DFA Models as a tool for solvency assessment

An application to Portuguese general insurance companies

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Abstract

This dissertation looks at *Dynamic Financial Analysis* (DFA) as a capital modelling methodology that can be very useful for regulators to assess the solvency position of general insurance companies, including the estimation of their capital requirements for a given degree of risk. Moreover, DFA models are regarded as providing a suitable framework for the risk-based capital standards and other more sophisticated approaches to regulation currently in active discussion under the EU Solvency II Project.

To illustrate its capabilities, a basic standardised DFA model is constructed and applied to the general insurance companies in Portugal. The financial results of the companies are projected considering different time horizons, allowing for the calculation of risk measures such as the ruin probability and the expected policyholder deficit. Finally, the impact of the change from the current EU factor-based solvency rules to a more risk-based approach is computed, anticipating a scenario of significant increase of the regulatory capital.

Acknowledgements

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1 Introduction

General insurance companies are vital for the development of the economy. They provide protection for everyday activity of individuals and companies worldwide. Without this protection most of the businesses could not be sustained, jeopardising the pace of human social and economic progress.

However, insurance is a risky business and its benefits are only possible if the providers do not default on their obligations. Due to its importance, it is vital that the policyholders feel reasonably confident that their insurance contracts will be honoured, otherwise they will not be willing to buy them.

These ideas are some of the main reasons to justify the presence of regulation on the insurance market. Besides many other complementary prudential measures, the fear of insolvency is dealt with by the regulator by setting minimum capital requirements for all insurance companies.

The European Union (EU) current solvency system is based on the following three pillars:

- Adequate and prudent technical provisions;
- Sufficient assets matching the technical provisions;
- Solvency margin, represented by a minimum amount of free funds, functioning as a cushion to allow for the uncertainty of the business.

In particular, the solvency margin is determined in an equivalent way for all general insurance companies, through a factor-based approach. The calculation method, although simple, has been criticized for not taking into account the risk characteristics of each individual company. Roughly speaking, the minimum capital is determined by applying fixed ratios to the amount of premiums or to the amount of claims incurred by the company. No differentiation is made for the riskiness of the insurance portfolio, the investment strategy of the assets supporting the liabilities, the reinsurance program and other risk management policies in place.

In response to this criticism, the European Commission is currently developing new risk-based capital standards to assess the overall solvency of insurance undertakings, under the so-called *Solvency II* project.

Many ideas on how to measure more efficiently the risk undertaken by a general insurance company have been proposed in the last years. For instance, some European countries, namely the UK, the Netherlands, Switzerland and Germany, have already implemented on their regulation activity[1] more sophisticated solvency measurement approaches.

A new technique that has received increasing attention in the last years is Dynamic Financial Analysis (DFA). The objective of a DFA model is to help the actuary on his task of forecasting the financial condition of a company, by considering the broad spectrum of the company's balance sheet. Basically, its purpose is to project the balance sheet and the operating statement of the insurer over a specified planning horizon. The main variables affecting the future financial position of a company are identified and generally treated as stochastic random variables. Both

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[1] Switzerland is still on the phase of development of the model.

the asset and liabilities are integrated in the model and the relationship between all variables is a very important input.

According to D'Arcy *et al* (1997), DFA can be defined as "the process of examining the entire financial position of an insurance company over time, considering both the interrelations among the various parts and the stochastic nature of the factors that can affect the results."

An adequate DFA model can be tailored to reflect the risk management and investment policies of a particular company, providing insight into the likelihood of future financial outcomes and allowing for the measurement of the impact of the change of some of those policies. The results generally consider a broad range of scenarios, from the most predictable to the more infrequent and even the catastrophic ones.

Therefore, a DFA model can be used by the regulators as a tool for solvency assessment of insurance and other financial companies, including the estimation of their capital requirements. Ideally, a DFA model should be tailored to reflect the experience and policies of each individual company. However, this task is likely to be hard for the regulators and some insurance companies, in particular the smaller and more recent ones. This is due to restrictions on the availability, quantity and/or quality of data, since the DFA model needs to be developed and calibrated through analysis of past data.

In particular, the regulator should be interested in a standardised model, i.e. a unique model which fits reasonably well all the supervised companies. A DFA model can still be considered; however it will be difficult to incorporate reinsurance and other risk management strategies that tend to be substantially different from company to company. A common solution to this problem is to consider a relaxed standardised approach that gives fairly conservative results for all companies, allowing and, in fact, encouraging companies to build suitable internal models with more accurate and expectably lower results.

The objective of the present study is to develop a fairly basic DFA model that can be used as a standardised model for the Portuguese general insurance companies. This model will then be used for the fulfilment of the second objective of measurement of the likely impact that the EU adoption of more risk-based orientated models should have on the capital requirements of Portuguese general insurance companies, relative to the current EU solvency system.

The data used was kindly provided by the Portuguese insurance regulator (ISP) and includes only the annual mandatory information that all companies must provide, namely the balance sheet and the profit & loss accounts per line of business. Section two will give a more detailed description of DFA models, including the presentation of the model used in this study. Section three will detail the results achieved for a chosen company, while Section four will extend the results to all general insurance companies operating in the Portuguese market. Section five will draw the conclusions, with a discussion of the merits and weaknesses of such a model and areas for future improvement.

2 DFA Models

The focus on DFA models started as a response to the increasing number of insolvencies verified on life insurers, due to unanticipated changes on the volatility of long term bond yields, one of the main investment categories of these companies. The repeated large interest rate increases on the late 70s changed the pattern of low and offsetting yield volatilities of the previous decades and prompted the companies, rating agencies and regulators to look more closely to the measurement of financial risk, in particular to the impact that differing economic conditions will have on surplus. The use of the traditional deterministic methodologies to get simple point estimates proved not to be adequate for this purpose. A few years later, these worries were gradually extended to the property-liability insurers.

2.1 Types of DFA Analysis

A DFA analysis can be performed using two approaches: scenario testing and stochastic simulation. The former method has been widely used by actuaries in the past, and involves the determination of the financial position of the company considering a number of selected potential scenarios (stress tests). The main scenarios that an actuary is interested are the generally infrequent ones that can put the company into serious financial distress. For instance, shocks in financial, insurance, demographic and other variables – sharp variations from the expectations on interest rates, lapse rates, mortality rates, loss frequency and severity, etc. However, a big drawback of the scenario testing method is that the likelihood or probability of the pre-selected scenarios is not estimated.

On the other hand, stochastic simulation is grounded on a theoretical framework where all the main variables identified as affecting the financial health of the company are treated as random variables with a suitable probability distribution, the parameters being estimated through analysis of the relevant past data. When appropriate, it is important that correlations between variables are incorporated in the model since treating all variables as independent can lead to underestimation of the potential losses. Simulation is then carried out, with the automatic generation of a substantially high number of scenarios, dependent on the probability distribution assumptions made on the variables, and the correspondent determination of the financial impact to the company. This method allows for the measurement of the likelihood of each type of scenario giving an important advantage over the scenario testing technique. For this reason, stochastic simulation is the approach more commonly used, especially with today's available computational power.

2.2   Insurance Risks

Part important of the process of developing a DFA model is to identify the variables that should be included as well as suitable probability distributions that describe them and the relationships between all variables. But, before this task, it is important to understand the risks that general insurance companies face throughout their business. The International Actuarial Association (2004) categorized the insurance risks into the following major headings:

- Underwriting
- Credit
- Market
- Operational
- Liquidity

Underwriting risks include the risks arising from the perils covered by the insurance contracts – for instance, fire, flood, death, motor accident, personal accident, earthquake, etc. Risks due to the poor management of the processes associated with the conduction of the business are also included like, for example: (1) holding a poor insurance portfolio, i.e. acceptance of contracts which give rise to abnormal number or amount of unanticipated claims, (2) mispricing of contracts, (3) poor design of contracts, (4) claim frequency and/or severity higher than expected, (5) poor reserving policy, responsible for reserves being insufficient to cover the liabilities, (6) unexpected change of the economical or social environment or new legislation with an adverse impact on the business, etc.

Credit risk is associated with the default or downgrade of the credit quality of other parties which have in some way an impact on the insurance company's results. For instance, a default or downgrade of credit rating of an issuer of securities can lower or even set to zero the value of some of the insurance company's investments and ultimately endanger the payment of some of the insurance liabilities. Also, an insurer can end up having to pay the gross amount of liabilities if the reinsurance company to whom it had transferred part of the liabilities defaults in its obligations. In the same way, the default or downgrade of credit risk of counterparties to derivative contracts, deposits given and other financial contracts can adversely change the value of those contracts or deposits or even make them worthless.

Market risk is associated with the level and volatility of the market price of assets. The investment results can make a significant part of the results of a general insurance company, especially for long tailed lines of business. Therefore, it is essential to measure the likelihood of adverse fluctuations of asset prices, considering all the major asset categories the company invests in – equities, bonds, cash, and property. Financial variables such as interest rates, inflation, stock prices and exchange rates affect the asset values and so they should be taken into consideration on the construction of a DFA model.

Operational risk was initially defined in complementary terms, as including all non-underwriting risks other than market and credit risk, both with internal and external sources. The British Banker's Association defined it more precisely as "the risk of loss resulting from inadequate or failed internal processes, people, systems or from external events". This is a difficult risk to

measure due to the lack of data and collected experience but it is believed that it should amount to a significant volume in terms of capital requirements since it has been recognized that management shortfalls itself led to many EU insurer failures.[2]

Liquidity risk arises whenever the company does not have sufficient liquid assets, from among the investments supporting the liabilities, to meet the cash flow requirements arising throughout the time. This can happen because the company concentrated too much investments on illiquid assets, for instance property, or because the cash flows arising from the liabilities were higher than expected, for example due to a higher number of claims after a catastrophe. Whenever liquidity risk strikes, losses are generally incurred because the company is forced to sell illiquid assets in a short period and maybe during a bear market.

2.3   Construction of a DFA Model

The main objective of the DFA model is to forecast the financial position of a general insurance company. Several variables or factors will have to be incorporated into the model, reflecting the major sources of risk that are likely to affect the results of the company. However, not all risks can or should be modelled. Important considerations are the objective of the model and whether the factors are quantifiable. For instance, several types of operating risk are difficult to estimate, like, for instance, embezzlement by the company's own management. It may be more reasonable to leave some risks out of the model, to avoid spurious conclusions, and deal with them mainly on qualitative terms.

A model should be thought of as a simplified version of reality. It will never reflect reality in perfection because it is impossible to identify and explain all variables occurring in the real world. Far from being a 'crystal ball', a model is designed to provide some sort of measure of the likelihood, the inherent costs of future events and the impact of changes in actual conditions. Thus, instead of trying to incorporate all minor factors in a DFA model it is better to keep it as simple as possible, recognizing only the most relevant factors.

Considering the risks identified in the previous section (except operational risk), the following table gives some examples of factors that may be appropriate to include in a model designed for a general insurance company:

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[2] According to the so-called Sharma Report (2002) commissioned by the EU Insurance Supervisors.

Figure 2-1 – Examples of variables to include in a DFA model for a general insurance company

Underwriting	Credit	Market	Liquidity
Exposure	Securities issuers' credit rating	Inflation	Claims payment patterns
Loss frequency	Reinsurers' credit rating	Short term interest rates	Assets income
Loss severity	Assets credit risk premium	Yield curve	
Expenses		Equity risk premium	
Payment patterns		Dividend yield	
Reinsurance		Property rental yield	
		Property rental income	

The next step is to decide which factors should be treated stochastically, i.e. should be allowed to vary, and which should be treated deterministically. Again, it is recommended that only the most relevant factors are represented by random variables, to make the model easier to implement and understand. Additionally, it is important that reasonable and adequate probability distributions are assumed for the stochastic variables, in an effort to reflect the reality as close as possible.

The strength of a stochastic model is the fact that it allows for a covariance structure between the variables to be built within the model. Several of the variables considered in the model are likely to be interrelated. For instance, inflation affects the behaviour of interest rates as well as the yields on various asset classes. On the liability side, higher inflation can lead to higher claim amounts. Other example is the case of a catastrophe where the correlation between the losses frequency of two or more different lines of business may be close to one, changing a possible independence assumption held during normal conditions.

The existence of correlations between economic and/or insurance variables can have a significant impact on the forecasts of the model. For instance, assumed positive correlations between lines of business can give rise to higher costs than if independence was assumed. This is because the possibility that those lines of business will move simultaneously in an adverse way will be higher. Therefore, the study of correlations between the selected variables and the inclusion of a suitable correlation matrix is a step of the process that should not be neglected.

As a summary, the following steps are suggested for the construction of a DFA model:

1. Identify the main risks affecting the company in analysis;
2. Select a list of suitable variables reflecting the risks that should be incorporated in the model;
3. Determine which variables should be modelled stochastically and their respective probability distribution;
4. Determine a suitable correlation matrix between the chosen variables;
5. Develop the model, including all the relationships between the variables.

2.4   Description of the DFA Model Used in the Study

As referred before, the aim of this study is to develop a fairly basic DFA model that can be used as a standardised model from a regulator's point of view. The data used includes only the accessible economic and statistic information that general insurance companies in Portugal are obliged to present annually to their regulator. Roughly speaking, it includes the balance sheet for all operations, the profit and loss accounts disaggregated per line of business and the detailed composition of the companies' investment portfolios.

Unfortunately the quantity and quality of the data was the main constraint on the choice of the model. Data is only readily available from 1999 onwards and there are doubts on the quality of information of some companies, especially for the former years. This only gives six years of information to calibrate the parameters of the model which is far from being optimal. Besides that, the model was created and parameterised, benefiting, whenever possible, of sensible adjustments to the data. The final results achieved are believed to be fairly reasonable, although it is recognized that further work has to be done either on collecting new data and introducing new relevant variables.

The DFA model used tries to project the profit and loss accounts for the next years, while keeping in track the amount of total investments (divided by assets supporting the liabilities and free assets) and the total equity capital. The main structure is as follows:

Figure 2-2 - General structure of the DFA model

PROFIT & LOSS ACCOUNT
+   Earned Premiums
-   Claims Incurred
-   Expenses incurred
Underwriting Result
+   Representative Investments Income
-   Representative Investments Cost
Insurance Results
+   Free Investments Income
-   Free Investments Cost
Profit & Loss Account Results

Non-insurance results and tax were not considered, for simplification reasons. For its nature, non-insurance results, other than free investment returns, are generally unpredictable and of an extraordinary nature. These results, as well as dividends, are better treated in a deterministic way, if the management can reasonably predict their outcome in the next year. Also, it was decided not to incorporate the tax rules since they revealed to be too complicated to deal with for the time and data available.

An important limitation of the model is the no consideration of reinsurance. Even if the company had treaties in place, it was decided not to model the reinsurance results because of the lack of data and the difficulty on treating reinsurance in a standardised way for all companies. Therefore, all the variables introduced in the model – premiums, claims, expenses – are modelled and treated as gross of reinsurance.

The following sections give detailed information about the way that each of the above presented factors was modelled.

2.4.1   Lines of Business

Concerning the amount of premiums, claims and expenses, each line of business is considered individually from a total of eight. The selection of the list of eight lines of businesses assumed the partition evident in the Portuguese accounts' plan with some adjustments aimed to increase the stability of results. The final list used for the purpose of the model is presented below and the adjustments made over the original list are presented in Appendix A:

Figure 2-3 - Lines of Business considered in the model

1	Employer's Liability
2	Personal Accidents
3	Health Insurance
4	Property "All-purpose" Insurance (Home, Commercial & Industrial)
5	Fire & Other Damages
6	Motor
7	Goods in Transit
8	General Liability

Some types of insurance were excluded of the model because their results, in particular the loss ratios, showed unacceptably high instability and because they account for a low proportion of the majority of the companies' insurance portfolio.[3] This applies to the groups: Maritime & Other Transports, Aviation and Others.

2.4.2   Earned Premiums

The earned premiums are assumed to increase (or decrease) at a deterministic real rate indicated by the user. It is further assumed that their value increase in line with the rate of inflation, variable also simulated in the model (explained below on section 2.4.5). This is a simplification, since it is likely that price inflation, as measured by the Retail Price Index (RPI)

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[3] This last argument is not valid for five relatively small companies, which were excluded from the analysis.

may not the best indicator for certain lines of business. For instance, claims involving medical expenses have shown a tendency to increase over inflation and so it might be more suitable to use a higher rate whenever personal accidents and health perils are involved.

2.4.3   Claims Incurred

The projections of the amounts of claims per line of business were made indirectly: instead of the claim amounts, the loss ratio was modelled. For a given line of business, it was assumed that the loss ratio follows a lognormal distribution. The shape of the lognormal distribution is appealing for describing the loss ratio and it has been widely used in this context.

To estimate the parameters of the distribution, the mixed model proposed by El-Bassiouni (1991) was considered. This model is a mixed two-way analysis of variance with fixed company effects and random time effects, which takes into account the information of all companies exploring the same line of business. It is assumed that the variance is inversely proportional to the risk volume of each company, as measured by the earned premiums. This idea is also appealing, since it is expected for a company with a bigger insurance portfolio to be able to predict its results with a higher degree of accuracy and stability – *the benefits of diversification*. Appendix B gives a more detailed mathematical description of the model.

The fact that only six years of data per company are available does not allow for the assumption of the loss ratio following a lognormal distribution to be properly tested. This limitation should be taken into consideration when evaluating the results of the model at a latter stage. Concerning the assumption that the variance is inversely proportional to the amount of earned premiums the following graphs, arguably, do seem to show some consistency:

Figure 2-4 - Dispersion graph between loss ratios and earned premiums – Employer's Liability

Figure 2-5 - Dispersion graph between loss ratios and earned premiums – Personal Accidents

Figure 2-6 - Dispersion graph between loss ratios and earned premiums – Health Insurance
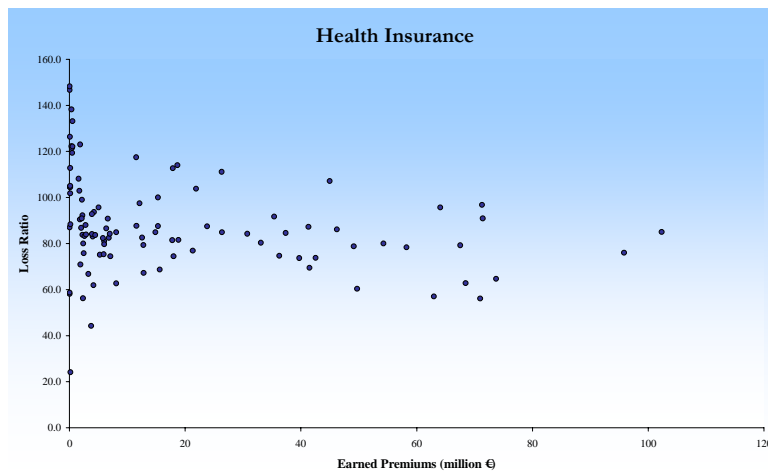
Figure 2-7 - Dispersion graph between loss ratios and earned premiums – Property 'All-purpose' Insurance

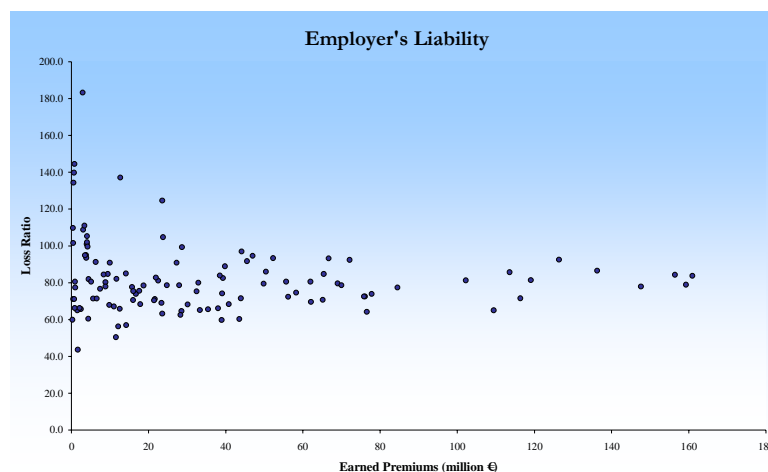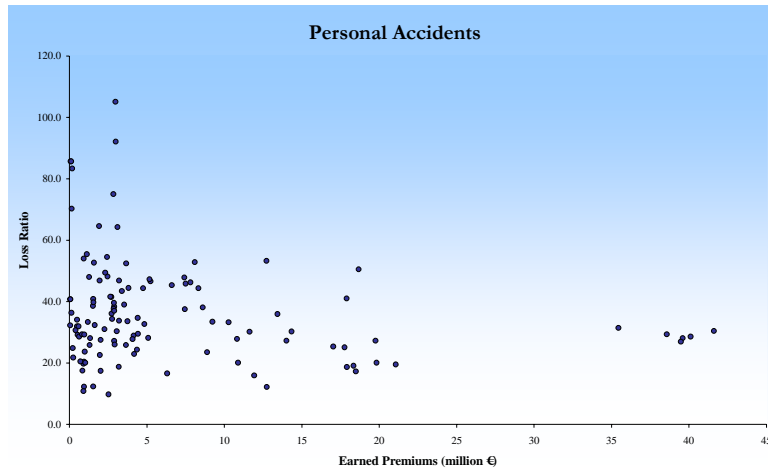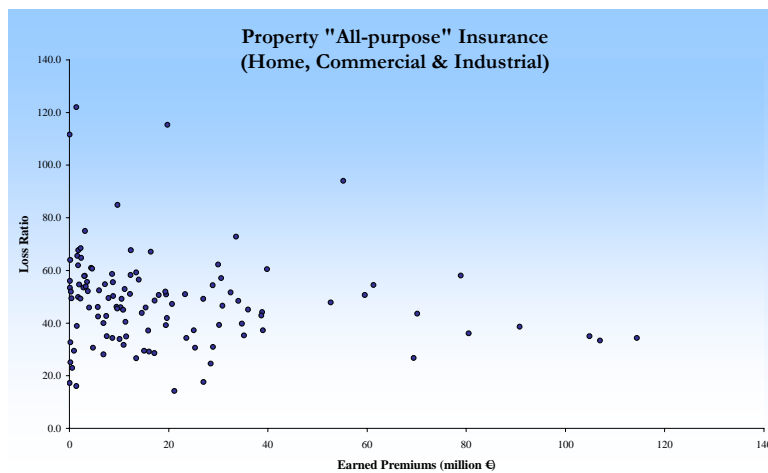**Figure 2-8 - Dispersion graph between loss ratios and earned premiums – Fire & Other Damages**
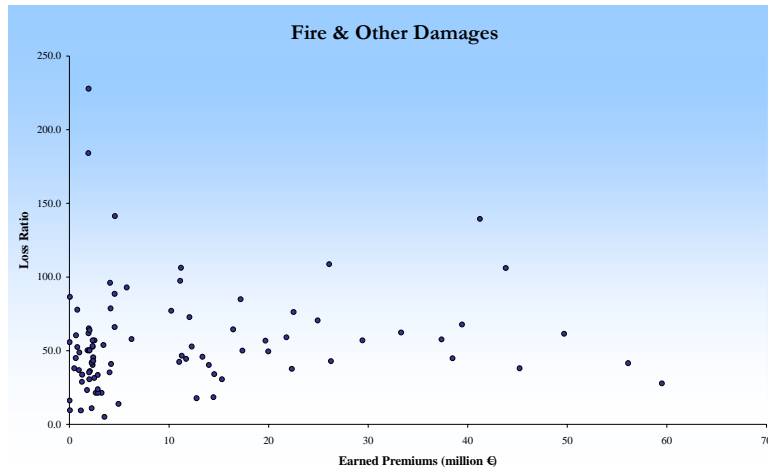
Figure 2-9 - Dispersion graph between loss ratios and earned premiums – Motor
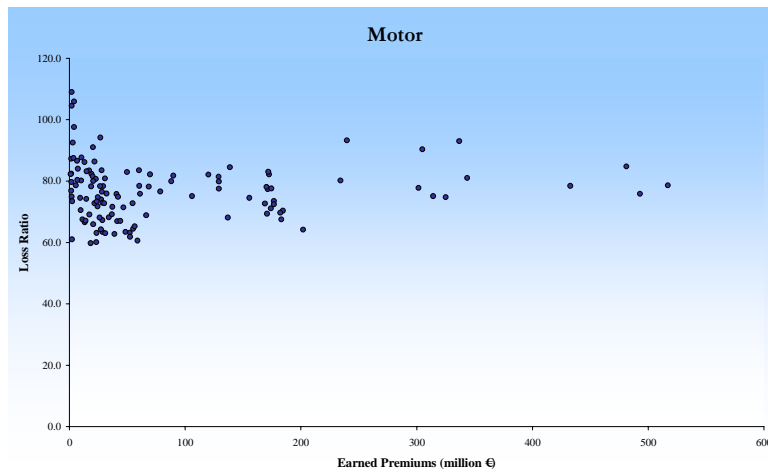
Figure 2-10 - Dispersion graph between loss ratios and earned premiums – Goods in Transit
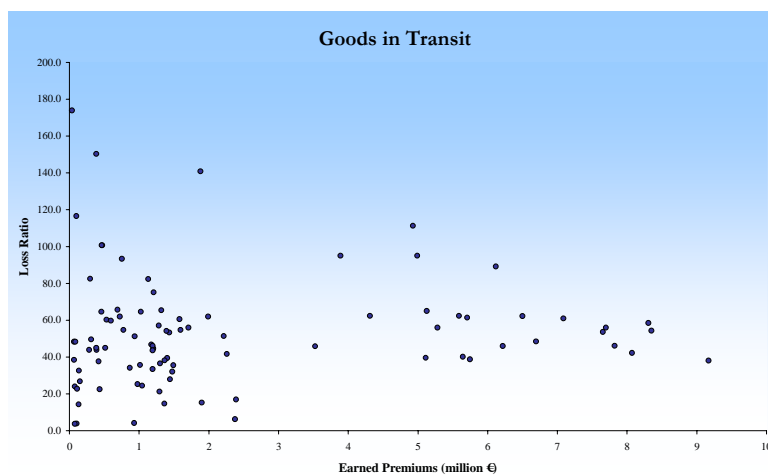
Figure 2-11 - Dispersion graph between loss ratios and earned premiums – General Liability
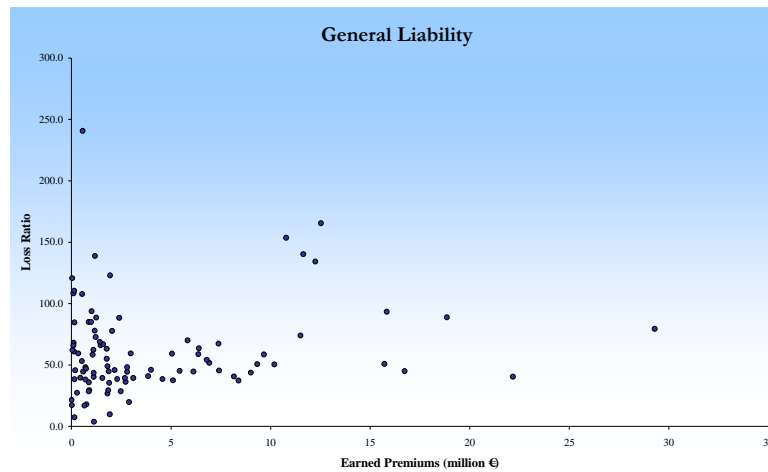
A probably more rigorous and accurate way to estimate the future claims amount would be to model the frequency and severity of individual claims separately, for instance assuming a Compound Poisson distribution and the Individual or Collective Risk Model frameworks. Another approach would be to simulate the loss ratio per year of accident, instead of a 'total' loss ratio which can be biased by differing claims management policies or extraordinary events occurred on a particular year. Although the previous approaches are recognized as more reliable, it was not possible to implement them in this study because of the lack of data.

As referred before, it is important to recognize the correlations between the variables of the model. In response to this, a correlation matrix between loss ratios of different lines of business was incorporated in the model. This matrix was estimated from the available past data and it is presented below. However, the reliability of the presented matrix is open to debate, since the low quantity and, in some cases, low quality of the data used does not allow for strong conclusions to be draw.

Figure 2-12 – Correlation Matrix between Loss Ratios of different lines of business

	Emp. Liab.	Pers. Acc.	Health	Property	Fire	Motor	Goods Tr.	Gen. Liab.
Emp. Liab.	**1.00**	0.23	0.30	0.25	0.03	0.58	0.39	0.56
Pers. Acc.	0.23	**1.00**	0.15	0.49	0.55	0.13	0.00	0.37
Health	0.30	0.15	**1.00**	0.33	0.02	0.00	0.00	0.54
Property	0.25	0.49	0.33	**1.00**	0.52	0.00	0.38	0.48
Fire	0.03	0.55	0.02	0.52	**1.00**	0.00	0.10	0.27
Motor	0.58	0.13	0.00	0.00	0.00	**1.00**	0.56	0.34
Goods Tr.	0.39	0.00	0.00	0.38	0.10	0.56	**1.00**	0.24
Gen. Liab.	0.56	0.37	0.54	0.48	0.27	0.34	0.24	**1.00**

Negative correlations were not allowed in order to increase the prudence of the model.

The correlations were implemented in the simulation process using the Normal copula technique proposed by Wang (1998). More details on this technique are available in Appendix C.

2.4.4 Expenses

In a similar way to the claims incurred, the expense ratio was modelled. Analysis of past data showed that a fixed ratio was probably not the best option for a number of companies. Therefore, a 'speculative' assumption that the expense ratio follows a Normal distribution was considered to introduce additional volatility on the model.

A more rigorous way to model the company expenses would start by the separation between fixed and variable expenses and between direct and indirect ones. Each of the individual categories could them be modelled in the most suitable way – either as a fixed ratio of the premiums or a fixed amount subject only to inflation. Again, no data was available to consider this level of detail on the study.

Independence was assumed between expense ratios of different lines of business and between pairs of expense ratios and loss ratios, mainly for practical reasons. Arguably, there are no strong intuitive views that substantially contradict these assumptions, although these issues should be further investigated.

2.4.5 Investments

The total investment portfolio of a company is separated into two components: the assets supporting the liabilities and the free assets. Generally, it is expected that the free assets will be invested in a more aggressive way in order to increase the expected return, while prudence is paramount for the assets backing the liabilities. Differently from the premiums, claims and expenses, the investments were not divided by lines of business since that is not required by the Portuguese regulator.[4]

The objective here is to estimate the expected asset returns for the next years. It is thus necessary to break down the portfolios into individual asset categories. The categories considered were equities, long-term bonds, cash, property and non-earning assets. Price inflation was an additional variable that had to be modelled, given that it impacts on the asset returns of most of the indicated asset classes and also, as explained in a previous section, on the rate of nominal premium growth.

A notional portfolio approach was thus followed, instead of the consideration of the actual assets held by the company. This is a common approach followed by actuaries aimed to ease the calculations. Therefore, some assumptions had to be made regarding the classification of the assets by nature. These are identified in more detail in Appendix D. The proportions on

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[4] With the exception of the Employer's Liability business. Even so, it was decided not to use this single separation.

each asset category used were taken from the real position of the companies as at the 31st of December of 2004.

Just a remark regarding the non-earning assets: these include items such as reinsurance debtors, credits over policyholders and other entities, etc. so long as they are accepted for solvency purposes under the Portuguese legislation. In particular, the deferred acquisition costs were rejected as making part of the investment portfolios. To increase the prudence, these items were assumed to be static across time[5] for the purpose of the model. Additionally, it is worth mentioning that derivative securities were not considered for simplification and because their proportion on the portfolios of all companies is significantly low.

The asset model was incorporated in the DFA model to generate returns for all the mentioned asset categories. The Wilkie standard stochastic asset model was used for this purpose, considering the parameters estimated by Wilkie for the UK market (data period 1924-1994) indicated in his 1995 paper. The main objective here is to introduce some sort of randomness on the asset returns which will impact on the results of the final DFA model. It is understood that a rigorous approach would involve the calibration of the parameters to reflect the Portuguese market and/or other financial indices better suited for the investment strategies of the Portuguese general insurance companies. This was not done due to the complexity and time needed for the task. Nevertheless, this is recognized as an important area where the model can be improved.

Concerning the investment costs, after analysis of the past data available it was decided to assume a fixed cost ratio of 1% over the total value of investments on equities, bonds and property.

2.4.6 Risk Measures Used

The DFA model used was constructed as a stochastic simulation model. In particular Monte Carlo simulation was carried out, which involves the use of randomly generated uniform ([0,1]) variables and their conversion to the desired probability distribution by means of the inverse of the relevant distribution function.

The Profit & Loss Account was projected year on year, considering a maximum time horizon of 20 years, with the computation of the empirical distribution of each individual variable per year and the calculation of risk measures like the ruin probability, the expected policyholder deficit and the minimum capital required for a given ruin probability.

The ruin probability measures how frequently a company is likely to get into financial distress. In other words, it is the probability of the insurer's liabilities exceeding its assets in any given period. For instance, if $\Psi(t)$ represents the ruin probability and U_t the insurer's equity capital at the end of period t, the formula is:

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[5] Although, arguably, it could be more prudent to allow these items to have some downside risk – for instance, by consideration of the credit risk of the counterparties involved.

$$\Psi(T) = \mathrm{P}(N < \infty), \text{ where}$$

$$N = \inf\{t : U_t < 0\} \text{ stands for the first occurrence of ruin}$$

The variable *T* defines the set of time points *t*. For instance, the determination of the *n*-period context ruin probability requires that *T* = {1, 2, …,*n*}.

Capital is defined as the excess of assets over liabilities, as demonstrated by the figure below, which represents a company's balance sheet structure. Capital represents the owner's stake or equity in the firm. A company is said to be on a technical insolvency position whenever its liabilities exceed the assets or, equivalently, when the capital, or surplus, drops to zero or below. When this happens, it is assumed that ruin occurs.[6]

Figure 2-13 – Balance sheet structure of an insurance company
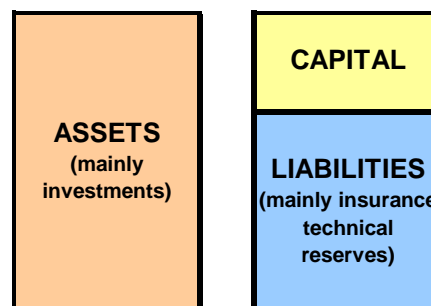
The ruin probability is a useful measure to assess the degree of risk of a particular company. However it does suffer from important shortcomings. For instance, as criticized by Butsic (1994) and Powers (1995), this measure does not consider the severity of insolvency. The severity of insolvency may not be important from a shareholders' perspective,[7] since their loss is capped, but it is of extreme importance for the policyholders and other creditors of the company. Certainly, the regulator will be interested in minimizing the potential losses to the policyholders and so, the use of a risk measure which doesn't appreciate the severity of losses (for instance the ruin probability) for solvency assessment purposes should not be done without caution.

The expected policyholder deficit (EPD) is a risk measure that obviates this problem, since the severity of losses is clearly taken into consideration. The EPD is calculated as the expected costs of the ruin or, equivalently, is equal to the product of the conditional expectation of the loss given ruin times the ruin probability. The mathematical formula considering a one time period is as follows:

$$\mathrm{EPD} = \mathrm{E}\left[\mathrm{Max}\{0 - U_1, 0\}\right] = \mathrm{P}\{U_1 < 0\} \cdot \mathrm{E}\left[0 - U_1 \mid U_1 < 0\right]$$

---

[6] It may be theoretically possible for an insurer to operate beyond the point of technical insolvency, in particular if cash inflows and outflows are sufficiently lagged, but this would be a risky situation and unlikely to be reached without previous regulatory intervention.

[7] A different opinion exists when talking about mutual companies, since in this case the policyholders are also the providers of capital, i.e. are in a position similar to shareholders.

To allow this measure to be comparable between two or more companies of different sizes, it is convenient to use the EPD ratio, taken from the EPD divided by the expected value of the liabilities.

However, there are also problems with this measure. As Barth (2000) showed, assuming that a minimum EPD ratio is fixed as a standard for solvency purposes, scope remains for a relatively high ruin probability to be accepted for a larger company, i.e. one with a high expected value of liabilities. Since larger companies represent a major threat to the insurance market in case of insolvency, this situation clearly lowers the credit of this risk measure.

After considering the limitations of the two mentioned risk measures – the ruin probability and the EPD ratio – Schmeiser (2004) concluded that none can be considered fundamentally superior to the other. Therefore, in this study, it was decided to compute both measures which are presented on the practical examples of the next chapters. However, some preference was given to the ruin probability for practical reasons.

Instead of calculating and quoting the risk measures on a real example of a company with a given amount of equity capital, the inverse approach can be used, i.e. the value of the risk measure can be fixed *a priori*, reflecting the desired degree of prudence, and the minimum amount of equity capital necessary to satisfy that level of risk can be obtained. This technique was tried in this study, but only considering the ruin probability measure, also for practical reasons. The results achieved are presented on the next chapters.

Other important question relates to the time horizon that should be considered to evaluate the risk measures, especially when these are used for solvency assessment purposes. According to the International Actuarial Association (2004), "a supervisor must take into account the time horizon between the date as of which company financial statements are prepared and the expected date by which a supervisor could take control of the insurer if this was deemed to be necessary." This is because the delay between those two dates can be quite significant, maybe several months or more. In the meantime, the company will probably continue to operate even if its position is weak. Therefore, the projections of a model should be large enough to allow the regulator to anticipate the financial position of the company and to intervene as soon as possible. Again, quoting the above mentioned international body: "it would be rare to assume this time horizon could be considerably shorter than one year."

3 Practical Application to a General Insurance Company

3.1 Overview of the Company

The results of the studied model were applied to a company exploring general insurance business and will be detailed in this chapter as an example to enhance understanding of the capabilities of a DFA model. The selected company is a significant player on the Portuguese insurance market with exposure to all the lines of business.[8] About half of the total earned premiums of company XYZ[9] in 2004 relate to the motor insurance business, followed by significant exposures in employer's liability and 'all-purpose' property insurance business. General Liability and Goods in Transit have the lowest concentrations, although still significant in amount when compared with other Portuguese companies.

The analysis of six years of data reveals that the company has systematically achieved negative underwriting results which were more than compensated by the investment returns, except for a couple of years when the markets were depressed and the investment returns were very poor.

Considering the capital requirements calculated according to the regime still in force in the EU, the company's equity capital available at the end of 2004 (223 million euros) allows for a very comfortable position, with three times the required solvency margin (73 million euros).

3.2 Parameterisation of the Model

The inputs necessary to run the DFA model for the company XYZ are presented and commented below:

- The earned premiums per line of business, taken from the Profit & Loss Accounts of 2004. Additionally, deterministic real rates of premium growth were necessary.

- Mean and standard deviation of the lognormal distribution assumed for each individual loss ratio.
  The following table summarises the parameters estimated for this company after application of the El-Bassiouni (1991) methodology:

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[8] Roughly 5% of the 2004 premiums account for the rejected lines of business mentioned in Appendix A and thus will not be modelled.

[9] For confidentiality reasons, the company will not be identified and it will be referred to as company XYZ. Additionally, all amounts in Euros were modified.

Figure 3-1 – Mean and Standard Deviation of the Loss Ratios

t	Line of Business	Lognormal Mean	Lognormal Std. dev.
1	Employer's Liability	84.7	7.0
2	Personal Accidents	23.7	3.9
3	Health Insurance	94.6	9.2
4	Property "All-purpose" Insurance	54.7	11.3
5	Fire & Other Damages	66.6	24.9
6	Motor	78.9	4.5
7	Goods in Transit	61.3	18.5
8	General Liability	54.2	16.1

Although all the companies exploring general insurance business were used to calibrate the individual parameters, each company has an estimate of the mean which reflects primarily its own experience. It is thus possible for two companies to have relatively different means in the same line of business. This makes sense since one company can have a better underwriting policy allowing for the building of an insurance portfolio composed of low claims generating contracts and it should not be penalised for the bad practices of other companies.

However, the variance of the loss ratios of different companies is linked, since the same parameters (θ_1 and θ_2)[10] are used to estimate it. The differentiation is made on the basis of the amount of earned premiums, with larger companies having lower variance due to the expected benefits of diversification.

From the table above, it can be seen that volatile results will be expected mainly for the Fire & Other Damages, Goods in Transit and General Liability businesses. This is a reflection of the overall volatile results registered in the market on these lines of business. Motor and Personal Accidents are the lines of business where more stable results were achieved benefiting also from the relatively high amount of earned premiums compared with the market.

▪ The correlation matrix, reflecting the correlations between the loss ratios of the different lines of business is the one indicated in Figure 2-12.

▪ The expense ratio per line of business, which is assumed to follow a Normal distribution. The following table presents the mean and standard deviation, as estimated from the available past data:

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[10] For completeness, see Appendix B.

Figure 3-2 – Mean and Standard Deviation of the Expense Ratios

t	Line of Business	Normal Mean	Normal Std. dev.
1	Employer's Liability	27.5	0.9
2	Personal Accidents	32.4	2.1
3	Health Insurance	22.9	1.8
4	Property "All-purpose" Insurance	44.1	1.6
5	Fire & Other Damages	33.4	2.0
6	Motor	31.3	1.2
7	Goods in Transit	25.3	2.3
8	General Liability	51.5	2.7

The parameters of the Normal distribution assumed for the expense ratios are estimated considering only the experience of the company in analysis. The non-consideration of a 'market factor' does not seem to be too important here as it is for the loss ratio. In fact, a typical assumption on other models is to assume a deterministic expense ratio. In this case, a Normal distribution is assumed to allow for additional volatility in the model. However, this is considered as a 'speculative' assumption since the fitting of such a distribution was not properly tested.

The results show relatively stable expense ratios for company XYZ with a tendency for higher stability on the lines of business where the company is more concentrated.

- The standard Wilkie model applied to the selected asset categories requires 30 parameters which were taken from Wilkie's 1995 paper and reflect the results estimated for the UK market. The suitability of such parameters was already discussed in previous chapters

- The investment portfolios of the company, divided by assets supporting the liabilities and free assets. The proportion invested in each of the selected notional asset categories is presented in the graphs below:

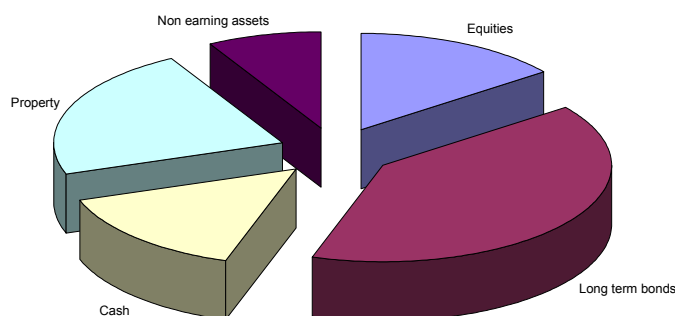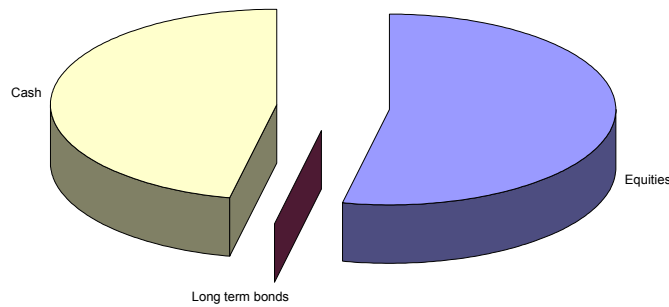**Figure 3-3 – Portfolio Structure of the assets backing the liabilities**

Figure 3-4 – Portfolio Structure of the free assets

- The amount of equity capital available as at 31st of December of 2004, which amounted to 223 million Euros.

3.3 Results of the Model – One-year Projection

This section will detail the results of the model when the inputs were as detailed in the previous section (3.2). Concerning the rates of premium growth, it was considered a real rate of 0% for all lines of business, i.e. the nominal amount of earned premiums grow only in line with the rate of price inflation (variable also simulated by the model). There is some difficulty in estimating these rates from past data given that no trends are evident. In fact, the rates of premium growth are generally more dependent on management decisions and expectations.

A word of caution is needed at this point. As will be said on the conclusions of this report, the results presented and in particular the amounts should be taken only as a basis for further work. Additional diagnostic tests and parameter fitting are needed to ensure that the model studied becomes accurate enough to be used professionally for solvency measurement purposes.

The table below shows the mean projected underwriting results for the end of the next year (2005) as well as the simulated coefficient of variation, i.e. the ratio of the standard deviation over the mean.

Figure 3-5 – Projected Underwriting Results per line of business

	Employer's Liability	Personal Accidents	Health Insurance	Property 'All-purpose'	Fire & Other Damages	Motor	Goods in Transit	General Liability	**TOTAL**
U/W result	**-11.4**	**11.8**	**-4.1**	**0.9**	**0.0**	**-24.0**	**1.2**	**-0.8**	**-26.4**
	59.0%	*11.0%*	*54.3%*	*774.0%*	*15513.7%*	*46.7%*	*134.1%*	*293.3%*	*90.3%*

million Euros

The *box-and-whiskers* plot below illustrates the same results with the focus on the dispersion of amounts:[11]

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[11] The represented points are: minimum, quantile 5%, average, quantile 95% and maximum.

Figure 3-6 – Projected Underwriting Results per line of business (*Box-and-whiskers*)
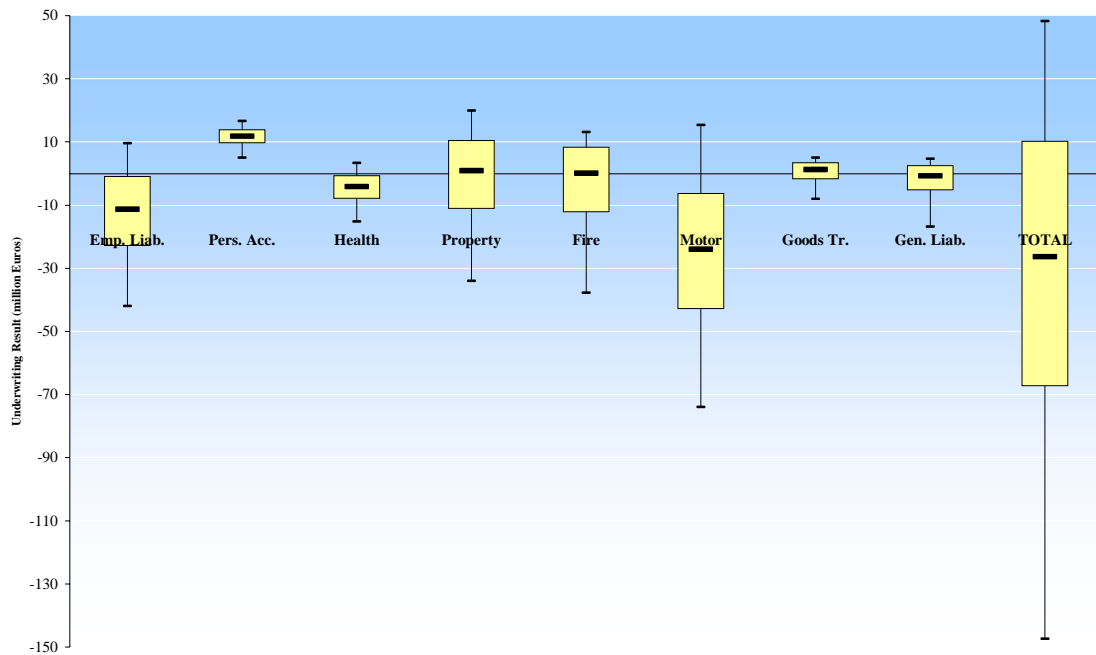
As expected, a comparison of Figure 3-5 and Figure 3-1 shows a relationship between the volatility of the loss ratio and the volatility of the underwriting result. The impact in amounts is expressed in Figure 3-6 revealing a relatively high uncertainty on the total underwriting result with a high downside potential reaching 150 million euros. However, only with 5% of probability the underwriting loss is expected to be worse than 70 million euros.

The investment results calculated on an aggregate basis, the transition to the P&L Account result and its representation using a *box-and*-whiskers plot are presented below:

Figure 3-7 – Simulation of the P&L Account Result

U/W result	**-26.4**	
	90.3%	
Investment return	**84.4**	
	63.1%	
Investment costs	**7.4**	
	0.0%	
Insurance result	**50.7**	
	115.8%	
Free invest. return	**0.2**	
	109.6%	
Free invest. costs	**0.0**	
	0.0%	
P&L Acc. result	**50.9**	
	115.7%	
	million Euros	
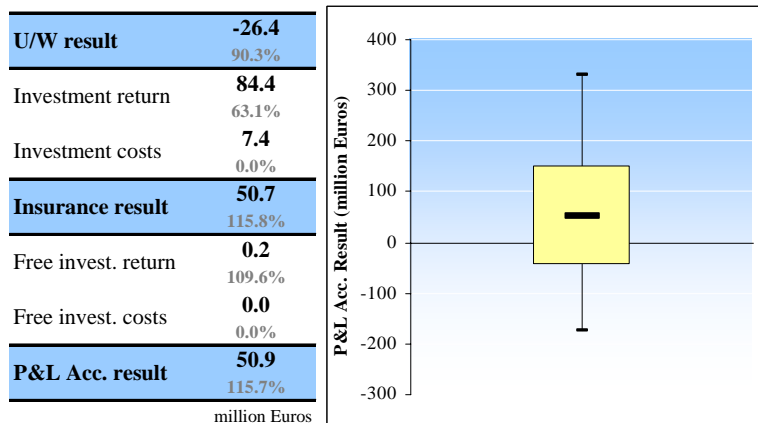
It can be seen that, in terms of mean values, the investment results are sufficient to more than compensate the negative underwriting results.[12] Also, the introduction of an extra variable stochastically modelled – the investment results – causes the increase of volatility from the underwriting results to the P&L Account results.

Finally, the next table details the mean and the coefficient of variation of the equity capital projected for the end of 2005:

Figure 3-8 – Projection of the Equity Capital

	2004	2005
Equity Capital	222.9	273.7
		21.5%

million Euros

The following graph shows the empirical distribution of the projected equity capital.[13]

Figure 3-9 – Empirical distribution of the one-year projection of the equity capital
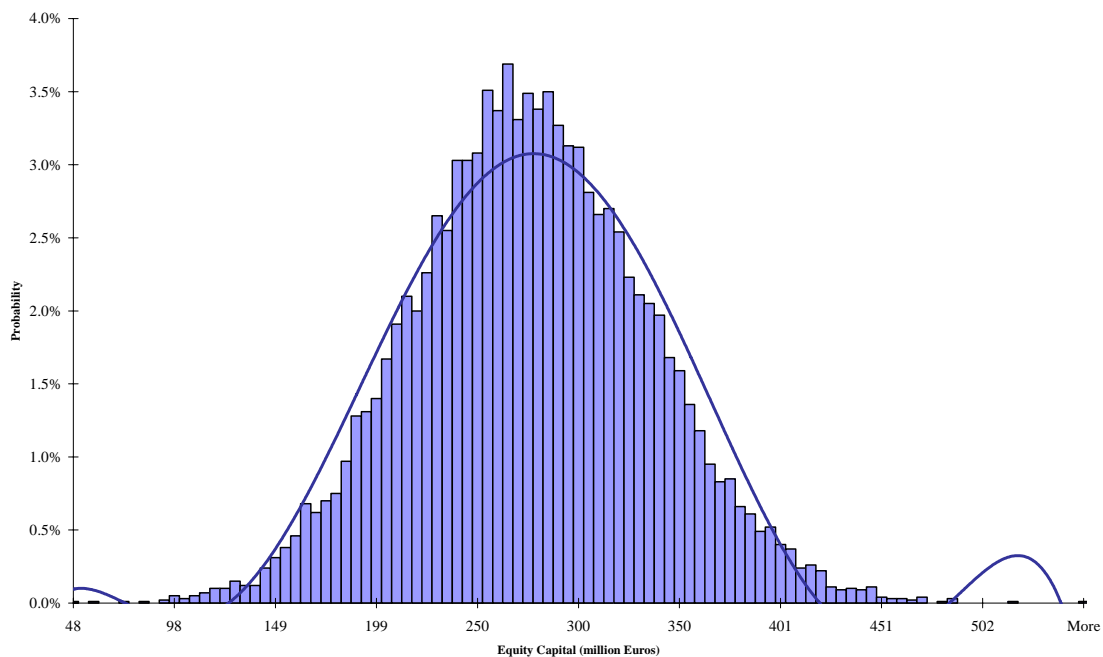
The holding of an equity capital of 223 million Euros at the end of 2004 seems to be more than sufficient to ensure the solvency of the company in one-year's time. In fact, none of the 10,000 simulations showed a result were the equity capital would end up negative, i.e. where company XYZ would be in a technical insolvency position. As said before, even for the current solvency standards (EU solvency regime in force) the company is considered to be in a comfortable position, holding three times the minimum amount required.

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[12] The free investment results are almost zero since the company XYZ has a relatively low amount of free assets.

[13] The trend line is a polynomial approximation of sixth order.

Therefore, the ruin probability, as well as the EPD, (both measures were introduced in section 2.4.6) are zero in the present case, assuming a time horizon of one-year only. This is theoretically impossible, since there is always a possibility for an insurance company to go insolvent – for instance, a massive and very unlikely catastrophe or a major fraud scandal.

The following graph shows the simulated ruin probability for a given starting equity capital:

Figure 3-10 – Ruin probability vs. starting Equity capital
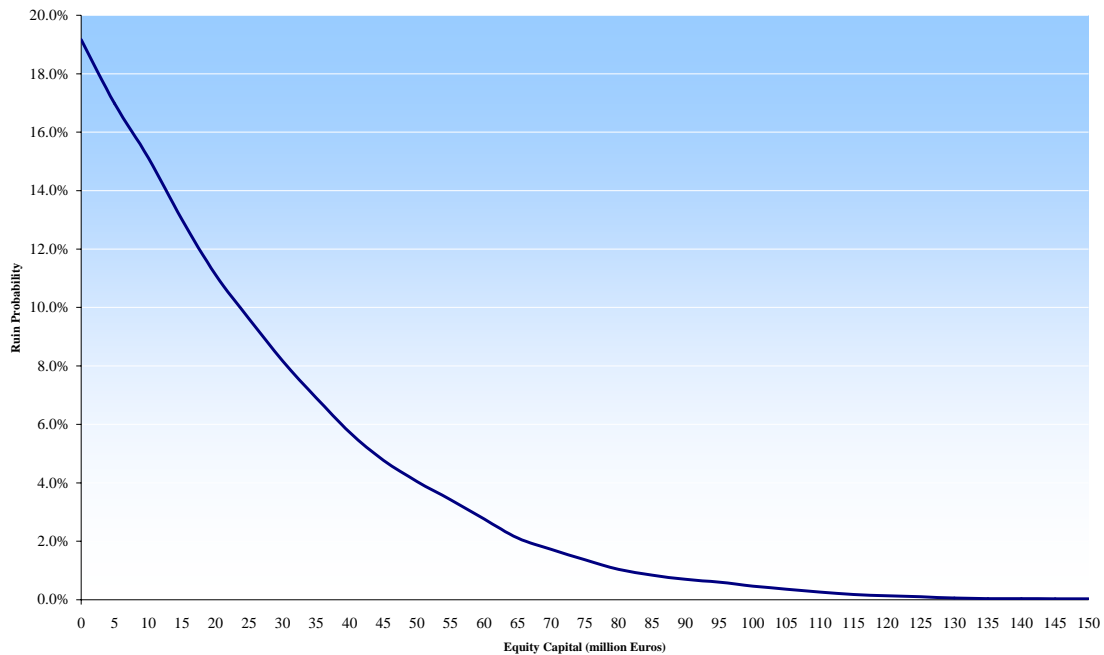
This approach allows for the determination of the capital requirement for a given confidence level. For instance, if a maximum one-year ruin probability is set to 1%, the minimum capital that the company should hold at the end of 2004 would be around 82 million Euros.

For curiosity, the holding of an amount equal to the solvency margin required by the current solvency system (73 million Euros) has a ruin probability attached of 1.5%, according to the model. However, in these circumstances, the expected value given ruin would reach 94 million Euros, i.e. a deficit of 21 million Euros, revealing a thicker tail of the P&L Account Result distribution.[14]

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[14] The shape of the P&L Account Result empirical distribution is equivalent to the shape of the projected Equity Capital empirical distribution (see Figure 3-9) given that the one-year projected capital is equal to the projected P&L Account Result minus the equity capital available at the end of 2004 (which is a constant).

3.4   Sensitivity of Assumptions – One-year Projection

One of the main capabilities of DFA models is the analysis and quantification of the impact of changes on the actual conditions. Depending on the way the model is constructed, it may be possible to analyse some or all of the following:

o   Changes on the expected rates of growth on premiums – for instance, the management may be interested in analysing the impact of focusing on a particular line of business and/or divesting on one or more lines of business

o   Different investment strategies

o   Different reinsurance policies

o   Other risk management policies, like for instance derivative securities or alternative risk transfer techniques

In this section, this capability will be explored on the particular model studied. It will be analysed the impact of variations on the following input:

▪   Rates of premium growth

▪   Correlation matrix between the loss ratios of different lines of business

▪   Structure of the investment portfolios

3.4.1   Change of the Rates of Premium Growth

The following graph plots the one-year ruin probability against the starting equity capital. Several scenarios of nominal premium growth are considered, from -10% to 10%, including the case studied on a previous section, i.e. growth in line with price inflation. For simplicity, the rates of growth were assumed to be equal for all lines of business.

Figure 3-11 – Ruin probability vs. starting Equity capital for different premium growth scenarios
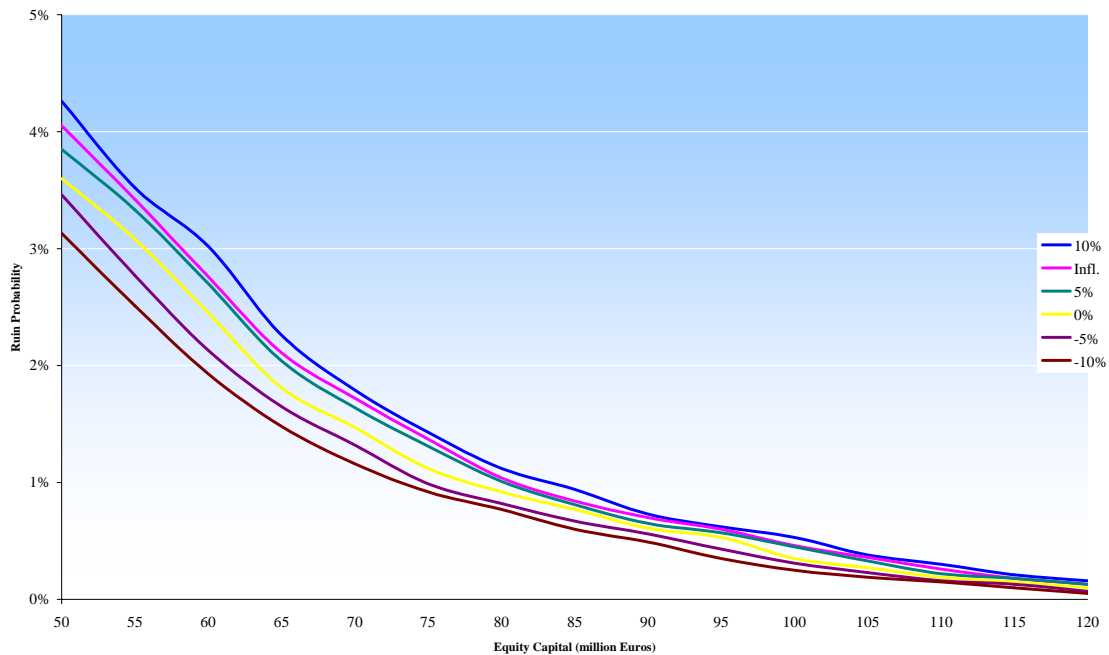
As can be noticed from the above graph, no matter the amount of equity capital held at the beginning of the period, the ruin probability tends to be lower for small increases or even decreases of the premiums. This seems, however, not to be a reasonable result from an intuitive point of view. In fact, an overall reduction of premiums can be expected to cause at least two damaging effects:

▪ The expenses will be spread over a lower amount of premiums, causing the expense ratio to increase and, thus, the underwriting results to be lower.

▪ In particular for long-tailed classes of business, the loss ratio should be expected to be higher,[15] given that the evolution on outstanding claims from previous years will not be affected by the decrease on premiums.

None of these effects is explained by the studied model. In fact, the modelled expense ratio does not depend on the amount of premiums and the modelled loss ratio does not make distinctions between the claims originated on the current year and on previous ones.[16]

In conclusion, the studied model is not adequate to analyse the sensitivity of the change of assumption regarding the rates of premium growth. Some ideas to improve the model on this context are:

▪ Divide the expenses in categories and incorporate each one as a variable in the model. For instance, consider (1) a fixed amount reflecting the fixed expenses of the company, i.e. expenses that are not particularly sensitive to the amount of premiums and/or claims (e.g. rent of the main premises, support services' staff salaries); (2) a proportion of the premiums

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[15] The loss ratio considered here is similar to the concept used in the studied model – it includes all claims paid, irrespective of the year of origin.

[16] With lower premiums, the modelled loss ratio is expected to become more volatile (see Appendix B). However this effect does not seem to be sufficient to have an impact on the underwriting results.

to reflect the expenses directly related to the amount of premiums written (e.g. acquisition costs) and (3) a proportion of the claims paid to reflect costs originated during the settlement of claims (e.g. investigations, legal expenses for some lines of business).

In case of an overall decrease of premiums, the ratio of total expenses to premiums could be expected to increase, given that only expenses of type (2) would be directly affected.

▪ Introduce a loss ratio per year of accident as variables in the model and project the claims considering a claims development pattern assumption, which is expected to be different for each line of business.

An overall decrease of premiums would impact only on the loss ratio correspondent to the year of decrease. For all the past accident years, the nominal amounts of claims should not be significantly changed from expectations and thus the overall loss ratio (irrespective of accident years) would increase.

Sensitivity tests involving changes on the rates of premium growth of individual lines of business were not carried out. The results of such tests are not expected to be so straightforward since they should take into account the risk volumes and the riskiness of the individual lines of business (as measured by the volatility of the assumed loss ratio and expense ratio distributions).

3.4.2   Change of Correlation Matrix

For simplicity, only four scenarios were considered: (1) the matrix presented on Figure 2-12 which was already used in the previous sections; (2) a matrix where the correlations between pairs of loss ratios are assumed to be zero, i.e. independence assumption; (3) a matrix where all the correlations are assumed to be one, i.e. perfect correlation assumption and (4) a matrix where all the correlations are assumed to be -0.14,[17] i.e. a relatively weak negative correlation assumption.

The following graph plots the results of the tested scenarios, namely the ruin probability against the starting equity capital:

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[17] A smaller value could not be used because of constraints on the mathematical formulas underlying the Wang's algorithm.

Figure 3-12 – Ruin probability vs. starting Equity capital for different correlation scenarios
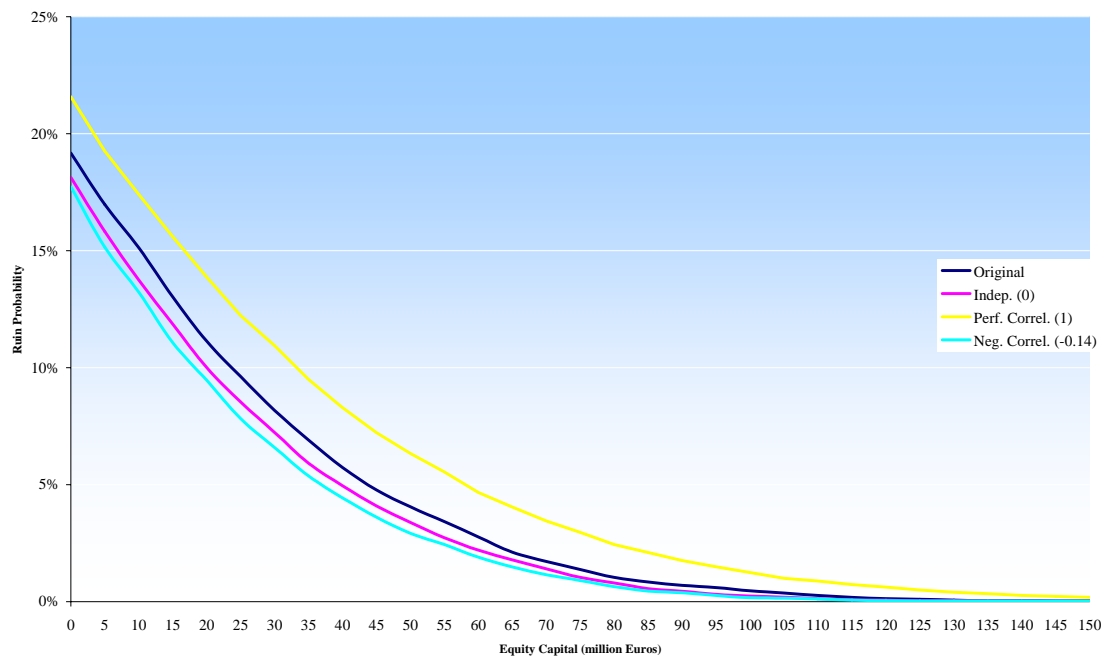
The results point out to the conclusion that, irrespective of the starting equity capital, the higher the correlation between the loss ratios of different lines of business, the higher the expected ruin probability.

These results make sense from an intuitive point of view. For instance, high positive correlations generally mean that if a given line of business performs badly, then the chance of the other lines of business performing badly as well is increased, when compared with the independence assumption. Therefore, the combined result of all lines of business can be more negative for positive correlations.

On the other hand, reverse results are expected for negative correlations. If two lines of business are highly negatively correlated, then the underperformance of one is expected to be compensated by the outperformance of the other, i.e. it is more likely for the two variables to develop in opposite ways. This property would be regarded as very desirable by insurers, due to its contribution for the stability of results and for the lowering of the regulatory capital. However, in general insurance business, it is hard to find two or more lines of business where such negative correlation markedly exists.

3.4.3   Change on the Structure of the Investment Portfolios

The sensitivity tests were performed through alterations on the portfolio of assets backing the liabilities only. The amount of free assets is comparably very low and so it is unlikely that it will cause any significant distortions.

The structure used for the analysis on chapter 3.3 was the one indicated by Figure 3-3, which reflects the real position of the company at the end of 2004. This structure and five other

different scenarios are plotted on the graph below. For simplicity, only extreme scenarios were considered, i.e. scenarios where all is invested in one asset category (except for the real scenario). These are:

1. 100% on non-earning assets
2. 100% on cash
3. 100% on long-term bonds
4. 100% on property
5. 100% on equities
6. 'Real' scenario

A parenthesis should be open here: apart from the original scenario, all the other scenarios are not reasonable to be adopted in real life, especially for liquidity reasons. For instance, 100% in property would raise important liquidity risk which, although it is not measured by this model, it should not be forgotten – this risk alone is sufficient to impair the financial position of a company. In practice, an investor will consider a diversified portfolio including some or all of the asset categories, weighted according to the risk and return appetite and the need for liquidity.

Figure 3-13 – Ruin probability vs. starting Equity capital for different investment strategies
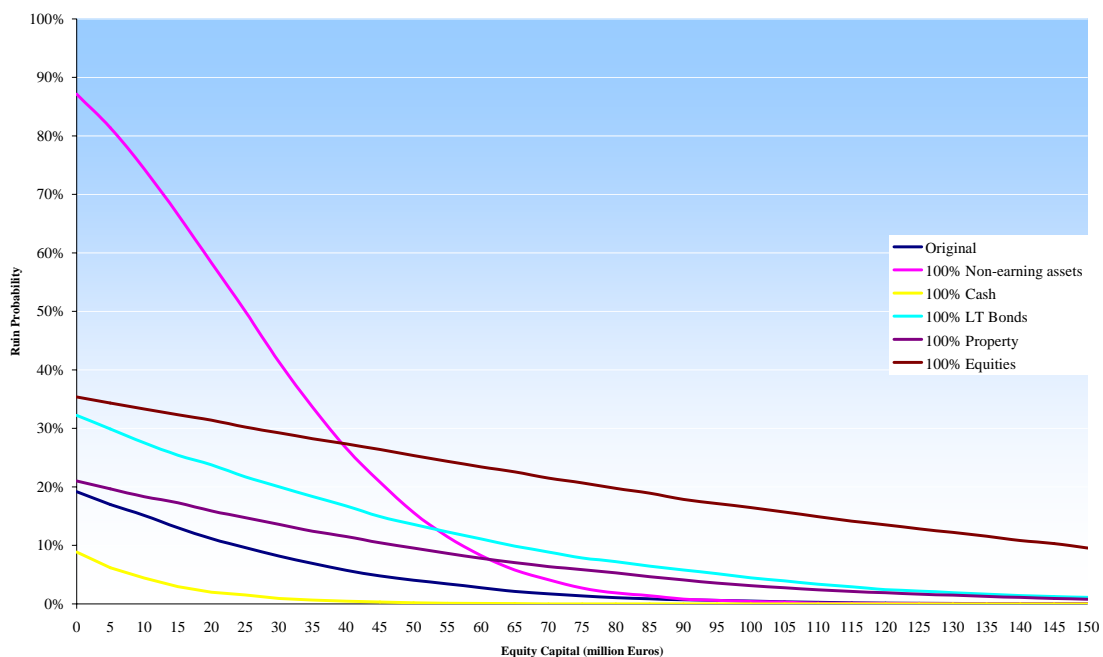
The graph demonstrates the importance of the investment strategy and the sensibility of the ruin probability to different investment scenarios. The impact depends on the relative riskiness of the assets.

This comparison would not be completed without consideration of the expected investment return on each scenario. The next table presents the expected value and the standard deviation of the investment ratio and the return on equity ratio:[18]

Figure 3-14 – Investment ratio and ROE ratio for different investment strategies

	Original	100% Non-earn. assets	100% Cash	100% LT Bonds	100% Property	100% Equities
Investment Ratio	**9%**	**0%**	**6%**	**8%**	**14%**	**12%**
	5.6%	0.0%	0.0%	8.0%	11.6%	19.4%
Return on Equity	**23%**	**-12%**	**15%**	**17%**	**42%**	**37%**
	26.4%	10.7%	10.7%	36.1%	51.1%	84.6%

The investor's decision is not based only on risk, but also on the expected return – if this was the case, the company was undoubtedly better off investing all the assets in cash, but, as the above table shows, with relatively poor expected asset returns. As expected, the original scenario, reflecting a diversified portfolio with all asset categories seems to achieve the best risk/return trade-off (for instance, this is evident when compared with the 100% LT Bonds scenario). There are probably other diversified portfolios that may provide a more optimal trade-off for the company. The DFA model will thus allow the company to look for it, while keeping an eye on the impact on the ruin probability or other risk measures.

3.5   Results of the Model - Multi-year Projection

This section extends the analysis made on section 3.3 into a wider time horizon, namely a maximum of 20 years. As before, the parameters considered are the ones detailed in section 3.2 and the nominal amount of earned premiums is assumed to grow in line with price inflation.
A simplifying assumption of independence between periods was assumed in order to extend the projections of the model. It is important to note that the ignored dependencies are very likely to exist – for instance, the impact of the insurance cycle where theoretically the profits/losses per line of business develop according to a cyclical behaviour dependent on the underlying economic conditions and competition. The relaxation of this independence assumption is likely to lead to higher volatility of results and thus to higher capital requirements. Therefore, the non-consideration of these dependencies should be understood as a limitation of the model studied.
The following graph shows the increasing 'funnel of doubt' generated when the underwriting results are projected for several years. The quantiles considered were 99.9% and 99% and, as before, 10,000 simulations were on the basis of the results.

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[18] The investment ratio is calculated as the division of the simulated asset returns by the value of the portfolio at the end of the previous period. The return on equity is the division of the simulated P&L Account result by the equity capital available at the end of the previous period.

Figure 3-15 – Projected underwriting results for a maximum of 20 years
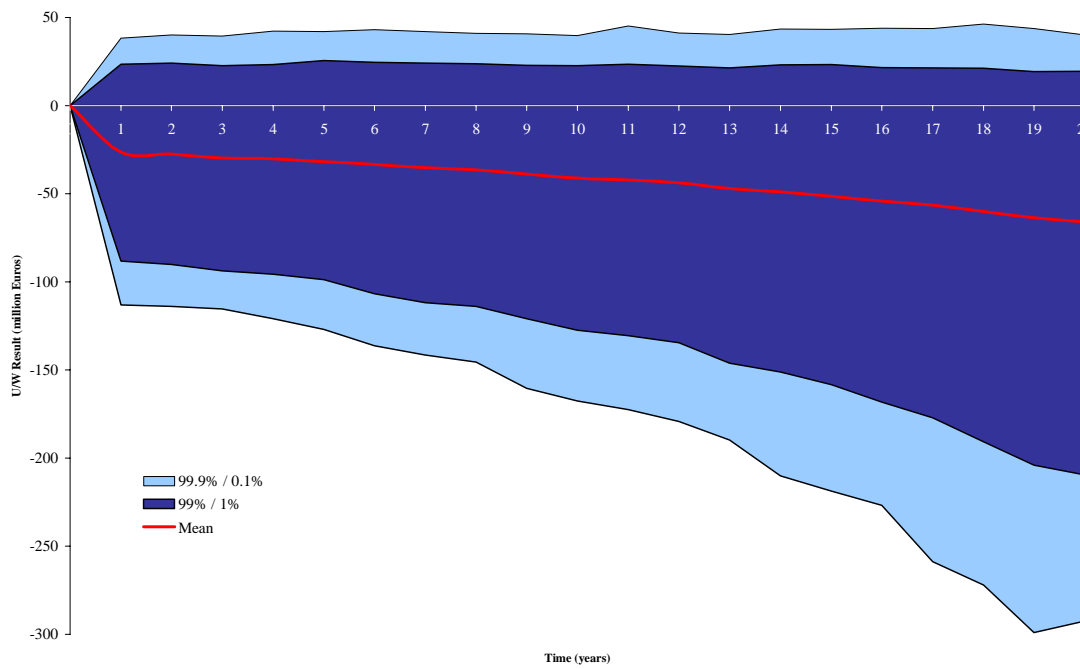
As expected, the volatility of results increases significantly year after year. It is interesting to notice the relatively higher increase on the downside risk. This is due to the use of the right-tailed lognormal distributions to describe the loss ratios.

Concerning the P&L Account results, i.e. the underwriting results plus the investment returns, there is also increasing volatility when the projections are extended into the far future. However, the downside risk noticed for the underwriting results is significantly weakened. The graph below illustrates this point:

Figure 3-16 – Projected P&L Account results for a maximum of 20 years
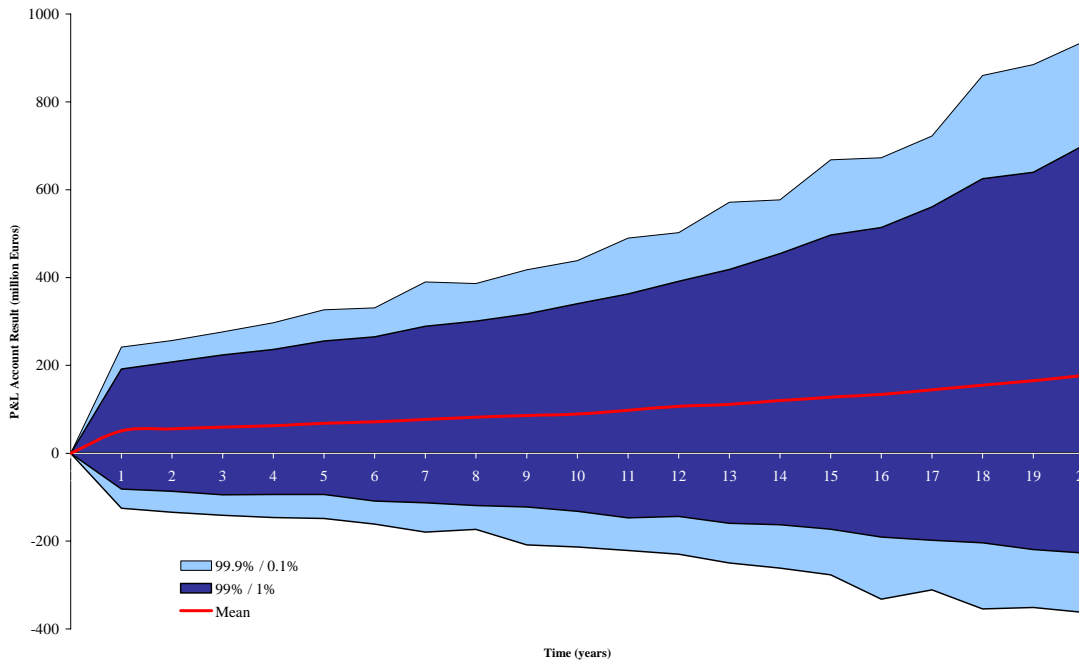
From here, it is evident the importance of the investment results for an insurance company. Finally, the evolution of the equity capital is also presented:

Figure 3-17 – Projected Equity Capital for a maximum of 20 years
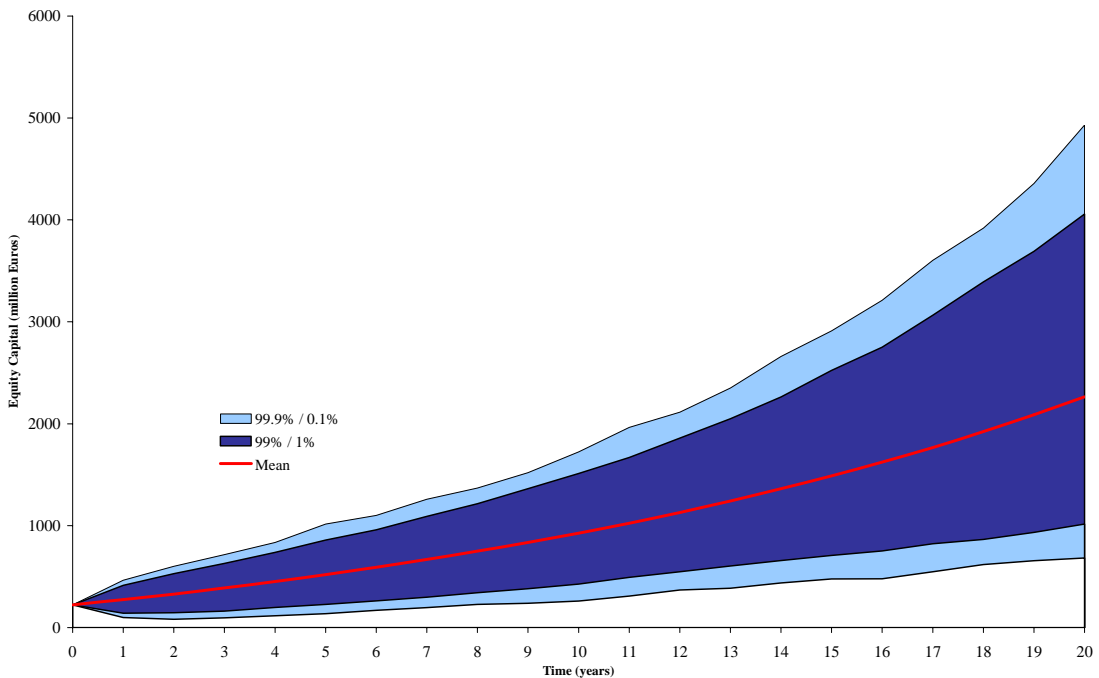
In mean terms, the equity capital is expected to rise across time. However, this happens also with substantial increase on the volatility.

As said before, company XYZ seems to be well capitalised for a one-year time period. With the extension of the timeframe, the conclusion remains unchanged. In fact, considering a conservative degree of risk of 0.1%, the projected equity capital remains positive for all the period in analysis. However, a very small ruin probability of 0.02% exists, being triggered at the 3rd year.

In this particular example, it is not very evident,[19] but the ruin probability does increase with the time horizon of the projection and decrease with the equity capital available, as expected. In particular, the time horizon considered can have a major impact on the capital requirements. For instance, as mentioned in section 3.3, the minimum capital required for a degree of risk of 1% and for a one-year period is 82 million Euros. The ruin probability associated with this amount increases to 1.6% for a two-year analysis and to 2% for a four-year analysis.

Assuming an unknown starting equity capital, it is possible to use the results of the model to determine the minimum capital that should be hold for a given maximum ruin probability. The following graph plots the minimum capital required for a ruin probability of 1% against the time horizon used in the calculation of the risk measure:

Figure 3-18 – Minimum capital required for a 1% ruin probability considering different time horizons
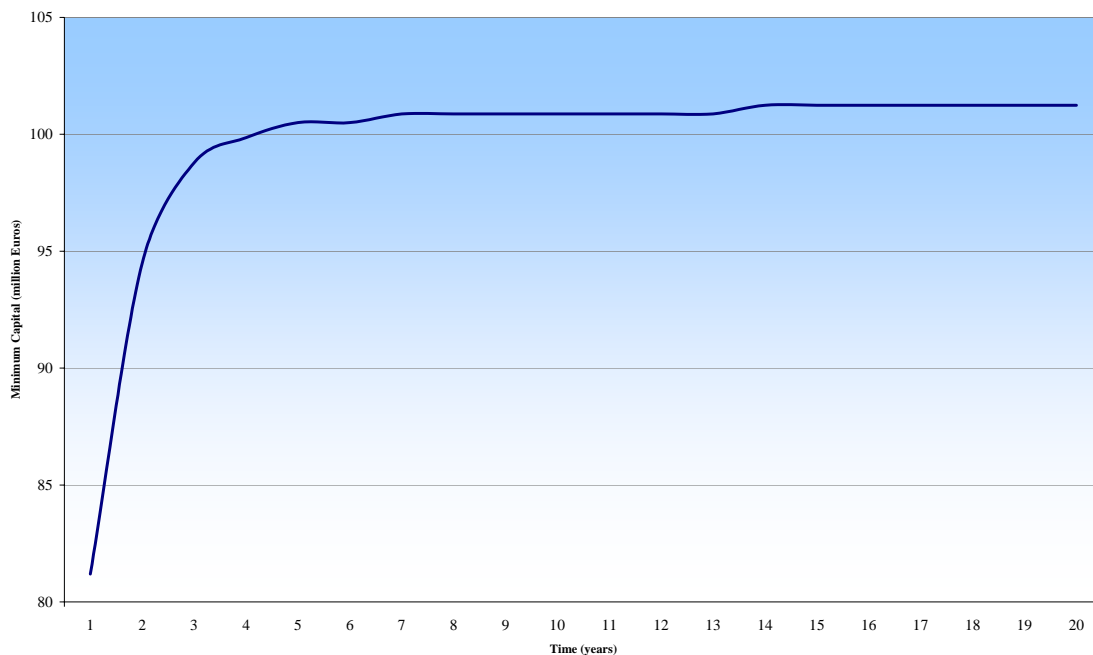
For long-term horizons, the starting equity capital needed tends to a constant value around 102 million Euros. The assumption of independence between periods contributes to this trend: the bad P&L Account results of some years will be compensated by the good performance on other years. The accumulation of successive bad years is more likely in the short term, explaining the sharp increase verified until the 4th year.

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[19] Again, because of the relatively high starting equity capital of company XYZ.

Although it will not be fully demonstrated, tests were run showing that the investment strategy also plays an important role on this behaviour and different investment strategies can alter it significantly. For instance, the consideration of a different investment strategy such as the reduction on equities and increase on non-earning assets, gives rise to a demand for more initial capital for longer term scenarios, as can be seen in the graph below.[20]

Figure 3-19 – Minimum capital required for a 1% ruin probability – modified investment strategy
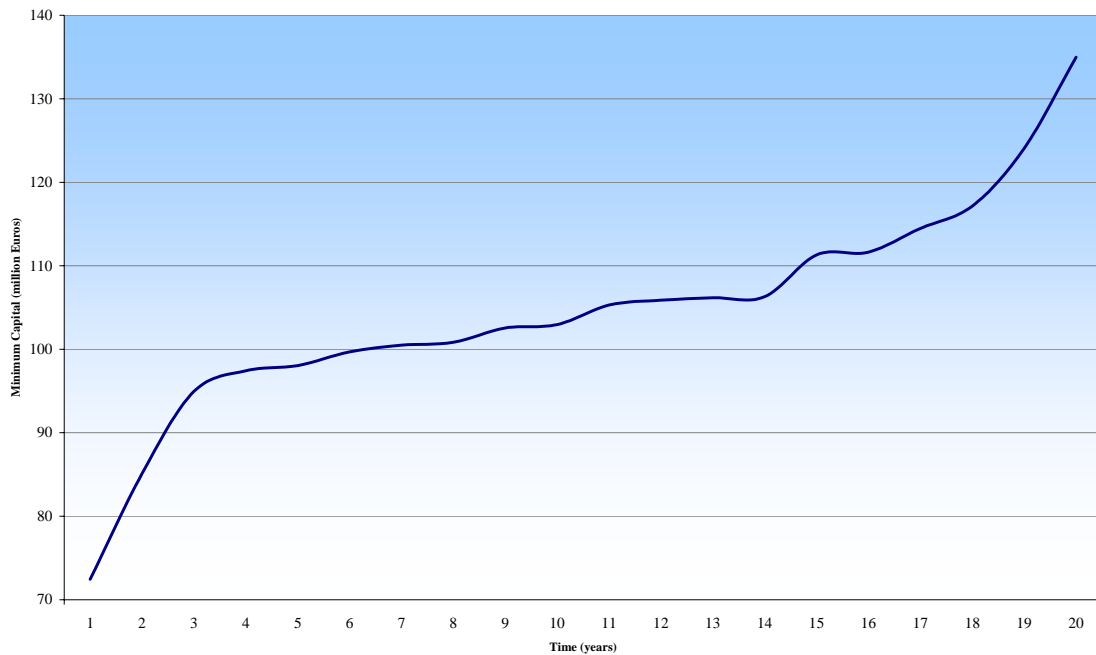
This change of behaviour may be due to problems on the expected returns being too low relative to the increase on the amount of claims and expenses.[21]

Some interesting remarks can be made from the comparison of the last two graphs:

▪ For shorter term horizons, higher capital is required for the original scenario. This may be due to the fact of the original investment portfolio containing equities, which are risky assets. On the other hand, the non-earning assets were assumed to maintain their nominal value through time, and therefore they pose no volatility risk.

▪ From the 6th year ahead, the modified scenario requires increasingly more capital than the original one. The explanation may be that the higher investment returns provided by equities allow for the meeting of the increasing liabilities (both claims and expenses) while the non-earning assets lose real value across time.

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[20] The graphs represents a situation where the percentage invested on equities is set to zero with the one invested on non-earning assets rising accordingly.

[21] Recall that the underwriting results are, on average, negative (see Figure 3-5).

4 Results for the Portuguese General Insurance Market

As at 31st of December of 2004, the Portuguese general insurance market was composed of 27 insurance companies, of which 2 are mutual companies and 6 explore both life insurance and general insurance business.[22]

The studied DFA was applied to a total of 20 companies. The majority of the rejected 7 companies were withdrawn from the analysis because their insurance portfolio was highly concentrated on the lines of business excluded from the model.[23]

However, the ruin probability and the EPD ratio were only calculated for 14 companies. The remaining 6 companies are the ones that explore also life insurance business and thus, a method to separate the total equity capital between the life insurance and the general insurance business is needed to proceed with the analysis. This task is out of the scope of the study.

The model parameters for each company were estimated in an analogous way to what was described for company XYZ in section 3.2. In particular, all companies were assumed to be expecting the same rates of premium growth – growth in line with price inflation – for comparability reasons.

The next graph plots the simulated ruin probability and the expected loss given ruin for the set of 14 general insurance companies:[24]

Figure 4-1 – Ruin probability and Expected loss given ruin for 14 Portuguese general insurance companies

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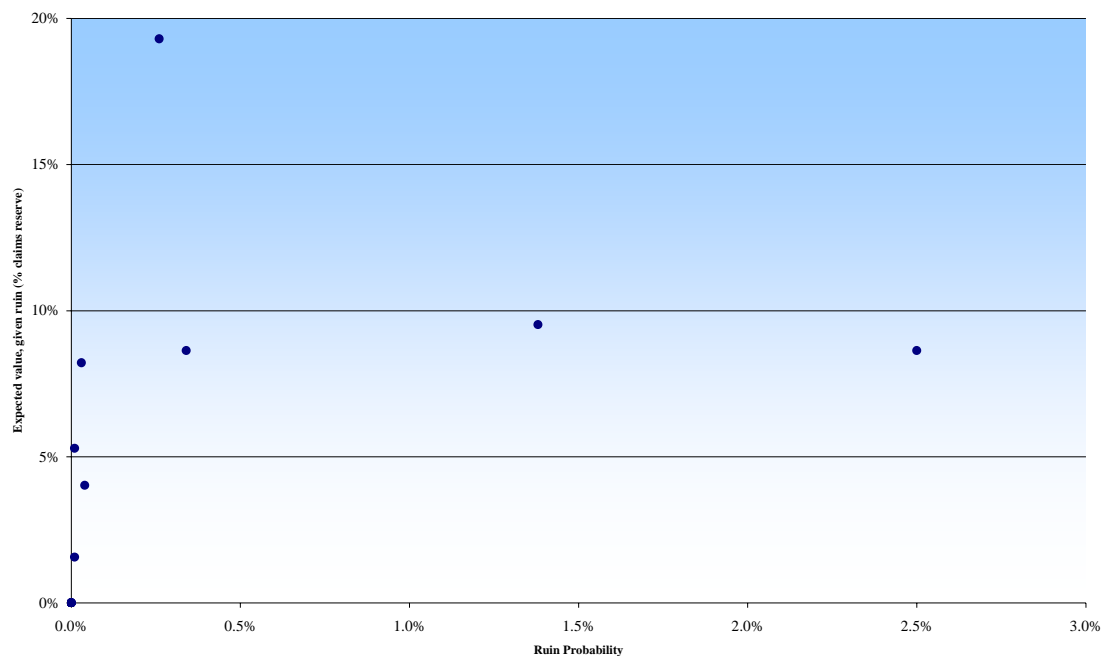
[22] The current number of companies is smaller due to mergers occurred after the mentioned date.

[23] For further details, see Appendix A.

[24] Only 9 points are visible because 6 companies are represented by the point of coordinates (0,0).

The results of the model point out to relatively low values of ruin probability, with all companies registering percentages below 0.5% except two more worrying cases: one company with almost 1.5% and other with 2.5%.

In terms of the expected amount of loss assuming that ruin occurs, the results generate higher preoccupations, especially for the company with the expected loss estimated on almost 20% of its claims reserve.

The next graph introduces the correspondence between the two risk measures estimated, i.e. the ruin probability and the EPD ratio:[25]

Figure 4-2 – Ruin probability and EPD Ratio for 14 Portuguese general insurance companies
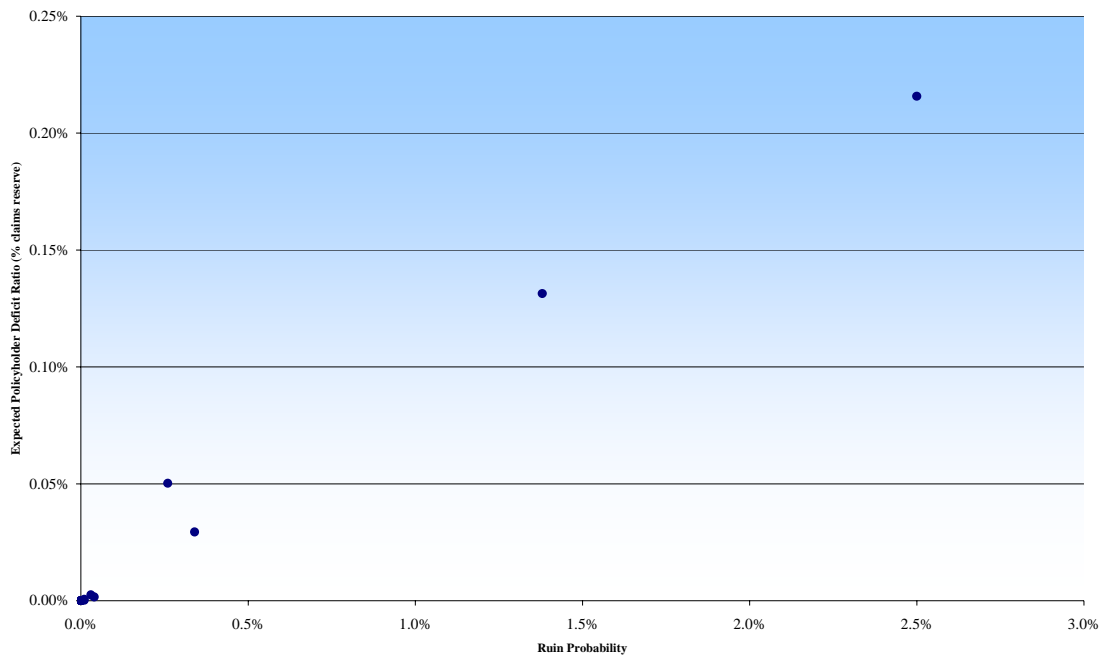
Based only on the EPD ratios, the more worrying cases now seem to be only the two companies with higher ruin probability. The EPD ratio is a better measure than the expected loss given ruin since it takes into account not only the amount of expected loss but also the likelihood of the ruin occurring.

Generally speaking, the results seem to be fairly tolerable. However, to draw a final conclusion it is necessary for the regulator to decide on the benchmarks or limits of risk that is willing to accept for the supervised companies. It is important to notice that risk can never be withdrawn altogether, but the results given by the model can provide very useful alerts both for the regulator and the company.

As referred before, the model can be used to estimate the amount of capital that a company should retain for a given level of risk. This estimation was carried out for all the 20 companies[26]

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[25] The EPD ratio was calculated assuming that the expected value of the liabilitites of a company is equivalent to the amount of its claims reserve.

[26] The analysis here included the six companies exploring both life and general insurance business.

and a comparison was made with the amount of required capital from the current EU solvency system (the solvency margin). The results are represented on the graph below, considering the degrees of risk of 0.1% and 1% and the one-year ruin probability as the reference risk measure:

Figure 4-3 – Comparison between the capital estimated by the model and the current solvency margin
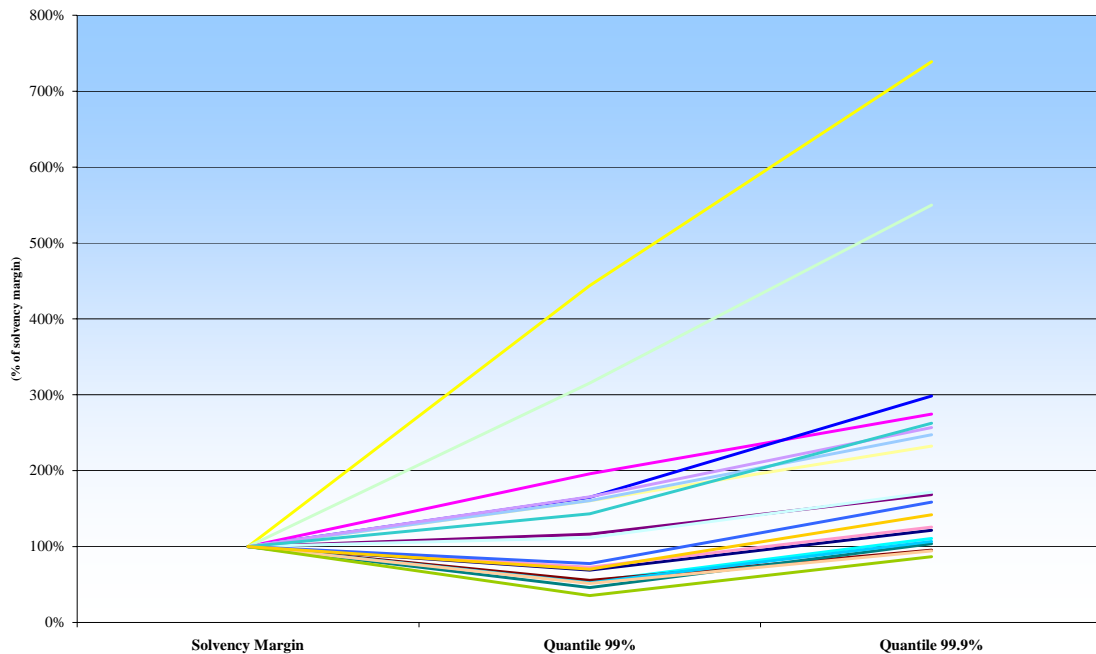
The results show that the consideration of the higher degree of risk (0.1%) implies that almost all companies should retain more capital than the amount taken from the current solvency system. For some companies there is a dramatic increase.

It is interesting to notice that the increase (or decrease in few cases) of the capital requirements happens with different slopes for each company. This highlights the importance of risk-based capital standards and the idea that the current solvency system does not have a strong correspondence with those standards (otherwise the slope of different companies would be fairly similar).

It is noticeable the impact that the choice of the degree of risk has on the estimated amount of capital. In the particular example of the model studied, it is prudent to consider the higher degree of risk because (1) the model was constructed to function on a standardised and thus conservative basis and (2) some weaknesses and limitations were already identified in this model and so the final results should not be taken blindly but subject to discussion.

5 Conclusions

Extensive research and discussion on more sophisticated approaches to insurance regulation, including the construction and implementation of risk-based capital models are currently on the agenda, namely for the EU commission and other international actuarial and regulatory bodies. Amongst many other European companies, Portugal will be affected by the adoption of such new risk-based solvency standards. Therefore, it is now of interest to anticipate the likely impact that the alterations to the current solvency system will cause to the supervised general insurance companies, especially in an attempt to smooth possible drastic effects.

The objective of the present study was to introduce the Dynamic Financial Analysis Models and to demonstrate that these can be a very useful tool for the solvency assessment of general insurance companies. The particular case of the Portuguese general insurance companies was of interest. Based on the information available to the regulator, a fairly basic DFA model was constructed and implemented for those companies, allowing for the risk quantification with measures such as the ruin probability and the expected policyholder deficit.

The studied model was focused on the measurement of the so-called solvency margin, i.e. the cushion of free assets that should be held to allow for the inherent volatility of the insurance business. However, besides this cushion, it is very important to ensure that the other two pillars referred to on the introduction of this report are respected: (1) the technical provisions should be adequate and prudent and (2) sufficient matching assets should be in place. It is worth noticing that these two pillars were not evaluated in the current study. Therefore, if any of the mentioned two conditions fail, so does any conclusions reached solely from the studied DFA model.

Assuming that those two conditions are met, the results achieved by the model confirm the idea that the introduction of more risk-orientated models should cause an increase in the capital requirements relative to those calculated on the basis of the current EU solvency system. According to the results of the model, some of the increases can be quite dramatic. Considering only the 14 companies exploring exclusively general insurance business, the capital held at the end of 2004 would be sufficient to meet the requirements set by the model for: (1) all companies except two, for a degree of risk of 1% and (2) all companies except four for a degree of risk of 0.1%. These numbers are based on an analysis using the one-year ruin probability as the relevant risk measure.

The decision making concerning the solvency assessment of insurance companies needs the setting of an appropriate risk measure and time horizon for the analysis, possibly by the regulator. Both the ruin probability and the EPD are suitable risk measures. However, they do have some important limitations, previously identified on this report, which should be taken into account. Concerning the time horizon, expert opinions recommend a minimum of one-year, to allow for the probable delays of the regulator's intervention on a financial impaired company.

Care should be taken when drawing conclusions from the studied DFA model. It is recognized that it has some weaknesses and limitations, the main ones being:

- The model does not include any variables that directly quantify important risks such as operational risk and liquidity risk. These risks are too important to be neglected and therefore, the analysis of the solvency position of a company, including the capital requirements, should always be complemented by addressing the likelihood and severity of losses associated with those risks, in a quantitative or even qualitative way.

- The model does not consider the impact of reinsurance. The regulator should be interested in encouraging companies to make good use of reinsurance and other risk management techniques. Therefore, a 'positive discrimination' should be allowed for companies that efficiently reduce their risks through these arrangements and this attitude should be evident from the results of the model.

- The model ignores possible cash outflows that affect the P&L Account projections, like for instance non-insurance results, tax and dividends.

- The model assumes independence between periods when the projections are made to a time horizon higher than one year. If this assumption does not hold, than the projected losses are likely to be underestimated.

- Finally, it should be understood that any model, no matter how complex it is, will always be an approximation of reality. Therefore, the results of any model should not be taken blindly, without judgement, as if one were on the presence of a 'crystal ball'.

However, despite its weaknesses and limitations, the studied model does provide a suitable framework on which the risk of different companies can be quantified and compared. This comparison is particularly important for the regulator because the identification of the companies with more worrying and/or uncertain positions can alert the regulator to redirect more attention and resources to those companies in order to enhance their solvency as soon as possible.

The model studied, although not optimal, can be used as a basis for further work. For instance, the following points provide ideas to what can be improved in the model in order to increase its reliability and capabilities:

- Continuous collection of data relevant to the analysis of the major risk factors and calibration of the parameters using higher quantity and, if possible, higher quality of data. If available, external data may be considered to perform this task, if it is credible enough and representative of the Portuguese reality.

- Goodness of fit tests to the results of the model, including statistical and intuitive analysis of the assumptions made – in particular, the verification of the reasonableness of the assumed lognormal distribution for the loss ratios, the normal distribution for the expense ratios and the correlation matrix between loss ratios of different lines of business.

- Calibration of the parameters needed for the asset stochastic model, since the ones estimated by Wilkie (1995) may not be suitable for the reality of the Portuguese general insurance companies.

- If possible, the projection of the claims incurred amounts should be tried through separated estimation of the frequency and severity of claims or through separation of the 'total' loss ratios per accident year.

- Also, if the data allows it, the projection of the expenses should consider the breakdown of expenses per type, considering the likely relationship of each type with the amounts of premiums written, claims incurred and/or inflation.

- Whenever possible, incorporate the impact of reinsurance in the model. This may be possible for 'standardised' types of reinsurance, where the terms and conditions are usually similar to all companies.

- Study and incorporation of additional and relevant correlations between variables. For instance, it may make sense to consider correlations between loss ratios of different periods, in particular if the impact of the insurance cycle is evident.

- Introduce a model for catastrophic events. In particular, this may imply that the change of the 'normal' correlation structure when a major catastrophe is simulated. This is in recognition of the idea that a rare catastrophic event may adversely affect several lines of business at the same time, thus pushing the assumed 'normal' correlations to values approximate to one (perfect positive correlation).

- Overall, the model should reflect all the major factors that affect the future financial position of an insurance company. However, it is always recommended to maintain the model as simple as possible. Simplicity will help on the flexibility and understanding of the model and so, a reasonable balance between accuracy and simplicity is needed.

As mentioned before, further work is needed, and this is especially true now that the EU discussions on risk-based capital models are on its peak. Additionally, other developments are currently in force that will impact on the solvency assessment of insurance companies. For instance, the evaluation of assets and liabilities using a 'fair value' approach and the harmonisation of the accounting standards within countries are likely to change the perspective on which solvency is looked. These issues pose additional challenges for the development of suitable risk-based capital models for which both the companies and the regulators should be prepared.

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Appendix A – Lines of Business Selected for this Study

The general insurance companies in Portugal are required to present the Profit & Loss Account disaggregated per line of business, according with the following list of insurance groups:

1. Accident & Health
   a. Employer's Liability
   b. Personal Accidents
   c. Carried Persons
   d. Health
2. Fire & Other Damages
   a. Fire & Natural Hazards
   b. Agriculture – Fire
   c. Agriculture – Crops
   d. Cattle
   e. Theft
   f. Crystals
   g. Loss of Refrigerated Goods
   h. Machinery Breakdown
   i. 'All-purpose' Property Insurance (include Home, Commercial and Industrial)
   j. Others
3. Motor
   a. Land Vehicles – Property
   b. Goods in Transit
   c. Land Vehicles – Liability
   d. Carried Persons
4. Marine & Other Transports
   a. Railroad Vehicles – Property
   b. Railroad Vehicles – Liability
   c. Boats – Property
   d. Boats – Liability
   e. Goods in Transit
   f. Carried Persons
5. Aircraft
   a. Aircraft – Property
   b. Aircraft – Liability
   c. Goods in Transit
   d. Carried Persons
6. Goods in Transit
7. General Liability

    a.   Product Liability

    b.   Professional Liability

    c.   Business Liability

    d.   Hunters

    e.   Others

8. Others

    a.   Credit

    b.   Pledge

    c.   Financial Loss

    d.   Legal Protection

    e.   Assistance

    f.   Others

The selection of the list of insurance groups to be used in the study was aimed at obtaining a relatively stable history of loss ratios, without forgoing intuition.

The use of the original groups 1-8 revealed, in some cases, very unstable loss ratios. This may be due to different allocation criteria followed by different companies or other data quality problems. Several trials, with the analysis of different groupings taken from the above insurance types, allowed for the selection of the final list presented in Figure 2-3, which exhibited reasonable results. Below is presented the correspondence between the final selection and the original groupings indicated above:

▪ Employer's Liability	1. a)
▪ Personal Accidents	1. b), c)
▪ Health Insurance	1. d)
▪ Property 'All-purpose' Insurance	2. i)
▪ Fire & Other Damages	2. a) to h), j)
▪ Motor	3. all
▪ Goods in Transit	4. e); 5. c); 6.
▪ General Liability	7. all

The items Marine & Other Transports (4. all except e)), Aircraft (5. all except c)) and Others (8. all) were excluded from the analysis because their historic loss ratios were too volatile to be accepted and no reasonable grouping was found to improve this. This implies that 5 companies cannot be analysed, since their portfolio is very biased in the rejected lines of business. For all the other companies, the total portfolio that can be modelled ranges from 91.5% to 100%. The excluded lines of business will be assumed to have a deterministic combined ratio of 100% to allow the model to be run for the latter companies.

<u>Appendix B</u> – Description of the El-Bassiouni Model

The model proposed by El-Bassiouni (1991) was used to calibrate the parameters of the lognormal distribution assumed for the loss ratios of each individual line of business. The following paragraphs summarise the mathematics underlying the model:

Consider a given line of business explored by a total of *a* companies, each having available past data from years 1 to *b*. Let X_{ij} denote the loss ratio for company *i* in year *j* and p_{ij} the respective amount of earned premiums. It will be assumed that:

$$Y_{ij} = \ln(X_{ij}) = \alpha_i + \beta_j + e_{ij}, \quad i = 1,...,a; \quad j = 1,...,b$$

α_i are fixed constants, β_j and e_{ij} are mutually independent normal random variables, where:

$$\beta_j \sim N(0; \theta_2)$$

$$e_{ij} \sim N\left(0; \frac{\theta_1}{p_{ij}}\right)$$

Therefore, the logarithm of the loss ratio X_{ij} (Y_{ij}) is normally distributed with mean and variance:

$$Y_{ij} = \ln(X_{ij}) \sim N\left(\alpha_i; \theta_2 + \frac{\theta_1}{p_{ij}}\right)$$

The variance of the logarithm of the loss ratio is inversely proportional to the risk volume, as measured by the amount of earned premiums. This assumption agrees with the intuitive result that bigger companies are expected to have more predictable and stable results.
X_{ij} is thus lognormally distributed and its first two moments are given by:

$$E(X_{ij}) = \exp\left(\alpha_i + 0.5 \cdot \left(\theta_2 + \frac{\theta_1}{p_{ij}}\right)\right)$$

$$V(X_{ij}) = \exp\left(2 \cdot \alpha_i + \theta_2 + \frac{\theta_1}{p_{ij}}\right) \cdot \left(\exp\left(\theta_2 + \frac{\theta_1}{p_{ij}}\right) - 1\right)$$

There are (*a* + 2) parameters that need to be estimated – α_1, α_2, …, α_a, θ_1 and θ_2. Maximum likelihood estimation will be carried out. The full explanation of the estimation method can be found in El-Bassiouni (1991), in particular the construction of the necessary iterative process.

For reference, the iterative process is presented here in a summarised way and with the exact same notation:

1. Obtain initial estimates of θ_1 and θ_2 using the formulae:

    $$\theta_1^{(0)} = R_1 / [(b-1)(a-1)]$$
    $$\theta_2^{(0)} = [R_2/(b-1) - \theta_1^{(0)}]/a,$$

    where:

    $$R_2 = \frac{1}{a}\sum_{j=1}^{b}\left(\sum_{i=1}^{a}Y_{ij}\right)^2 - \frac{1}{a \cdot b}\left(\sum_{i=1}^{a}\sum_{j=1}^{b}Y_{ij}\right)^2$$

    $$R_1 = \sum_{i=1}^{a}\sum_{j=1}^{b}Y_{ij}^2 - \frac{1}{b}\sum_{i=1}^{a}\left(\sum_{j=1}^{b}Y_{ij}\right)^2 - R_2$$

2. Substitute $\theta_1^{(k)}$ and $\theta_2^{(k)}$ into the following formula to estimate the vector of parameters $\alpha^{(k)}$ (a×1):

    $$\alpha^{(k)} = H^{-1}h$$

    H is a *a×a* matrix whose elements are given by:

    $$H_{rs} = \begin{cases} \sum_{j=1}^{b} p_{rj} - \rho_j \cdot p_{rj}^2, & r = s \\ -\sum_{j=1}^{b} \rho_i \cdot p_{rj} \cdot p_{sj}, & r \neq s \end{cases}$$

    where ρ_i is defined as:

    $$\rho_j = \theta_2 \left/ \left(\theta_1 + \theta_2 \cdot \sum_{i=1}^{a} p_{ij} \right) \right.$$

    h is a *a×1* vector whose elements are given by:

    $$h_r = \sum_{j=1}^{b} p_{rj}\left[Y_{rj} - \rho_j \sum_{i=1}^{a} p_{ij}Y_{ij} \right]$$

3. Substitute $\theta_1^{(k)}$, $\theta_2^{(k)}$ and the vector $\alpha^{(k)}$ into the following formulas to obtain the next iteration, $\theta_1^{(k+1)}$ and $\theta_2^{(k+1)}$:

$$\theta_1^{(k+1)} = \frac{1}{a \cdot b - a}\left[\sum_{i=1}^{a}\sum_{j=1}^{b}p_{ij}Y_{ij}Z_{ij}^{(k)} - \sum_{i=1}^{a}\left(\sum_{j=1}^{b}p_{ij}Y_{ij}\right)\cdot\left(\sum_{j=1}^{b}p_{ij}Z_{ij}^{(k)}\right)\Bigg/\sum_{j=1}^{b}p_{ij}\right]$$

$$\theta_2^{(k+1)} = \sum_{j=1}^{b}\left[\beta_j^{*(k)}\right]^2 \Bigg/ \left[b - \mathrm{tr}\left(T^{(k)}\right)\right]$$

where:

$$\beta_j^* = \rho_j\sum_{i=1}^{a}p_{ij}\left(Y_{ij} - \alpha_i\right)$$

$$Z_{ij} = Y_{ij} - \beta_j^*$$

$$\mathrm{tr}(T) = \left(\theta_1/\theta_2\right)\left[\sum_{j=1}^{b}\rho_j + \mathrm{tr}\left(H^{-1}G\right)\right]$$

Note: tr(*W*) denotes the trace of a matrix *W*, i.e. the sum of its diagonal elements.
G is a *a×a* matrix whose elements are given by:

$$G_{rs} = \sum_{j=1}^{b}\rho_j^2 p_{rj}p_{sj}$$

4. If satisfying convergence is not achieved, go back to step 2.

<u>Appendix C</u> – Incorporating Correlations in the Simulation Process

Each individual loss ratio is assumed to follow a lognormal distribution with given parameters. Additionally, all pairs of loss ratios are assumed to be correlated according to the correlation matrix introduced in Figure 2-12.

The Monte Carlo simulation process functions by successively generating sets of loss ratios, randomly selected from their given probability distributions. However, it is important to guarantee that the generated sets, although random, still preserve the correlation structure assumed on the above mentioned matrix. To solve this problem, Wang (1998) suggested the Normal Copula technique as an efficient correlation model. The mathematical details can be found on his paper. Below is presented the practical procedure that was implemented:

1. Construct the lower triangular matrix *B* (8×8) using the formula:

$$b_{ij} = \frac{\rho_{ij} - \sum_{s=1}^{j-1} b_{is} \cdot b_{js}}{\sqrt{1 - \sum_{s=1}^{j-1} b_{js}^2}}, \quad 1 \le j \le i \le 8,$$

   where ρ_{ij} denotes the correlation between the loss ratios of the lines of business *i* and *j*.

2. Generate a column vector (8×1) of independent standard normal variables *Y*.

3. Let *Z* be the column vector (8×1) taken from the product between the matrices *B* and *Y* and:

$$u_i = \Phi(z_i),$$

   where z_i is the *i*-th element of the vector *Z* and Φ the standard normal (0 , 1) distribution function.

4. The desired set of randomly generated numbers from the required lognormal distributions and satisfying the assumed correlation structure are given by the formula:

$$X_i = F_{X_i}^{-1}(u_i)$$

   Note: F_{X_i} represents the assumed lognormal distribution function for the loss ratio of the *i*-th line of business.

Appendix D – Correspondence of Assets

Equities, long term bonds, cash, property and non-earning assets were the five categories used on the implemented DFA model. To arrive to only five categories, assumptions had to be made regarding the nature of the assets held by the Portuguese insurance companies. Below is presented the list of real asset classes that were included as part of each of the five notional categories:

EQUITIES

Equities

Investment funds - majority on equities

LONG TERM BONDS

Government bonds

Corporate bonds

Investment funds - majority on bonds

Secured and unsecured loans

PROPERTY

Land and Buildings

Property investment funds

CASH

Deposits

Certificates of deposit

NON-EARNING ASSETS

Reinsurance debtors

Credits on policyholders

Other credits