induces double the elation effect as winning the same amount at odds 3 to 1. This model has an effect that is remarkably similar to the Decision Weighting assumption of Prospect Theory: small probability, high gains cause high elation and thus are preferred to large probability, low gains of comparable expected value. This explains the observed risk seeking behaviour associated with low stakes, high gains gambles such as national lotteries. This version of Disappointment model can be presented in a way that isolates a function ***π(p)*** that plays a similar role to that of the Decision Weight function in Prospect Theory. A graph of a typical function ***π(p)*** deduced from a disappointment model is displayed in contrast with Prospect Theory, in Graph 5:

Graph 5



The general results of Disappointment models are similar to those of Regret models, explaining many of the paradoxical behaviours of risky choice that occur when the odds for winning or losing are asymmetric. The theory of Disappointment provides an alternative approach to the Decision Weighting effect of Prospect Theory, which is more intuitively relevant to human psychology - and not as arbitrary. It does not seek to replace Regret as the sole psychological effect of interest to the individual decision-maker, but rather it is a complementary effect. Both Regret and Disappointment can co-exist in many situations, and their combined effects can give an explanation for human behaviour that departs from the von Neumann-Morgenstern assumptions.

4. The Combined Risk Attitude Model

4.1. Overview

The Combined Risk Attitude Model, which is introduced by this paper, is a new descriptive model of utility under uncertainty. It incorporates elements from several of the aforementioned models, in an attempt to extend the scope of alternative utility analysis and modelling. It supports the ongoing process of replacing the traditional Expected Utility framework by more accurate models, which has arguably begun in earnest since the introduction of Prospect Theory and its variants. It also includes psychological effects that involve some sort of trade-off with the standard utility-of-money effect, in a manner similar to the various Regret and Disappointment models that have been recently developed.

A key element of Prospect Theory is the importance that is given to the assumption of overweighting and underweighting of probabilities in human choice. In and by itself, this concept seeks – and actually manages – to explain a wide number of observed phenomena, such as risk-seeking behaviour in the domain of gains and risk-averse behaviour in the domain of losses. The Combined Risk Attitude Model seeks to present this concept of probability weighting, in a way that is more intuitively clear and based on realistic assumptions. A lot of focus is given on the phenomenon of risk attitude reversal – that is, the circumstances under which an individual shifts from risk-averse to risk-seeking, or vice-versa. This phenomenon serves as a guiding line to modelling the effects that, when combined, can provide an accurate representation of utility under uncertainty. The process of combining different independent effects, which serve to shape an individual's risk attitude, lends its name to our model.

4.2. Theory and Analysis of Risk Attitude Reversal

The variability in observed risk attitudes was traditionally explained by classical economists in abstract terms that had little to do with the assumption of utility maximisation commonly associated with risk-free choice. According to Friedman and Savage (1948: pp.280) risk attitudes were justified *"by ignorance of the odds or by the fact that "young men of an adventurous disposition are more attracted by the prospects of a great success than they are deterred by the fear of failure"* [1]*, by "the overweening conceit which the greater part of men have of their own abilities"*[2]*, by "their absurd presumption in their own good fortune"* [3]*, or by some similar deus ex machina."*

According to the Bernoulli and von Neumann-Morgenstern theories of Expected Utility, an individual's attitude towards risk was assumed to be fixed for most levels of wealth – excluding extreme circumstances. The default assumption of diminishing marginal utility of wealth would necessarily imply a risk-averse attitude for individuals with "well-behaved" preferences, something clearly in sharp contrast with the wide evidence of risk-seeking behaviour in the real world. Risk-seeking was thus associated with a paradoxically convex utility-of-wealth function. As already mentioned earlier in this paper, variants of the Expected Utility model by Friedman and Savage (1948) and Markowitz (1952) introduced points of inflection to the utility function, in an attempt to model the reversal of risk attitudes.

[1] Phrase originally quoted from A. Marshall (1920: 554)
[2] Phrase originally quoted from A. Smith (1776: 107)
[3] Phrase originally quoted from A. Smith (1776: 107)

A closer look into the conditions that give rise to the risk attitude reversal phenomenon can provide useful insight for modelling the behaviour of utility under uncertainty. Markowitz (1952) performs an analysis of this sort, using the example of a prospect offering a 10% probability of a gain of X, with a 90% probability of no gain. This prospect is compared with a certain gain of 0.1X, equal to the expected value – in monetary terms – of the uncertain prospect. This expected value will be henceforth referred to as the "***stake***" of the prospect, for simplicity and clarity of argument. Markowitz argues that, for low stake prospects, individuals reveal a risk-seeking preference; they prefer the gamble to its expected value. As the stake becomes higher (in relation to the individual's level of wealth) the preference shifts to risk-averse.

A similar analysis is done in the domain of losses, where a reflection effect is observed. That is, for low stakes of loss, individuals prefer a certain loss to a risky one, but as the stakes increase they tend to favour the risky loss. Since the shape of the utility function is still used as the sole explanation behind risk attitudes, the observations of Markowitz produce the distinctively shaped function found in Graph 2.

An important consideration that is not addressed by the Markowitz analysis is whether – and to what extent – the probabilities of gains and losses in a risky prospect affect risk attitudes. By using an example prospect with a fixed probability of gain or loss of 10%, but varying the amount of the gain/loss, Markowitz avoids having to tackle this problem in his experiment. However, if we seek to get a more complete understanding of the way risk attitudes are formed in risky choice, we have to consider this second effect.

In order to do so, we can explore a similar scenario, keeping the stakes constant but varying the corresponding probabilities. Let's take, for example, a prospect offering a 1% chance of 10000 pounds; would most individuals choose it over a certain gain of 100 pounds? We can derive an affirmative answer if we look at real world evidence from gambling, national lotteries and other situations involving low probabilities of gain. On the contrary, a 50-50 chance of winning 200 pounds is generally not preferred to the certain gain of 100. In the domain of losses, a reflection effect typically occurs; a certain loss of 100 pounds would be generally preferred to a 1% chance of losing 10000 pounds – in fact, people sacrifice more than that in the form of insurance premiums to insure their properties against low-probability perils such as fire and earthquake. But for the same stake of -100 pounds, they might prefer to have a 90% chance of losing 110, so that they can keep a chance of avoiding the loss altogether.

The above results may seem familiar; they are indeed the same results associated with Prospect Theory, and the Disappointment and Regret models. Different rationale is used in each case: the over- and under-weighting of probabilities in the first scenario, and the psychological utility effects of Disappointment/Elation and of Regret/Rejoice respectively in the latter models. Whatever the reason may be for the observed behaviour, the case seems to be that probabilities do affect risk attitudes, especially when we face extreme odds, for or against the gain/loss in question.

We can see a different picture of the effect of varying probabilities in a prospect, in the case where the stakes are very high. Typically, if we had a choice between a certain gain of 10000 and an uncertain gamble with an expected value of 10000, we would always show an aversion to risk; we would choose the certain gain. What if we had a choice between the risky prospects ***A (11111, 0.90)*** and ***B***

(1000000, 0.01)? As stakes increase, it becomes more likely that we go for the more probable gain, rather than risk it all on a wild gamble. Thus, extreme odds can have two opposing effects: inducing risk-seeking for low probability gambles as long as the stakes are modest, but also inducing risk-aversion for high stakes; as always, the reverse applies for losses.

To summarise our analysis of Risk Attitude Reversal, we form a set of assumptions corresponding to the above general effects. We assume a simple risky prospect of ***A (X, p)*** and a certain gain ***C*** = ***Xp***. If we then compare the utility given by ***A***, with the utility given by the certain gain of ***C***, we get a measure of the risk aversion associated with ***A***. Our three assumptions are given below, for both gains and losses:

1. Holding the stake ***Xp*** constant at a low or moderate ***positive*** level, risk-aversion associated with ***A*** increases when we have an increase in ***p*** and a corresponding decrease in ***X***, and vice versa. Holding the stake ***Xp*** constant at a low or moderate ***negative*** level, risk-aversion associated with ***A*** decreases when we have an increase in ***p*** and a corresponding decrease in the absolute value of ***X***, and vice versa.

2. Holding the stake ***Xp*** constant at a high ***positive*** level, risk-aversion associated with ***A*** decreases when we have an increase in ***p*** and a corresponding decrease in ***X***, and vice versa. Holding the stake ***Xp*** constant at a highly ***negative*** level, risk-aversion associated with ***A*** increases when we have an increase in ***p*** and a corresponding decrease in the absolute value of ***X***, and vice versa.

3. Holding ***p*** constant, risk-aversion associated with ***A*** increases as the stake ***Xp*** increases, and vice versa. This effect also applies for negative prospects; as the stake goes down to negative values and decreases, risk aversion decreases.

We assume, of course, that when the measure of risk-aversion becomes negative it is equivalent to a measure of risk-seeking. Assumption 1 leads to the so-called *"fourfold pattern"* of risk attitudes predicted by Prospect theory. That is, risk-averse attitudes are consistent with prospects offering high-probability gains and low-probability losses, and risk-seeking attitudes are consistent with prospects offering low-probability gains and high-probability losses. Assumption 2 provides a counter-argument to the above, presenting a reverse "fourfold pattern" in the special case of high stake prospects. Further, Assumption 3 suggests that higher stakes lead to an overall increase in risk-aversion for gains, and in risk-seeking for losses. These three fundamental assumptions set the basic framework for the Combined Risk Attitude model, discussed in more detail in the following sections.

4.3. The Riskless Utility Function

The riskless utility function measures the utility of monetary outcomes under certainty, and forms the basis of our CRA model. Upon this base value, the additional effects that arise from the presence of uncertainty are accumulated to give a final value of utility for a risky prospect – along with the corresponding risk-averse or risk-seeking attitude. This function plays a similar role compared to the value function found in Prospect Theory, as well as the choiceless utility function found in Regret Theory. As such, it operates on a gains/losses framework, rather than the – arguably obsolete – final wealth framework upon which Expected Utility theory was formulated.

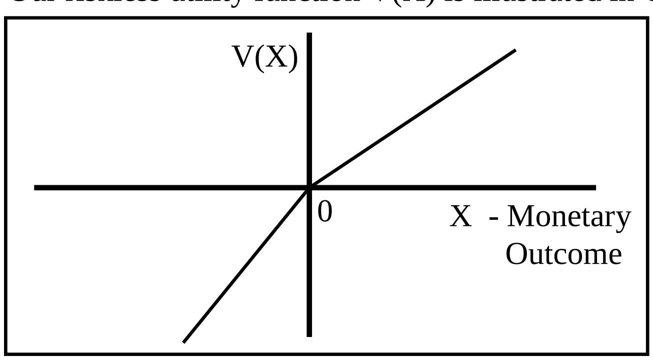
It is obvious that the shape of the riskless utility function should directly reflect the way utility is derived from monetary gains and losses. Prospect Theory presents a value function which is convex for gains and concave for losses – this follows from the assumption of diminishing marginal utility of wealth. The resulting effect is a general bias towards risk-aversion in the domain of gains and risk-seeking in the domain of losses. On the other hand, in Regret and Disappointment models a linear value function is often assumed, in order to give more focus to the respective psychological effects to utility. Given the descriptive nature of our model, two questions need to be asked. Is the utility of monetary gains and losses subject to the economic law of diminishing marginal utility? If so, is that a significant effect on the formation of risk attitudes?

The answer to the first question is rather ambiguous. The wide evidence supporting the law of diminishing marginal utility seems to favour the Prospect Theory treatment of the topic. However, a counter-argument would be that diminishing marginal utility is derived from the concept of satiation; as we consume more of a given good we become partly satiated, thus additional units of the good are less enjoyable. The unique quality of money is that it is not directly consumed, but instead substituted for other goods. This inherent flexibility enables consumers to split their monetary gains into different goods, to avoid satiation. This would serve to mitigate – or eliminate – the diminishing marginal utility effect in the special case of monetary outcomes.

The second question has also been subject to controversy. Diminishing marginal utility of money was originally introduced by classical economists as an explanation to the observed phenomenon of risk-aversion. People are risk-averse, they claimed, because wealth is subject to the law of diminishing marginal utility. However, Rabin (2000) argues against this common assumption. He proves that *"any utility of wealth function that doesn't predict absurdly severe risk aversion over very large stakes predicts negligible risk aversion over moderate stakes"* (Rabin, 2000: pp.1). He concludes that *"aversion to modest-stakes risk has nothing to do with the diminishing marginal utility of wealth"* (Rabin, 2000: pp.4).

Keeping in mind the above, we will use a linear riskless utility function. We will, however, incorporate the concept of loss aversion as given by Prospect Theory – thus, the utility function is steeper for losses than for gains. According to this, economic agents are generally risk-neutral in the absence of additional effects on utility. Our riskless utility function V(X) is illustrated in Graph 6:

<u>Graph 6</u>



4.4. The Long-Shot Effect: Modelling Disappointment, Regret and other psychological aspects

This section will revolve around the analysis and modelling of the effect of probabilities on the utility gained by a risky prospect. In general, it is observed that very low probability gambles, such as a National Lottery, tend to attract people to participate even though the payoffs are obviously actuarially unfair. On the other hand, high probability gambles are generally unattractive; even a 50-50 fair gamble is not something most people would accept for stakes of money of some significance. On the domain of losses, the reflection effect leads to the willingness of most people to insure their assets, at a premium, against low-probability hazards. In this paper, this effect will be referred to as the "Long-Shot" effect, since it operates mostly when the outcome in question is quite improbable.

In Prospect Theory, this effect is explained by the arbitrary assumption of "decision weighting", operating on the probabilities of a prospect. This works quite well to enable the Prospect Theory model to accurately represent observed preferences. For descriptive purposes, however, we will seek an alternative way to justify the Long-Shot effect in risky decision-making. Disappointment and Regret Theory provide a solid basis for our analysis. In most situations where this phenomenon is experienced, the individual is called on to make a choice – for example, whether to participate in a gamble or sign an insurance contract. In such a case, the regret and disappointment effects will operate in tandem to introduce an additional utility effect on the low-probability outcome of a prospect.

For example, in a scenario featuring a 10% probability gain in a gamble, the agent anticipates a significant amount of utility – due to rejoice and elation –

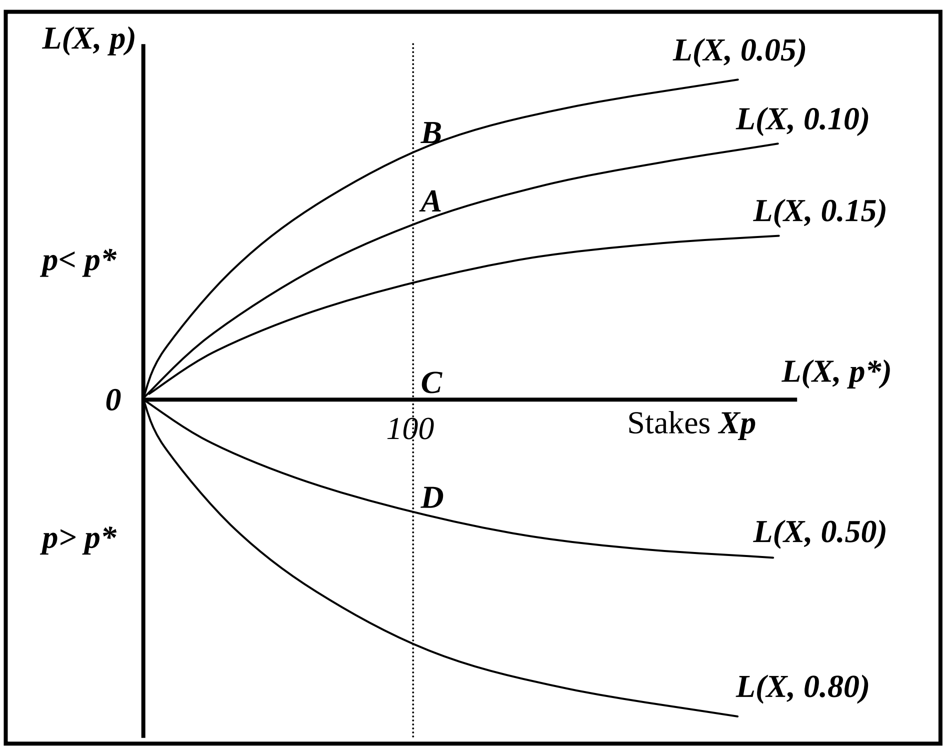
in the case when he wins. If he loses, the utility loss due to disappointment and regret is negligible, given the 90% chance of losing. The same argument operates in the domain of losses; the small probability of your uninsured house destroyed in an earthquake would mean an incredible amount of disappointment and regret if this actually occurs. Other psychological effects, such as a preference for positive skewness in a prospect's outcome distribution, together with an aversion to negative skewness, would be equivalent to the aforementioned tendencies. The anticipation of these effects functions to shift the ex-ante utility of the prospect towards the direction (positive or negative) of the most improbable outcome.

To model the above effect, we introduce what we will call a "Long-shot function" to be added to the riskless utility function, as a component of total utility. This will depend on the probabilities and outcomes of the prospect in question. In this paper, for simplicity, we will mainly focus on simple prospects offering a positive or negative payoff with a given probability, the only other contingency being a zero payoff. Thus, for a prospect $A(X, p)$ we will have a corresponding Long-shot function of $L(X, p)$.

For our modelling purposes, we make an assumption that there exists a level of probability p^*, $0 < p^* < 1$, for which the function $L(X, p^*)$ is always equal to zero. That is, if we face a probability p^* of gaining X, the anticipation of positive and negative psychological effects cancel each other out. Therefore, given our linear riskless utility function, we would be indifferent between the prospect of $A(X, p^*)$ and its certainty equivalent Xp^* for modest stakes (the additional effect that takes place when the stakes become high will be addressed in the following section). In terms of a Prospect Theory Decision Weight function, we would have $\pi(p^*) = p^*$; this is where the Decision Weight graph crosses the 45-degree line. According to a study done by Preston and Baratta (1948), this point corresponds, for most individuals, to an approximate probability level of 25%. Tversky and Kahneman (1992) present a slightly higher probability level, ranging between 30% and 45%.

For a prospect featuring an outcome X with a corresponding probability of ***p < p****, the function ***L(X, p)*** is strictly increasing as ***p*** decreases. When ***p > p****, the function ***L(X, p)*** is negative and decreases further as p increases. The function increases in magnitude as the outcome X increases in absolute value. In accordance with our fundamental assumption **(1)** we must also have that, when keeping the stakes constant but varying the levels of X and p in opposite directions, the probability effect dominates the outcome effect. We sketch an example of this result in Graph 7:

Graph 7



In the above example, the point ***C*** corresponds to a zero level of long-shot utility since it is associated with a probability of p*. For the same stakes of 100, point A shows us the positive effect of utility associated with a prospect offering 1000 with probability 10%. A 5% probability of getting 2000 is even more attractive, as given by point B which is higher than A. At point ***D***, the 50-50 gamble offering 200 is associated with a negative utility effect, as are all prospects with $p > p^*$.

4.5. The High Stakes Effect: Risk Aversion revisited

As we have previously mentioned, our model assumes a linear riskless utility of monetary gains/losses, which is not subject to the law of diminishing marginal utility. We have explained why this assumption is robust for prospects with low and moderate stakes. However, there is considerable evidence that as stakes increase, agents consistently tend to become risk-averse in the domain of gains and risk-seeking in the domain of losses. This is consistent to our fundamental assumption **(3)**; we will include this "High Stakes" effect in our model as an additional factor behind the formation of risk attitudes.

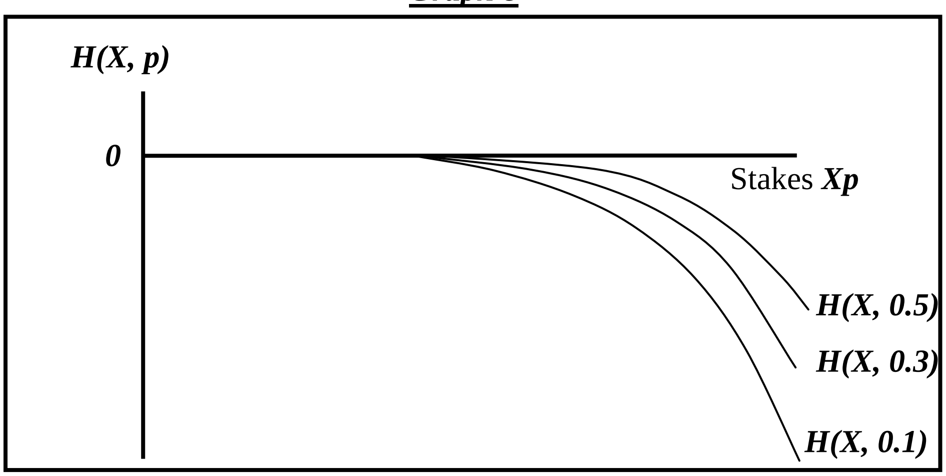
For simplicity, we will set aside the wealth differences of the agents and concentrate on the gains/losses framework that is more relevant for modelling utility of risky prospects. However, in accordance with the analysis of Markowitz (1952), we should keep in mind that the magnitude of stakes is actually relative to the wealth position of each individual. Thus, what is considered to be a very high stake for a poor individual would be only moderate for someone wealthier.

We can explain this effect in several possible ways, but perhaps the most obvious is to reintroduce the classical notion of risk-aversion, related to the law of diminishing marginal utility. When stakes become high we tend to become averse to any uncertainty associated to obtaining the high gain, because most of our important material needs would be satiated if we can secure it. Similarly, we would often go to great lengths to avoid a certain loss, when that loss is so high that it would mean a significant decline in our living standards. Therefore, after the stakes exceed a certain threshold, the high stakes effect kicks in, biasing our decision-making towards ensuring great gains and avoiding great losses.

An important point to examine is whether there is any interaction between the high stakes effect and the long-shot effect. This would be necessary for high-stake prospects, according to our assumption **(2)**. Recall our example with the risky prospects ***A (11111, 0.90)*** and ***B (1000000, 0.01)***. The long-shot effect would produce a bias favouring the risky prospect B. However, in reality, it is more likely that we would prefer the less risky prospect A. Thus, it follows that the high stakes effect can work to mitigate and even dominate the long-shot effect, when the stakes become very high. This is a result which the Prospect Theory formulation fails to address, as it was mostly based on empirical evidence from choices made between low to moderate stake prospects.

We continue our attempt at modelling utility by introducing the High Stakes function ***H(X, p)***, which has a weakly negative value for positive prospects and a weakly positive value for negative prospects. Thus, it gives a bias towards risk aversion for gains and risk seeking for losses – this is an effect similar to that given by a non-linear value function as in Prospect Theory, with the difference that it only comes into effect as the stakes increase. Graph 8 demonstrates the function ***H(X, p)*** for different levels of probability. We can see that as probabilities become smaller, the negative utility effect of a high stake prospect becomes more pronounced.

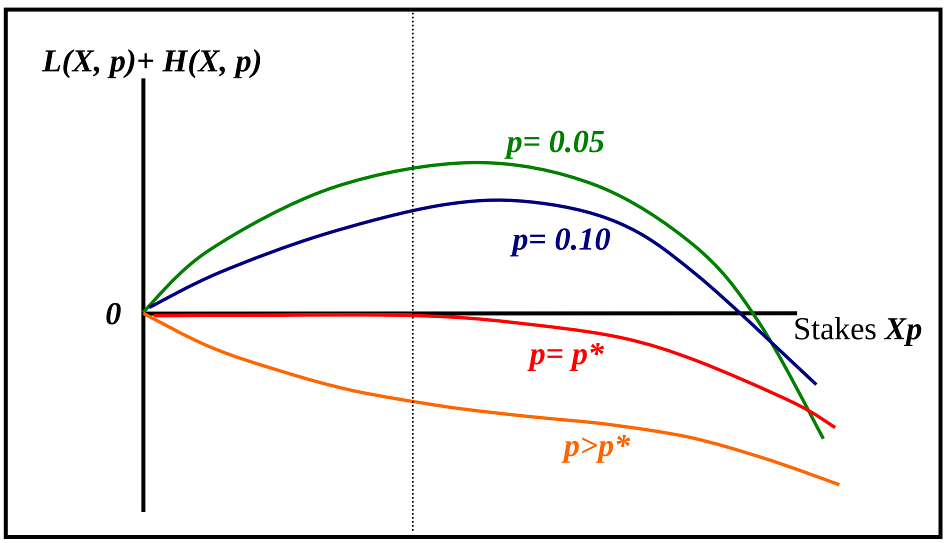
Graph 8



4.6. Combined Risk Attitude

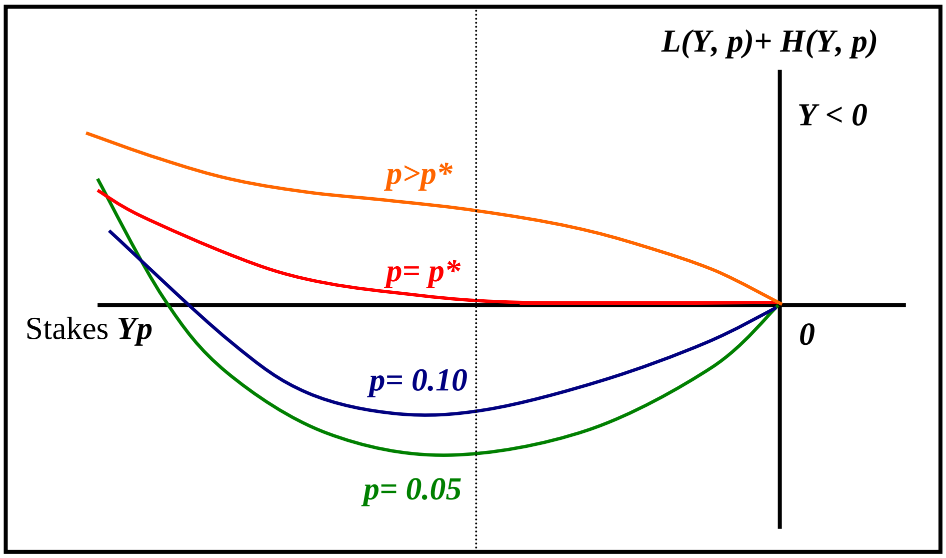
The previous two sections have analysed the two main effects that combine to produce an agent's attitude to risk. Plotting the two additional effects against the expected value of our prospect (i.e. the stake ***Xp***) directly gives us the direction and magnitude of the risk attitude associated with a given prospect. A positive value signifies that the risky prospect offers additional utility from these effects compared to its certainty equivalent, thus being consistent with risk-seeking behaviour. A negative value is similarly associated with risk-aversion. The sum of the functions ***L (X, p)*** and ***H (X, p)*** is given in Graph 9, for a prospect ***A (X, p)*** of positive nature and for various probability levels.

Graph 9



In the domain of losses, we have a mirror image of the above picture, with the effects being opposite in sign - consistent with the reflection effect. That is, ***L(X, p)*** is negative for low probabilities, producing risk aversion, and positive for high probabilities, producing risk seeking. ***H(X, p)*** is weakly positive and increasing as stakes become high in absolute value. Graph 10 illustrates these functions:

Graph 10



To put everything together, we add the above effects to our riskless utility function. Thus, for a prospect ***A(X, p)*** we have:

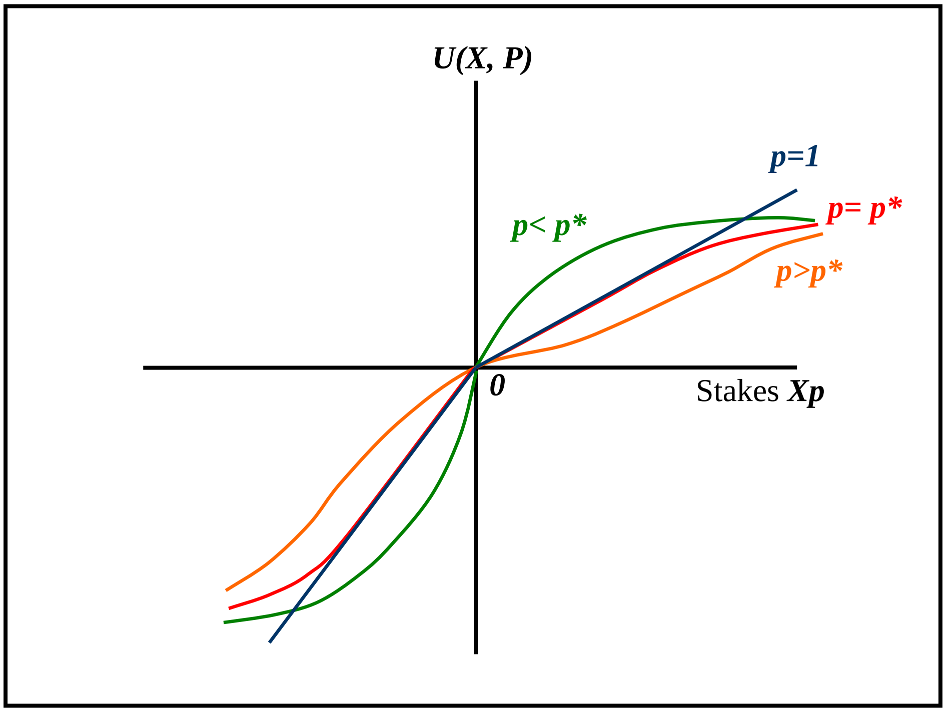
$$U(X, p) = pV(X) + L(X, p) + H(X, p)$$

Given the linear nature of ***V(X)***, this is equivalent to:

$$U(X, p) = V(pX) + L(X, p) + H(X, p)$$

Therefore we can graph both the risky prospects and their certainty equivalents on the same graph, with stakes on the X-Axis and total utility on the Y-Axis. The vertical difference between the riskless utility function and the total utility function for a given stake shows the degree of risk aversion or risk seeking associated with the corresponding prospect. An example is given in Graph 11 for negative and positive prospects, and different levels of probability:

Graph 11



The above diagram shows the formation of risk attitudes for different prospects as probabilities and stakes change. For low probability, low/moderate stakes we have a strong long-shot effect and a negligible high stakes effect. This translates to a risk-seeking attitude for gains and risk-aversion for losses. It is no coincidence that in this category we find most situations where we observe gambling and insurance contracts undertaken at an actuarially unfair cost. For high probability prospects we have the opposite attitudes; risk-aversion for gains and risk-seeking for losses. As stakes increase, the high stakes effect dominates and we find that low-probability gains become less attractive, while low-probability losses become more attractive, in comparison with more probable prospects of the same expected value.

So far, we have focused our analysis on examining simple prospects of the form ***A(X, p).*** The introduction of an editing phase, where prospects are simplified by decision makers before utilities are calculated, can allow our model to be applied to an extended range of situations. As in Prospect Theory, one operation that takes place in the editing phase is the breaking down of prospects into risky and riskless components. Thus if we have a prospect ***A(X, p; Y, 1-p), X>Y,*** this would be broken down to the certain gain ***Y*** and the risky prospect ***A(X-Y, p)****;* the utility associated with the latter can be analysed according to our model. The case of mixed prospects – including both negative and positive outcomes – is more complicated, but can be treated in a similar way at the cost of some accuracy, if we shift the reference point towards one of the two directions.

5. Applications of the Model

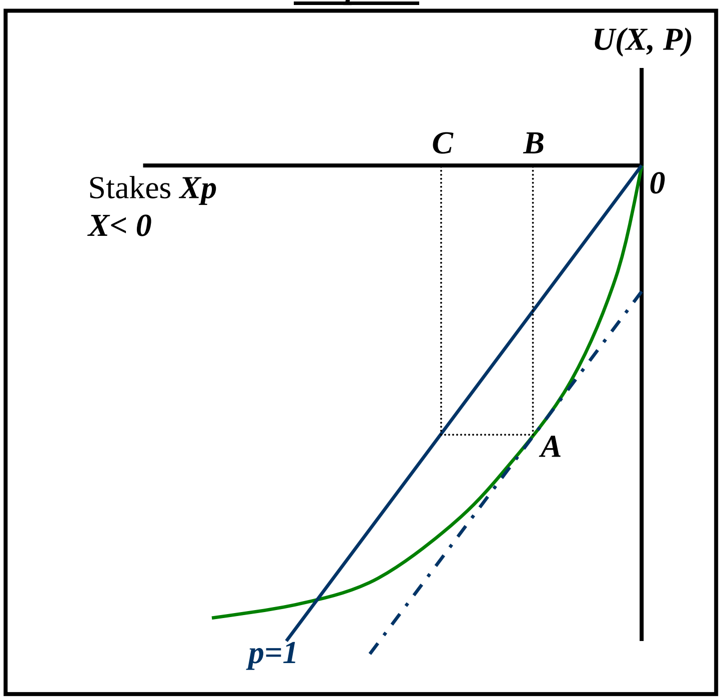
5.1. General Insurance

Insurance has always been associated with decision-making under uncertainty, as it provided evidence for the risk-averse behaviour predicted by the traditional Expected Utility model. According to Expected Utility, a risk-averse individual would always pursue full insurance against any risk, of any probability or magnitude, as long as the premium is actuarially fair. The phenomenon of underinsurance is explained by the fact that premiums tend to be higher than their fair level, to cover the costs and profit margins of insurance companies. Our model offers an alternative view of this issue.

The maximum level of premium that an insurance company can charge is determined by the amount of risk-aversion of the individual policyholder, associated with the hazard he faces. In our model, risk-aversion varies with the probability and magnitude of the possible loss. For a given probability level, the level of risk-aversion increases, then decreases, as the magnitude of the loss increases. Our model predicts that the highest risk-aversion is found in risks with very low probability of loss, as long as the stakes involved are not too high. In order to maximise their profit margin, insurers can focus on particular types of policies offering cover against hazards with probability levels and magnitudes that are consistent with very high risk-aversion.

In Graph 12, we can see that the point ***A*** that maximises risk-aversion is found where the gradient of the Total Utility function is equal to that of the linear Riskless Utility function. The actuarially fair premium for a corresponding insurance policy would be the expected loss ***B***. However, the policyholder would be willing to pay a maximum of ***C*** to insure himself against this loss.

Graph 12



Another result of this model, which is also shared with Prospect Theory and other alternative utility models, is the risk attitude reversal towards risk-seeking for high-probability prospects. In the case of insurance, this means that there are only particular types of risks that are insurable by a risk-neutral insurance company. Specifically, these risks must be associated with a loss probability less than ***p****; for those low probabilities of loss, our model predicts risk-aversion for low and moderate stakes. On the contrary, high probability risks are associated with risk-seeking, and thus individuals would not be willing to insure for a fair premium.

A common, empirically observed phenomenon in insurance is the prevalence of underinsurance policies. This is not inconsistent with our model. For a low-probability, insurable risk, as stakes become high our model predicts a reduction in risk-aversion. Once this risk-aversion becomes too low to compensate for the actuarially unfair premiums charged by the insurance company, an individual would prefer an underinsurance policy, for a lower premium that is consistent with his level of risk-aversion. The same effect also means that insurers can limit the amount of coverage they offer – especially for insurable risks that have a relatively high probability of loss – in order to increase their profit margins. On the other hand, it has to be taken into account that the underwriting and administrative costs of the insurer are mainly based on the number of policies underwritten and not their type – this would need to be balanced against the higher premium chargeable for limited coverage policies.

5.2. Further Applications

In the previous section we have mentioned the case of insurance. A similar, yet opposite in nature, application would be the case of gambling and lottery construction. The reflection effect predicts that what applies in the domain of losses is mirrored in reverse in the domain of gains. Casinos, sports-betting bookers and other gambling institutions can increase their profits by taking into consideration the conditions under which individuals express risk-seeking attitudes. Additionally, this can be applied to marketing strategies, where lottery-type contests are offered to promote the sales of a given product. Since gambling institutions usually have a direct control over the probabilities and outcomes that they offer, they can customise them according to the utility functions of a representative gambler. In that case, our model would predict an enhanced long-shot effect due to a higher elation/rejoice factor from extreme odds. This would lead to higher risk-seeking for low-probability gambles over moderate stakes, as well as a higher level of $\boldsymbol{p^*}$.

An important result found in our model, as well as the analysis of Markowitz, is the sensitivity of risk-attitudes of individuals to the magnitude of stakes in relation to their level of wealth. Insurance, gambling and other services are aimed at offsetting individuals' risk-seeking for gains and risk-aversion for losses, when it occurs. These attitudes are influenced heavily by the individual's wealth position. What is a high-stake prospect for an average individual would be moderate-stake for a wealthy one – he would thus experience the desired risk attitude. Our model predicts a possible opportunity for taking advantage of this factor. This would, in practice, correspond to the case of "V.I.P" services such as "Executive" insurance policies and "Elite" casinos, where high stake risky prospects are essentially traded at a premium. The "status symbol" factor involved with the exclusive nature of such institutions would make them additionally attractive to the target types of individuals.

Our model can also provide some insight to the area of Finance and Portfolio Management. So far, most financial models work on the basis of mean and variance analysis. High mean returns are generally desirable, while high variance is considered undesirable. The utility framework of our model suggests that this is not sufficient for a complete evaluation of an uncertain future gain. Specifically, higher moments – and especially the skewness – of the distribution of gains from different financial assets and investment opportunities should also be taken into account in order to maximise stakeholders' utility. Although our model does not address the type of complicated risky prospects found in finance, it does predict that positive skewness is generally desirable for low and moderate stake prospects – corresponding to the positive long-shot effect on utility. Additionally, the gains/losses framework of our model – shared with many other alternative models of utility – could apply in Finance by using the risk-free rate of interest as the "reference point" between the domains of gains and losses.

5.3. Comparison with other Models of Utility

The Combined Risk Attitude model has been formulated on the basis of similar assumptions as Prospect Theory and other non-expected utility models. It seeks to explain the same observed behaviours in risky decision-making. Thus, the important question is: what new does this model have to offer to the subject of modelling utility under uncertainty? How does it behave in comparison to other models, and what are its strengths and limitations?

A main feature of our model is that it places most emphasis on the comparison between same-stake prospects. Since all the main effects of our model are graphically presented with the stakes on the X-Axis, we can directly see how prospects with the same expected value offer different values of utility depending on the probability and levels of gain or loss they offer. This comes at the cost of making comparisons between different-stake prospects less clear. However, it can be argued that, under non-expected utility modelling, the latter type of comparison is not as significant. For example, a 10% probability of winning X is qualitatively different than a 60% probability of winning X; according to our model, the effects on the utility function are different in such a way that a direct comparison would not be particularly informative.

Another important focus of our model is found on the analysis of risk attitudes and the circumstances under which they can change. In Expected Utility risk attitudes are fixed by the shape of the utility functions – a treatment that is too rigid for the purposes of an analysis of this phenomenon. In Prospect Theory, Regret and Disappointment models, we can see risk attitudes changing as we move from the domain of gains to the domain of losses, as well as when we are dealing with extreme probabilities. Our model performs the same kind of analysis, but it further combines it with a treatment of the way different stakes can also affect risk attitudes – as featured in the model of Markowitz. Combining these different effects in our model shows how they can work together or in opposite directions, to form an individual's attitude to risk.

Our model takes a similar route as Regret and Disappointment models in the way it introduces separable effects, which are added on a basic utility function – in our case, the "Riskless" utility. On the other hand, the Decision Weighting effect of Prospect Theory and other models featuring subjective probabilities operate by altering the weight carried by the utility of different outcomes in decision making. It is yet unclear which treatment of non-standard utility effects is the most appropriate; perhaps further research might prove that the effects introduced by this model would fit observed choices better, if they are modelled in an alternative way.

Furthermore, the model introduces another departure from Expected Utility – risk attitudes are no longer produced by a convex or concave shaped value function, as is usually the case in traditional utility analysis. Although some other models – such as Bell's model of Disappointment – include linear riskless utility functions, this is mainly done for the purpose of simplicity. On the contrary, our model follows the arguments of Rabin (2000) and produces risk attitudes that are separable from the way utility operates under certainty. Once again, the extent of the validity of this assumption can only be proven by empirical research which is beyond the objectives of this paper.

5.4. Scope for Research and Improvement

The subject of utility and decision-making under conditions of risk and uncertainty is still at an early phase of development. Most models either provide a limited analysis from a specific point of view, or offer a generic overview of the issue. Our model is not an exception; although this paper has built the framework and basic foundations of an alternative theory and treatment of utility under uncertainty, there is still plenty of scope for research, improvement and refinement.

The obvious thing our model can make use of is a functional form for the different functions and effects on utility. This would require empirical studies done by a survey or otherwise, to compare risk attitudes for simple prospects as probabilities, outcomes and stakes vary. This would also enable us to specify the parameters which determine individual differences in risk attitudes – and estimate their approximate values. Such an analysis would shed more light on the circumstances under which our model can provide more robust and accurate results than other models of utility under uncertainty.

Another possible extension of our model would be to apply similar concepts and framework of analysis to the case where prospects are continuous, offering a probability distribution on a range of outcomes. Such an improvement would significantly increase the sophistication of our model and its ability to represent a wide range of observed behaviours and attitudes. The introduction of Cumulative Prospect Theory was a major step towards making Prospect Theory the predominant alternative to Expected Utility Theory; our model cannot possibly compete unless a similar extension is developed.

Finally, a major step towards increasing the robustness of our model would be to empirically examine and analyse whether some of its assumptions can be relaxed or modified to represent the desired effects on utility more accurately. For example, we can examine the behaviour of ***p**** - the probability of a prospect which corresponds to risk-neutral attitudes for low to moderate stakes. We can see whether it is constant for a given individual and how it varies between different people. Additionally, we can see whether there is sufficient interaction between the different effects on utility; if so, their treatment as separable and additive in our formulation could be altered in some way to represent that interaction.

6. Conclusion

This paper has undertaken the analysis of what is arguably the most abstract of topics in economics and related disciplines – utility under uncertainty. As such, to a considerable extent we may find that some of our model's assumptions and predictions are rather arbitrary in a way. This is not a novelty; in this area of research, most models share this common drawback, whether it is Prospect Theory with its non-linear decision weighting of probability and its multitude of framing effects, or Regret Theory with its focus on how choice itself can alter the utility effects of the action that is chosen.

For the purposes of this paper, our model has maintained an adequate amount of simplicity and clarity of presentation by avoiding rigorous mathematical analysis and instead focusing on graphical representation of the direction and relative magnitude of its effects and results. This was done both due to the inherent complexity in modelling non-linear functions and the lack of appropriate data from which we would derive an accurate functional form for our equations.

In terms of context, we share a lot of elements found in Prospect Theory and other alternative utility models. The reflection effect, the loss aversion effect, the four-fold pattern of risk attitudes and the initial editing phase – all are included in our framework of analysis. We replace the Decision Weighting concept of Prospect theory with a non-linear additive Long-shot effect on utility, representing an accumulation of Regret, Disappointment and other psychological factors through which extreme odds may affect risk attitudes. We also include a treatment of how varying stakes affect this picture; we adopt the view of Markowitz in deriving a High Stakes effect on utility. This amalgamation of the most realistic assumptions and concepts found in different alternative models of utility serves to increase the descriptive power of our model and make it less arbitrary and more intuitively attractive than each of the other models it is based upon.

Throughout this paper, we keep in mind that our model is merely an approximation of reality; it only claims that its predictions are sufficiently robust to represent the way utility is formed and decisions are made under uncertainty. The Combined Risk Attitude model is not here to replace the existing models of utility; rather, it builds upon the foundations set by these models, and offers a view of the topic from a slightly different angle. As such, it is appropriate for complementing the existing models while arguably being more suitable for specific types of analysis – especially where the variable of interest is the magnitude of risk-aversion or risk-seeking associated with a given prospect. Expanding the model and establishing a functional form for its effects, through extensive empirical research on the formation of risk attitudes, can further enhance both its descriptive and prescriptive power, and make it more accurate and robust.

7. Bibliography

Allais, M. (1953): “Le Comportement de l’Homme Rationel devant le Risque, Critique des Postulates et Axiomes de l’Ecole Americaine”, *Econometrica* **21:** 503-546.

Bell, D.E. (1982): “Regret in Decision Making Under Uncertainty”, *Operations Research* **30(5):** 961-981.

Bell, D.E. (1985): “Disappointment in Decision Making Under Uncertainty”, *Operations Research* **33(1):** 1-27.

Bernoulli, D. (1738): “Specimen Theoriae Novae de Mensura Sortis.” *Commentarri Academiae Scientiarum Imperialis Petropolitanae,* **Tomus V:** 175-192. Translated by L. Sommer (1954) as “Exposition of a New Theory on the Measurement of Risk.” *Econometrica* **22:** 23-36.

Bleichrodt, H., J.L. Pinto and P.P. Wakker (2001): “Making Descriptive Use of Prospect Theory to Improve the Prescriptive Use of Expected Utility”, *Management Science* **45(11):** 1498-1514.

Braun, M. and A. Muermann (2003): “The Impact of Regret on the Demand for Insurance”, *Journal of Risk and Insurance* **71(4):** 737-767.

Coombs, C.H. (1975): “Portfolio Theory and the Measurement of Risk”, in *Human judgment and decision processes, pp.63-86.* Edited by M.F. Kaplan and S. Schwartz. Academic Press, New York.

Ellsberg, D. (1961): “Risk, Ambiguity and the Savage Axioms”, *Quarterly Journal of Economics* **75:** 643-669.

Friedman, M. and L.J. Savage (1948): “The Utility Analysis of Choices Involving Risk”, *Journal of Political Economy* **56:** 279-304.

Handa, J. (1977): “Risk, Probabilities, and a New Theory of Cardinal Utility”, *Journal of Political Economy* **85(1):** 97-122.

Harbaugh, R. (2003): “Skill signaling, prospect theory, and regret theory”, *Claremont Colleges Working Papers* **2002-03**, Claremont Colleges.

Harbaugh, W.T, K. Krause and L. Vesterlund (2002): “Prospect Theory in Choice and Pricing Tasks”, *University of Oregon Economics Department Working Papers* **2002-02**.

Hershey, J.C. and P.J.H. Schoemaker (1980): “Risk-Taking and Problem Context in the Domain of Losses – An Expected Utility Analysis”, *Journal of Risk and Insurance* **47(1):** 111-132.

Jia, J., J.S. Dyer and J.C. Butler (1997): “Generalized Disappointment Models”, *Journal of Risk and Uncertainty* **22(1):** 59-78.

Kahneman, D. and A. Tversky (1979): “Prospect Theory: An Analysis of Decision under Risk”, *Econometrica* **47:** 263-291.

Karmarkar, U.S. (1978): “Subjectively Weighted Utility: A Descriptive Extension of the Expected Utility Model”, *Organizational Behavior and Human Performance* **21:** 61-72.

Loomes, G. and R. Sugden (1982): “Regret Theory: An Alternative Theory of Rational Choice Under Uncertainty”, *Economic Journal* **92:** 805-824.

Loomes, G. and R. Sugden (1983): “A Rationale for Preference Reversal”, *The American Economic Review* **73(3):** 428-432.

Loomes, G. and R. Sugden (1986): “Disappointment and Dynamic Consistency in Choice under Uncertainty”, *Review of Economic Studies* **53:** 271-282.

Machina, M.J. (1982): “Expected Utility Analysis without the Independence Axiom”, *Econometrica* **50(2):** 277-324.

Machina, M.J. (1987): “Choice Under Uncertainty: Problems Solved and Unsolved”, *Journal of Economic Perspective* **1:** 121-154.

Markowitz, H. (1952): “The Utility of Wealth”, *Journal of Political Economy* **60:** 151-158.

Marshall, A. (1920): *Principles of Economics.* 8th Edition. Macmillan & Co. Ltd, London.

Preston, M.G. and P. Baratta (1948): “An Experimental Study of the Auction-Value of an Uncertain Outcome”, *American Journal of Psychology* **61:** 183-193.

Rabin, M. (2000): “Diminishing Marginal Utility of Wealth Cannot Explain Risk Aversion”, *Economics Department, University of California, Berkeley, Working Paper* **E00-287**.

Schoemaker, P.J.H. (1982): "The Expected Utility Model: its Variants, Purposes, Evidence and Limitations", *Journal of Economic Literature* **20(2):** 529-563.

Smith, A. (1776): *An Inquiry into the Nature and Causes of the Wealth of Nations.* Fifth edition. Edited by E. Cannan (1904), Methuen and Co. Ltd, London.

Sykes, I. (2004): "Application of Behavioural Finance", *Finance and Investment Conference, June 2004, Brussels*.

Taylor, N. (2000): "Making actuaries less human: Lessons from behavioural finance", *SIAS Working Paper.*

Tversky, A. and D. Kahneman (1981): "The Framing of Decisions and the Psychology of Choice", *Science* **211:** 453-458.

Tversky, A. and D. Kahneman (1992): "Advances in Prospect Theory: Cumulative Representation of Uncertainty" *Journal of Risk and Uncertainty* **5:** 297-323.

Varian, H.R. (2002): *Intermediate Microeconomics: A Modern Approach.* 6th edition. W.W. Norton & Company.

von Neumann, J. and O. Morgenstern (1947): *Theory of games and economic behaviour.* Second edition. Princeton University Press, Princeton, NJ.

Wakker, P.P., R.H. Thaler and A. Tversky (1997): "Probabilistic Insurance", *Journal of Risk and Uncertainty* **15:** 7-28.