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Long term care and capital accumulation: the impact of the State, the market and the family*

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Abstract

The rising level of long-term care (LTC) expenditures and their financing sources are likely to impact savings and capital accumulation and henceforth the pattern of growth. This paper studies how the joint interaction of the family, the market and the State influences capital accumulation in a society in which the assistance the children give to dependent parents is triggered by a family norm. We find that, with a family norm in place, the dynamics of capital accumulation differ from the ones of a standard Diamond (1965) model with dependence. For instance, if the family help is sizeably more productive than the other LTC financing sources, a pay-as-you-go social insurance might be a complement to private insurance and foster capital accumulation.

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1 Introduction

The purpose of this paper is to analyze the joint impact that alternative ways of financing long-term care (LTC) may have on capital accumulation.

LTC consists of nursing care (as opposed to health care) for people who depend on help to carry out daily activities such as eating, bathing, dressing, going to bed, getting up or using the toilet.[1] The demand for LTC is expected to increase. More than two out of five people aged 65 or older report having some type of functional limitation (sensory, physical, mental, self-care disability, or difficulty leaving home). In the EU, the relative importance of people aged 65+ will more than double by 2050, while the relative importance of people aged 80+ will more than triple.[2] Not only the relative number of dependent elderly will increase but also the costs because of the growing prices of services (the so-called Baumol disease).

On the supply side, the main provider of LTC is the family. Yet, in particular in a context of weakening family ties, individuals may also rely on the market of private insurance and on social policy. Even though the role of the family dominates that of the State and of the market, the relative importance of these three sources of provision varies across countries and over time.[3]

The present paper acknowledges both the importance of LTC and the diversity of its financing sources.[4] It uses a two period OLG model with risk of dependence to assess the influence of the various ways of financing LTC on capital accumulation. A casual look at the problem may lead one to think that the effect of LTC is not going to be different from the effect of any other type of increasing needs in old age, namely a call for more saving. In that respect LTC is expected to stimulate capital accumulation. This reasoning is surely correct

[1]For a recent survey on the economics of LTC, see Cremer et al. (2012) or Siciliani (2013).

[2]Source: European Commission (2013a).

[3]For more details, see European Commission (2013b).

[4]Brown and Finkelstein (2011) provide an overview of the economic and policy issues surrounding insuring LTC expenditure risk. They also discuss the likely impact of recent LTC public policy initiatives.

as long as LTC is financed by saving or private insurance. It is incorrect if LTC services are provided by the State or the family. Intuitively, as long as social insurance and family solidarity operate according to a pay-as-you-go principle, each of these two financing sources depresses capital accumulation. We show that, when different sources of LTC financing coexist, crowding out may lead to surprising results. The most interesting one is that, if family help is taken into account, a pay-as-you-go social LTC insurance may be a complement to private insurance and foster capital accumulation.

Our model rests on three key assumptions. First, we assume that the main motive for children's assistance is a family norm.[5] This idea is pervasive in sociology and gerontology. As an example, Lowenstein and Daatland (2006) study the impact of filial norms on the exchange of intergenerational support between adult children and older parents across five European countries. The effect of filial norms on help provision by children is shown to be moderate but significant and variable across countries, appearing more prescriptive in the South than in the North.[6]

Second, we focus on a particular type of assistance, consisting of an investment that children make before knowing whether their parents are dependent or not. This *ex ante* investment can concern housing or children's location and occupation choices. It is made with the idea that it will be particularly useful in the case parents become dependent. As an example, children may build a house with facilities that are relevant for dependent people.

The third key assumption is that parents prefer their children's help over other sources of LTC at least to a certain level. The idea that parents prefer being taken care by their children than by unknown formal caregivers (see Pauly 1990)[7] is standard and sometimes used to

[5]One generally distinguishes three motives for children helping their dependent parents: altruism, quid pro quo exchange or family norm. The crucial impact of social norms (family ties) for solutions to the LTC financing problem is emphasized by Costa-Font (2010).

[6]See also Silverstein et al. (2006).

[7]According to Pauly (1990), families rationally decide to forego the purchase of LTC insurance due to intrafamily moral hazard. Instead of purchasing insurance, parents will rely on the bequest motive to induce children to provide care. Under complete or incomplete information, Jousten et al. (2005), Pestieau and

explain why parents avoid purchasing private insurance.[8] In this paper, the dependent parents value particularly the effort and time that children put in earning the resources that they devote to filial help. For the early stages of dependence, this assumption seems to be particularly compelling. In more severe cases, such as heavy dementia, the role of children might be less valuable for the dependent parents.

Our analysis will focus not only on the steady state but also on the dynamics along the equilibrium path. In our analysis, the role of the State is restricted to provide a social insurance without aiming at social optimality. Its role is thus quite passive, and our approach mainly positive. We first present what we call the benchmark model, that is, a model à la Diamond without family help but with the possibility for the individuals to purchase private LTC insurance. In such a setting a pay-as-you-go social insurance scheme has a consistently depressive effect on capital accumulation. Also, there is no switch in the insurance behavior along the equilibrium path: individuals either always insure, or they never insure.

These two features do not hold any more when we introduce the family norm. The pay-as-you-go social insurance, when combined with the family norm, can surprisingly have an enhancing effect on capital accumulation. This is due to the fact that social insurance reduces family help; since family help is particularly valuable in case of dependence, individuals might react to its reduction by increasing savings and private insurance coverage. Thus, public insurance might actually be a complement of private insurance in presence of a family norm. This sheds a new light on the debate, initiated by Brown and Finkelstein (2008), about the crowding out effect of public LTC coverage on private insurance. Furthermore, we show that, as the family help increases or decreases over time, switches in insurance can appear; namely, along the dynamic path one can have first private insurance and then not

Sato (2008), and Kuhn and Nuscheler (2011) study the optimal design of a LTC policy in an heterogeneous setting.

[8]The empirical evidence on the crowding out of private insurance by family help is mixed. For instance, Mellor (2001) does not find substantial crowding out effects. Conversely, Costa-Font (2010) provides evidence of a negative correlation between familistic cultures and LTC insurance coverage.

at all, or the other way around. Finally, we show that the strength of the family norm on assistance has a depressive effect on capital accumulation, and that the probability of dependence affects capital accumulation in a non-monotonic way.

Economists have hardly treated the relationship between long-term care expenditures and capital accumulation.[9] The closest works are the ones on the effect of health care and of social security on growth. The literature on health investment, longevity and growth, is extensive, but has a different emphasis.[10] The literature on social security and growth finds that unfunded pension schemes have a depressive effect on capital accumulation relative to fully funded pensions or standard saving. The results of this literature are different from ours in that it always finds that old age family arrangements have the same effect as pay-as-you-go pensions except that they imply much larger incentive effects on either fertility or longevity.

The rest of the paper is organized as follows. In the next section we present the model. In Section 3, we study the benchmark case without family norm. We allow for a family norm in Section 4, and we study how the parameters of the economy affect the dynamics and the steady state of capital accumulation. We conclude in Section 5. We present proofs and analytic developments in several technical appendices.

2 The economy

We consider an overlapping generations model where time is assumed to be discrete. All agents (individuals and firms) are price-takers, and all markets are competitive. Individuals live two periods and, without loss of generality, the size of the population is assumed to be constant. An individual born in t supplies one unit of labor in the first period and receives the market wage w_t. In the second period he is retired and is dependent with probability $p \in (0, 1)$. In this case, he needs LTC.

[9] Some exceptions can be found in Ehrlich and Lui (1991) and Hemmi et al. (2007).

[10] See the literature review of Chen (2007) and Gong et al. (2012).

2.1 The financing of LTC needs.

LTC needs can be financed through different channels: the market, the State and the family.

The market. Individuals can use the market to provide for their LTC needs. First of all, they can self-insure through precautionary savings. By saving s_t in their young age they receive $R_{t+1}s_t$ in their old age, where R_{t+1} is the interest factor. Note that this way of financing LTC is not efficient, since ex-post savings are too high if the individual is not dependent. Alternatively, individuals can purchase an amount $i_t \geq 0$ of private LTC insurance in the first period. Then, they get an insurance allowance $R_{t+1}i_t/p$ in case of dependence in the second period. We thus assume that the insurance contract is actuarially fair.[11] In the following, we will say that individuals insure whenever they purchase private LTC insurance. Of course, even if they do not insure, they might (partially) self-insure through precautionary savings.

The State. The government may provide social LTC insurance through a pay-as-you-go system, by setting a linear tax $\tau \in [0, 1)$ on the labor income of the young in order to finance a transfer to the dependent. Then, each dependent elderly born in t receives a transfer $\tau w_{t+1}/p$. We thus assume no loading factor in social LTC insurance.

The family. The family can provide help to the dependent. In each period t, young individuals devote a fraction $x_t \in [0, 1-\tau]$ of their income to their parent.[12] This fraction is chosen before children know whether or not their parents are dependent.

The fraction of income devoted to parents depends on the past filial help behavior. Each individual observes the fraction $Z_{t-1} = x_{t-1} + \tau \in [0, 1)$ which his parent was willing to devote to his grandparent and the evolution[13] of Z_t across time follows the process $Z_t = \pi Z_{t-1} + \pi(1-\pi)$, where $\pi \in [0, 1]$ captures the intensity of transmission of the family norm.

[11] We showed in an earlier draft, considering a loading factor on the insurance premium does not qualitatively modify the analysis.

[12] The underlying assumption is that children are credit constrained.

[13] This reduced form is in the spirit of the ones generally used in education models which consider that the dynamics of human capital accumulation follows a known (exogenous or endogenous) process.

Thus, Z_t is increasing in Z_{t-1}, and it is equal to 0 if the family norm is not transmitted (i.e. $\pi = 0$), and equal to Z_{t-1} if the norm is perfectly transmitted (i.e. $\pi = 1$). In words, π can be viewed as the intensity of intergenerational imitation (or of transmission of the family norm). Since $Z_t = x_t + \tau$, the evolution of the voluntary help x_t follows a linear process:

$$x_{t+1} = \max\left\{0, \psi(x_t)\right\} \quad \text{with} \quad \psi(x_t) = \pi x_t + (1-\pi)(\pi - \tau) \tag{1}$$

This (linear) reduced form is consistent with the "demonstration effect" developed by Cox and Stark (2005) who state that parents who desire being helped in the future have an incentive to make transfers to their own parents in order to instill appropriate preferences in their children.[14] They posit that the demonstration is not perfect by assuming that with probability ϖ a child will simply imitate his parent's action, while with probability $1 - \varpi$ he will choose an action to maximize his expected utility, aware though that his own child may be an imitator. Applying this approach to our dynamic settings leads to obtain a linear process for the evolution of family help.

Another feature of our specification is that the parents weight the help they receive from their children more than any other transfers (from savings, private and/or social insurance). Indeed, an individual does not merely value his child's help as $x_{t+1}w_{t+1}$, but as $x_{t+1}^{\sigma}w_{t+1}$ with $\sigma \in (0, 1)$ measuring the importance of filial help for the parent. The lower σ, the higher the evaluation of x_{t+1} is (with respect to the other sources of income) for the parent. This captures the fact that, at least in the early stage of dependency[15], the elderly prefer being taken care by relatives rather than by unknown caregivers (see Pauly, 1990). Since an increase in children's help is less valuable if the help is already high, our formulation also takes into account the fact that the parent gets a psychological benefit from filial help, but might feel guilty to receive too much of it. In the limit, if the children devote all their income

[14] For an application of this method to a model of LTC financing without capital accumulation, see Canta and Pestieau (2013).

[15] There exist several stages of dependence that can be characterized by the dependent elderly's ability to perform in different areas of cognition and functioning: orientation, memory, judgment, home and hobbies, personal care, and community.

to their parents (i.e. $x_{t+1} = 1$), the latter do not get any psychological gain from filial help, and evaluate $x_{t+1}w_{t+1}$ as a mere monetary transfer.

Since our goal is to analyze the role of family help in presence of (potential) LTC needs, the family help is here most valuable for the parents in case of dependence. However, this help is not necessarily sunk: if the parent does not turn out to be dependent, he weights the family help by a parameter $\gamma < 1$.

2.2 The production process.

In any period t a single good is produced using two factors, capital K_t and labor L_t. Production occurs according to a Cobb-Douglas technology $AK_t^{\alpha}L_t^{1-\alpha}$ with $A > 0$ and $\alpha \in (0, 1)$.

Equilibrium Prices. As markets are perfectly competitive, each factor is paid its marginal product. Assuming that capital fully depreciates after one period we obtain:

$$w_t = A(1-\alpha)k_t^{\alpha} \quad \text{and} \quad R_t = A\alpha k_t^{\alpha-1} \tag{2}$$

where $k_t = K_t/L_t$ is the capital stock per worker in period t.

Inter-temporal equilibrium. As the endowment of capital at each period is equal to the resources that were not consumed in the preceding period, the capital stock in period $t+1$ is financed by precautionary saving s_t and private LTC insurance i_t. Since the size of the population is constant, we have:

$$k_{t+1} = s_t + i_t \tag{3}$$

In words capital accumulation depends on optimal individual decisions.

2.3 The optimal individual behavior.

In order to understand individual behavior, we first define individual welfare. We then solve the individual optimization program, and study insurance decisions.

Individual welfare. In the first period, each young individual devotes a fraction x_t of his wage w_t to his elderly parent, and a fraction τ to the government.[16] He devotes his remaining income to consumption c_t, precautionary savings s_t, and private LTC insurance i_t. In the second period, he consumes $R_{t+1}s_t$, receives the help from his child and, in case of dependence, receives also the benefits of both the private and the social LTC insurance, respectively $R_{t+1}i_t/p$ and $\tau w_{t+1}/p$.

The welfare $\mathcal{W}_t$ of an individual born in t is:

$$\mathcal{W}_t = u(c_t) + \beta\Big\{(1-p)H^{\text{not dep}}(s_t, x_{t+1}) + pH^{\text{dep}}(s_t, i_t, x_{t+1}, \tau)\Big\}$$

with $\beta \in (0,1)$ is the psychological inter-temporal discount factor and

$$c_t = (1 - \tau - x_t)w_t - s_t - i_t$$

The function $H(.)$ corresponds to second-period utility and is given by

$$H^{\text{not dep}}(.) = u\big[R_{t+1}s_t + \gamma x_{t+1}^{\sigma} w_{t+1}\big]$$

if the individual is not dependent and otherwise by

$$H^{\text{dep}}(.) = (1+\xi)u\left[R_{t+1}\Big(s_t + \frac{i_t}{p}\Big) + \Big(\frac{\tau}{p} + x_{t+1}^{\sigma}\Big)w_{t+1}\right] - D$$

with $\xi > 0$ and $D > 0$.

The function $H(.)$ takes into account the fact that the individual does not attribute the same value to the voluntary transfer received from his child as to other means of financing LTC. Importantly, $H^{\text{dep}'}(\kappa) > H^{\text{not dep}'}(\kappa)$ captures the fact that dependent individuals have higher needs.[17] The parameter $D > 0$ measures the utility loss implied by dependence and

[16] We thus implicitly assume that child's help is subject to payroll taxation exactly like precautionary saving and private LTC insurance. The alternative implying $c_t = (1-\tau)(1-x_t)w_t - s_t - i_t$ would not have conducted to much different results.

[17] The assumption that $H^{\text{dep}'}(\kappa) > H^{\text{not dep}'}(\kappa) = u'(\kappa)$ may be disputed (see, for instance, Finkelstein et al. 2009, 2013), since some goods may substitute or complement good health. Our assumption remains reasonable up to a certain wealth level, and we implicitly assume in this paper that this wealth threshold is not reached.

is assumed to be high enough to ensure that $H^{\text{dep}}(\kappa) < H^{\text{not dep}}(\kappa)$ for any feasible value of κ.[18]

Finally, for the sake of tractability, the instantaneous utility function $u(.)$ is assumed to be logarithmic.

The optimization problem. From now on, we use the indicator function $\mathbb{1} \equiv \mathbb{1}_{\pi>\tau\geq 0}$ to encompass the benchmark case without family help where $\mathbb{1} = 0$ (i.e. $\pi = \tau = 0$ and $\pi = 0 < \tau < 1$) and the case with family help where $\mathbb{1} = 1$ (i.e. $\pi > \tau \geq 0$).[19] Then, using (1) an individual born in t solves the following problem:

$$\max_{s_t,i_t} \mathcal{W}_t = \max_{s_t,i_t} \Bigg\{ \ln\left[(1-\tau-x_t\mathbb{1})w_t - s_t - i_t\right] + (1-p)\beta\ln\left[R_{t+1}s_t + \gamma\psi^{\sigma}(x_t)\mathbb{1}w_{t+1}\right] + p\beta(1+\xi)\ln\left[R_{t+1}\left(s_t + \frac{i_t}{p}\right) + \left(\frac{\tau}{p} + \psi^{\sigma}(x_t)\mathbb{1}\right)w_{t+1}\right] - p\beta D\Bigg\}$$

under the non-negative constraints $s_t \geq 0$ and $i_t \geq 0$.

To avoid unrealistic corner solutions in which individuals do not self-insure through precautionary savings (and then rely exclusively on family help if they are not dependent), we will make the following assumption:

Assumption 1. $\gamma < 1/(1+\xi)$.

As it is shown in Appendix A, Assumption 1 is sufficient (but not necessary) to have a positive s_t. Intuitively the weight γ of the family help is perceived by the parent as being low and incite him to self-insure through precautionary savings.

Hence, the first order condition (FOC) with respect to s_t is:

$$\frac{-1}{(1-\tau-x_t\mathbb{1})w_t - s_t - i_t} + \frac{\beta p(1+\xi)}{s_t + \dfrac{i_t}{p} + \left[\dfrac{\tau}{p} + \psi^{\sigma}(x_t)\mathbb{1}\right]\dfrac{w_{t+1}}{R_{t+1}}} + \frac{\beta(1-p)}{s_t + \left[\gamma\psi^{\sigma}(x_t)\mathbb{1}\right]\dfrac{w_{t+1}}{R_{t+1}}} = 0 \quad (4)$$

[18]Since we always obtain bounded steady states solutions, the resources of the economy are always finite and consumption is bounded by a threshold κ_{max}. Since $H^{\text{dep}}(\kappa) < H^{\text{not dep}}(\kappa) = u(\kappa)$ for any $\kappa < \widetilde{\kappa} = u^{-1}[D/\xi]$, it is sufficient to assume that D is such that $\kappa_{max} < \widetilde{\kappa}$.

[19]If $0 < \pi < \tau$, family help may vanish along the equilibrium path. We thus rule out this case (see Assumption 2).

Remark that without transfers from external sources (i.e. $\tau = \pi = 0$), this FOC would not depend on R_{t+1}.

Insurance behavior. The unconstrained solution for i_t could be negative, leading to a corner solution. Conversely, when individuals insure the FOC with respect to i_t is:

$$\frac{-1}{(1-\tau-x_t\mathbb{1})w_t - s_t - i_t} + \frac{\beta(1+\xi)}{s_t + \dfrac{i_t}{p} + \left[\dfrac{\tau}{p} + \psi^\sigma(x_t)\mathbb{1}\right]\dfrac{w_{t+1}}{R_{t+1}}} = 0 \tag{5}$$

Then, we formally obtain the following optimal level for insurance (see Appendix A.1):

$$i_t = \begin{cases} 0 & \text{if} \quad \delta\xi \leq \varepsilon(x_t) \\ \dfrac{[\delta\xi - \varepsilon(x_t)]s_t}{\delta/p + \varepsilon(x_t)} & \text{if} \quad \delta\xi > \varepsilon(x_t) \end{cases} \tag{6}$$

with $\delta = \alpha/(1-\alpha)$ and $\varepsilon(x_t) = \tau/p + \big[1 - \gamma(1+\xi)\big]\psi^\sigma(x_t)\mathbb{1}$.

Depending on the values of τ, p, ξ, γ, σ, π and x_t, $\varepsilon(x_t)$ can take any values in $[0, +\infty)$ while $\delta\xi$ can take any values in $(0, +\infty)$. This will lead to different dynamics of capital accumulation depending on the relative importance of sources of LTC financing.

3 Benchmark case: absence of family help

In order to understand the role of the family, we will first study an economy where the family help is not operative, i.e. individuals cannot directly help their elderly parents. We will denote this case with the subscript d, since it corresponds to the model of Diamond (1965) adapted to allow for dependence.

3.1 The market.

First, consider the case where the government does not intervene, i.e. $\tau = 0$. Then, there are no intergenerational transfers, $\varepsilon(x_t) = 0$, and individuals only provide for dependence in old age through precautionary savings or private LTC insurance. According to (6), individuals insure and this decision is independent of the capital stock. Based on equations (2) to (6), the dynamics of capital accumulation are given by (see Appendix B.1):

$$k_{t+1} = \zeta_p k_t^\alpha \quad \text{with} \quad \zeta_p = \frac{A(1-\alpha)(1+p\xi)\beta}{1+(1+p\xi)\beta} \tag{7}$$

Intuitively, individuals always transfer a share $A^{-1}\zeta_p/(1-\alpha)$ of their wage w_t to the second period using precautionary savings and private LTC insurance. According to (7), there exists a unique positive steady state capital stock,

$$k_{d|\tau=0} = \left[\frac{A(1-\alpha)\beta(1+p\xi)}{1+(1+p\xi)\beta}\right]^{\frac{1}{1-\alpha}}$$

which is globally stable in $\mathbb{R}_+^\star$, i.e., for all $k_0 \in \mathbb{R}_+^\star$, the optimal path $\{k_t\}_{t\geq 0}$ converges monotonically to $k_{d|\tau=0}$.

Remark that, in the case without dependence, we find the well-known dynamics of the standard Diamond's model i.e: $k_{t+1} = \zeta_0 k_t^\alpha$ with $\zeta_0 = A(1-\alpha)\beta/(1+\beta)$. Compared to this standard model, we have introduced the probability of dependence. Since $k_{d|\tau=0}$ increases in p, the probability of dependence has a positive impact on capital accumulation (see Appendix B.2). The higher the probability of dependence, the higher the savings and/or insurance coverage.

3.2 The market and the State.

We now consider the case where the government intervenes through the social (unfunded) LTC insurance described in Section 2.1, i.e. $\varepsilon(x_t) = \tau/p$. Individuals take this into account when choosing how much private LTC insurance to purchase and how much to save. According to (6), they insure if and only if $p\delta\xi > \tau$.

Three remarks about this condition can be made. First, the decision to insure is invariant through time and is not affected by the capital stock. Second, if the tax rate is relatively high individuals do not insure. The social LTC insurance crowds out the private LTC one. Third, individuals trade-off between the return of precautionary savings R and the one of private LTC insurance R/p. Thus, the higher is p the higher the attractiveness of private LTC insurance.

According to equations (2) to (6), the dynamics of capital accumulation can be described by (see Appendix B.1):

$$k_{t+1} = \vartheta_d k_t^{\alpha} \tag{8}$$

with:

$$\vartheta_d = \begin{cases} \eta_d \equiv \dfrac{A(1-\alpha)(1-\tau)\beta[\alpha p(1+p\xi)+(1-\alpha)(1-p)\tau]}{\alpha p[1+(1+p\xi)\beta]+(1-\alpha)[1+(1-p)\beta]\tau} & \text{if} \quad p\delta\xi \leq \tau \\ \\ \mu_d \equiv \dfrac{A(1-\alpha)(1-\tau)\alpha\beta(1+p\xi)}{\alpha[1+(1+p\xi)\beta]+(1-\alpha)\tau} & \text{if} \quad p\delta\xi > \tau \end{cases}$$

Since the sign of $p\delta\xi - \tau$ is time-independent, no switch in the insurance behavior is possible: individuals choose either to insure or not to insure in all periods. We can thus identify two regimes, characterized by the presence (or absence) of private LTC insurance along the optimal path $\{k_t\}_{t\geq 0}$. The existence of these two different dynamics is due to the presence of the social LTC insurance. As we have shown above, the insurance behavior does not affect capital accumulation when the government does not intervene (i.e. $\eta_d|_{\tau=0} = \mu_d|_{\tau=0} = \zeta_p$).

According to (8), there exists a unique positive steady state capital stock,

$$k_d = \begin{cases} k_d^n \equiv \left(\dfrac{A(1-\alpha)(1-\tau)\beta\left[\alpha p(1+p\xi)+(1-\alpha)(1-p)\tau\right]}{\alpha p[1+(1+p\xi)\beta]+(1-\alpha)[1+(1-p)\beta]\tau}\right)^{\frac{1}{1-\alpha}} & \text{if} \quad p\delta\xi \leq \tau \\ \\ k_d^i \equiv \left(\dfrac{A(1-\alpha)(1-\tau)\alpha\beta(1+p\xi)}{\alpha[1+(1+p\xi)\beta]+(1-\alpha)\tau}\right)^{\frac{1}{1-\alpha}} & \text{if} \quad p\delta\xi > \tau \end{cases}$$

which is globally stable in $\mathbb{R}_+^{\star}$, i.e., for all $k_0 \in \mathbb{R}_+^{\star}$, the optimal path $\{k_t\}_{t\geq 0}$ converges monotonically to k_d.

Remark that the steady state is such that $k_d = \max\left\{k_d^i, k_d^n\right\}$, where the superscripts "n" and "i" denote "no insurance" and "insurance", respectively.

We can now look more closely at the effect of τ on capital accumulation (see Appendix B.3 and Figure 1).[20] The capital stocks k_d^i and k_d^n are both decreasing functions of τ. When the

[20] In all figures illustrating the comparative statics with respect to the steady state capital stock k, we always assume, that k varies in a convex way. However, depending on the cases and the parameters specifications, k may also vary in a concave way.

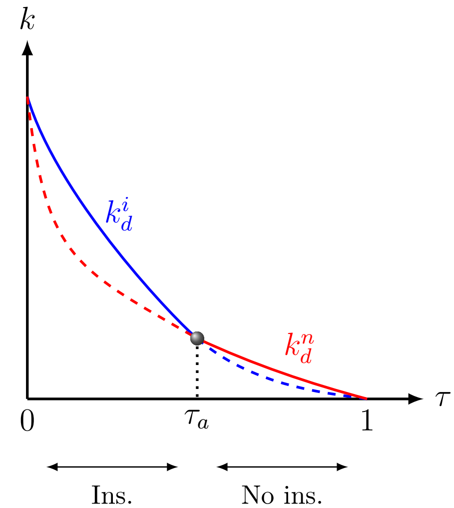


Figure 1: The steady state capital stock k_d as a function of τ.

tax rate increases, the disposable income decreases and this income effect reduces savings and capital accumulation. Furthermore, individuals insure if and only if $\tau < \tau_a = \min\{p\delta\xi, 1\}$. Intuitively, as τ increases, individuals get more social LTC insurance, which discourages precautionary savings and private LTC insurance.

To conclude, our results are standard, and mirror Diamond's model with the only difference that we introduced dependence and LTC insurances. They can be summarized as follows:

Proposition 1 – *Without family help, the capital stock k_t converges monotonically to k_d. The steady state capital stock decreases as the tax rate increases. The insurance behavior is time invariant. If the tax rate τ and/or the probability $1-p$ are sufficiently low (resp. high), individuals always insure (resp. never insure).*

We will now study whether these results are robust to the introduction of the family.

4 The State, the market, and the family

We now consider the case where children can help their parents. As described previously, family help is triggered by a norm imposing that a certain fraction of children's earnings is devoted to the parents.

In the following we characterize first the dynamics of voluntary family help, then the dynamics of capital accumulation. Finally, we study the effect of the intensity of intergenerational imitation, the probability of dependence, and the tax rate on the steady state capital stock.

4.1 The dynamics of family help.

We here want to focus on the case where the family help is always operative. Henceforth we restrict our study as follows.

Assumption 2. $\tau < \pi$ and $x_0 < 1 - \tau$.

Assumption 2 ensures that $x_t \in (0, 1-\tau)$.[21] Then, the dynamics $\{x_t\}_{t \geq 0}$ of family help described by (1) and represented in Figure 2 converge monotonically to $\widetilde{x} = \pi - \tau$.

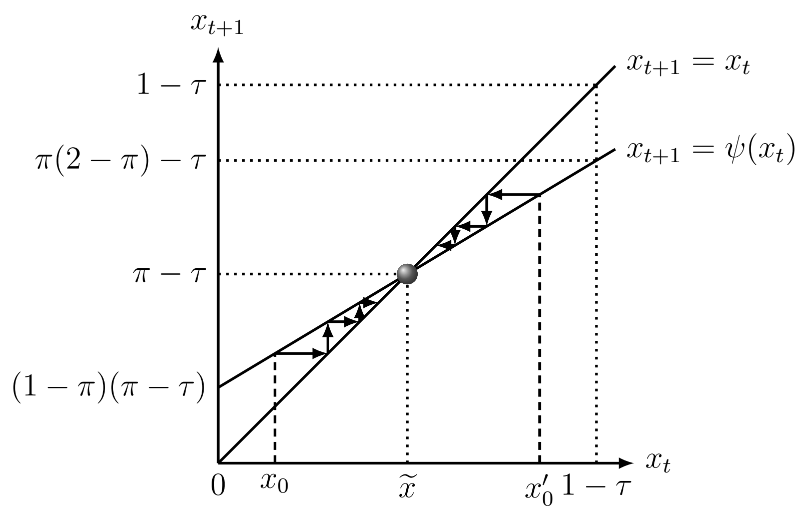


Figure 2: The dynamics of family help $x_{t+1} = \psi(x_t)$.

Remark that the fraction x_{t+1} is linear in x_t, and non-linear in π. Indeed, given x_t, x_{t+1} is a concave function of π, increasing up to $\bar{\pi}_t = (1 + x_t + \tau)/2$ and decreasing afterwards. Then, the parameter $\pi \in (\tau, 1]$ measures the intensity of imitation, but also its imperfection. This allows the individuals to devote a bigger or a smaller share with respect to the past generation. If $\pi \in (\tau, x_t + \tau)$, the imitation is weak and children transmit a smaller fraction

[21] If $\tau > \pi$, then there exists a date after which $x_t = 0$, and we would be in the case studied in Section 3. The case $x_0 > 1 - \tau$ has already been excluded because children are credit constrained.

than their parents did (i.e., $x_{t+1} < x_t$). If $\pi \in (x_t + \tau, 1)$, the imitation is strong and children transmit a bigger fraction than their parents did (i.e., $x_{t+1} > x_t$). Finally, in the limit case where $\pi = 1$, imitation is perfect, and $x_{t+1} = x_t$.

4.2 The global dynamics.

Insurance behavior. According to (6), individuals insure if and only if $\delta\xi > \varepsilon(x_t)$. Here, it is important to emphasize that $\varepsilon(x_t)$ depends on x_t and is then time-dependent. Thus, contrary to the benchmark case, changes in the insurance behavior over time are possible.

Assumption 1 ensures that $\varepsilon(x_t)$ increases in x_t. Then, $\varepsilon(x_t)$ and $\delta\xi$ cannot cross more than in one point denoted by $\hat{x}$. Consequently, individuals insure for any $x_t < \hat{x}$ and do not insure for any $x_t > \hat{x}$. Since the dynamics of x_t are monotonic (increasing if $x_0 < \widetilde{x}$ and decreasing if $x_0 > \widetilde{x}$) and independent of k_t, two cases can arise. When $\delta\xi$ is neither too high nor to low, $\hat{x}$ belongs to $\mathcal{I} = [\min\{x_0, \widetilde{x}\}, \max\{x_0, \widetilde{x}\}]$. Then, there exists a unique period T after which the sign of the sequence $\{x_t - \hat{x}\}_{t \geq 0}$ changes. Individuals change their insurance behavior after period T. When $\delta\xi$ is very high or very small, $\hat{x}$ does not belong to $\mathcal{I}$ and individuals either never insure or always insure. Under Assumptions 1 and 2, one (and unique) switch in the insurance behavior is possible and we can thus distinguish four regimes characterized by the insurance behavior along the equilibrium path (see Appendix C.4). When $\delta\xi \leq \min\{\varepsilon(x_0), \varepsilon(\tilde{x})\}$, there is no insurance in any period (Regime I). When $\delta\xi > \max\{\varepsilon(x_0), \varepsilon(\tilde{x})\}$, there is positive insurance in any period (Regime II). When $\varepsilon(\tilde{x}) < \delta\xi \leq \varepsilon(x_0)$ the dynamics display a switch from no insurance to insurance along the equilibrium path (Regime III). Finally, when $\varepsilon(x_0) < \delta\xi \leq \varepsilon(\tilde{x})$, the dynamics displays a switch from no insurance to insurance (Regime IV).

The dynamics of capital accumulation. Importantly, the impact of the parameters of interest on the insurance behavior, described above, is independent qualitatively of the level of γ. Thus, without loss of generality but for the sake of tractability, we illustrate the dynamics of capital accumulation when $\gamma = 0$. In this case, $\varepsilon(x_t) = \tau/p + \psi^{\sigma}(x_t)$ can be interpreted as the transfer that an individual in $t + 1$ receives in case of dependence from external sources (the State and his child). According to equations (2) to (6), these dynamics are given by (see Appendices A.2 and A.3):

$$k_{t+1} = \vartheta(x_t) k_t^{\alpha} \tag{9}$$

with:

$$\vartheta(x_t) = \begin{cases} \eta(x_t) \equiv \dfrac{A(1-\alpha)(1-x_t-\tau)\beta[\alpha(1+p\xi)+(1-\alpha)(1-p)\varepsilon(x_t)]}{\alpha[1+(1+p\xi)\beta]+(1-\alpha)[1+(1-p)\beta]\varepsilon(x_t)} & \text{if } \delta\xi \leq \varepsilon(x_t) \\ \\ \mu(x_t) \equiv \dfrac{A(1-\alpha)(1-x_t-\tau)\beta\alpha(1+p\xi)}{\alpha[1+(1+p\xi)\beta]+(1-\alpha)p\varepsilon(x_t)} & \text{if } \delta\xi > \varepsilon(x_t) \end{cases}$$

Using $\varepsilon(\tilde{x}) = \tau/p + (\pi-\tau)^{\sigma}$ and according to (9), there exists a unique positive steady state capital stock,

$$\widetilde{k} = \begin{cases} \widetilde{k}^n \equiv \left(\dfrac{A(1-\alpha)(1-\pi)\beta[\alpha(1+p\xi)+(1-\alpha)(1-p)\varepsilon(\tilde{x})]}{\alpha[1+(1+p\xi)\beta]+(1-\alpha)[1+(1-p)\beta]\varepsilon(\tilde{x})}\right)^{\frac{1}{1-\alpha}} & \text{if } \delta\xi \leq \varepsilon(\tilde{x}) \\ \\ \widetilde{k}^i \equiv \left(\dfrac{A(1-\alpha)(1-\pi)\beta\alpha(1+p\xi)}{\alpha[1+(1+p\xi)\beta]+(1-\alpha)p\varepsilon(\tilde{x})}\right)^{\frac{1}{1-\alpha}} & \text{if } \delta\xi > \varepsilon(\tilde{x}) \end{cases}$$

Remark that the steady state capital stock is such that $\widetilde{k} = \max\left\{\widetilde{k}^i, \widetilde{k}^n\right\}$.

The dynamics of $\{k_t\}_{t\geq 0}$ are more complex with family help than in the benchmark case, because $\eta(x_t)$ and $\mu(x_t)$ depend on x_t.

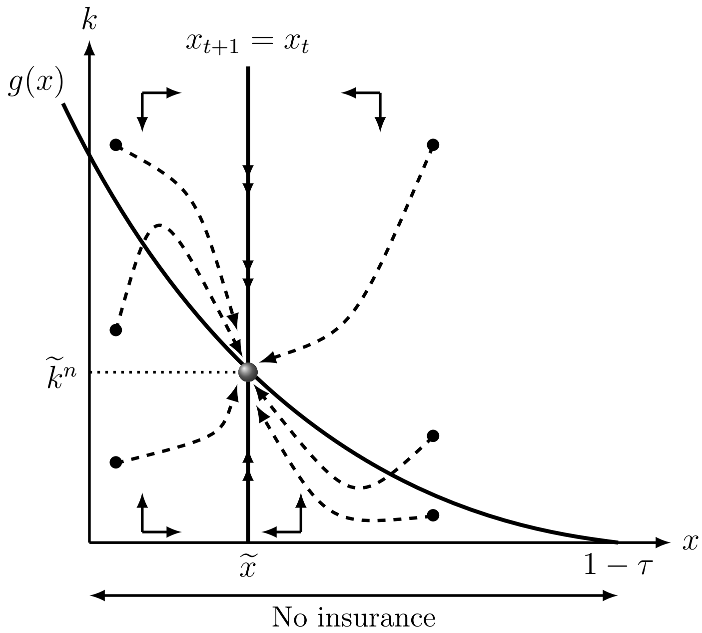
The global dynamics. According to (1) and (9), the dynamic system (k_t, x_t) is described by:

$$\begin{cases} k_{t+1} = \max\left\{\eta(x_t), \mu(x_t)\right\} \times k_t^{\alpha} \\ x_{t+1} = \psi(x_t) = \pi x_t + (1-\pi)(\pi-\tau) \end{cases}$$

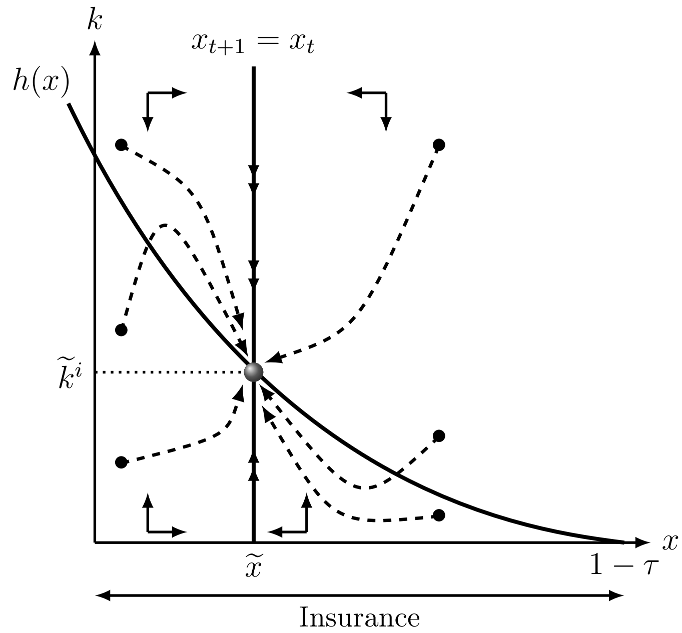
It is globally asymptotically stable and converges to a unique steady state: the pair $(\widetilde{k}, \widetilde{x})$. We can thus distinguish the four regimes characterized by the insurance behavior along the equilibrium path evoked in the beginning of this section (see Figure 3).[22]

Our main results established Appendix C can be summarized as follows:

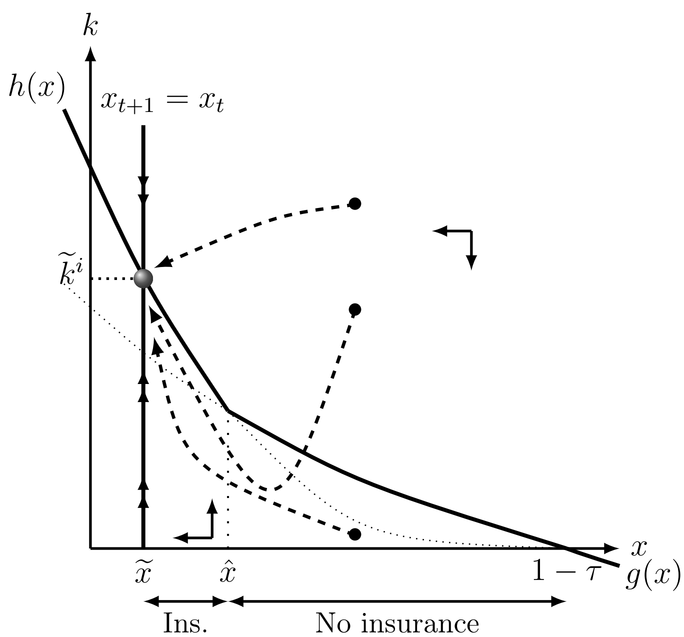
[22]In Figure 3, $g(x) = \eta(x)^{\frac{1}{1-\alpha}}$ whereas $h(x) = \mu(x)^{\frac{1}{1-\alpha}}$.



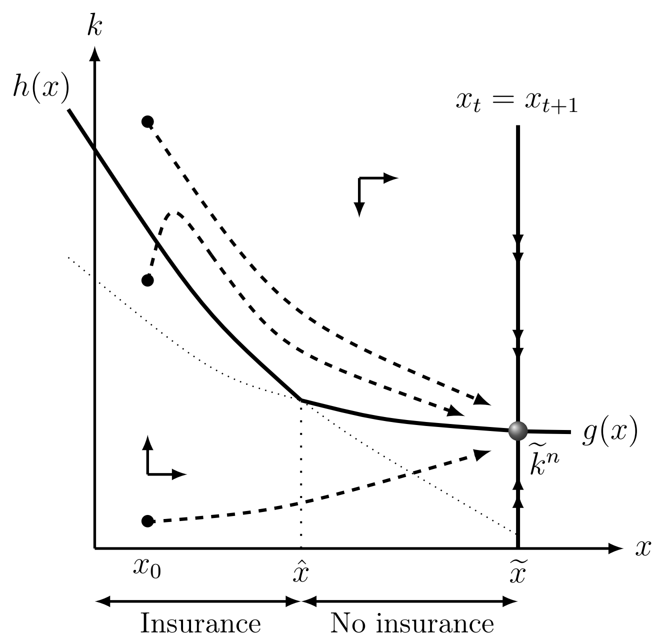
A – Regime I: $\delta\xi \leq \min\{\varepsilon(x_0), \varepsilon(\tilde{x})\}$



B – Regime II: $\delta\xi > \max\{\varepsilon(x_0), \varepsilon(\tilde{x})\}$



C – Regime III: $\varepsilon(\tilde{x}) < \delta\xi \leq \varepsilon(x_0)$



D – Regime IV: $\varepsilon(x_0) < \delta\xi \leq \varepsilon(\tilde{x})$

Figure 3: The global dynamics with family norm.

Proposition 2 – *The dynamic system* (k_t, x_t) *is defined by* (9) *and* (1). *For all* $(k_0, x_0) \in \mathbb{R}_+^\star \times \mathbb{R}_+$, *this system is globally asymptotically stable and converges to* $(\widetilde{k}, \widetilde{x})$. *The dynamics of capital accumulation are not necessarily monotonic.*

Individuals always (resp. never) insure if $\delta\xi$ is sufficiently high (resp. low). For intermediate values of $\delta\xi$ and $x_0 < \widetilde{x}$ (resp. $x_0 > \widetilde{x}$), individuals insure (resp. do not insure) up to a certain period, and then decide not to insure (resp. insure). Thus, one (and unique) switch in the insurance behavior is possible.

Contrary to the benchmark case without family help, the dynamics of capital accumulation are not necessarily monotonic and might be characterized by switches in the insurance behavior. Intuitively, since the family help and the private LTC insurance are substitutes, the dynamics of family help affect the insurance behavior over time. If family help increases over time, individuals might reduce the purchase of private LTC insurance. After a certain period, the market for private LTC insurance completely disappears. To the contrary, if family help decreases over time, the market for private LTC insurance might emerge after a certain period.

4.3 Comparative statics.

We now study the impact of the parameters of the economy on insurance behavior and long run capital accumulation.

Insurance behavior. First of all, we can analyze how variations in π, p, and τ affect the insurance regime in the steady state (see Appendix D.1). Since private LTC insurance occurs at the steady state if and only if $\delta\xi > \varepsilon(\tilde{x})$, it is sufficient to study how $\varepsilon(\tilde{x})$ varies with these parameters. As $\varepsilon(\tilde{x})$ is increasing in π, there exists a threshold π_a such that individuals insure if and only if the degree of intergenerational imitation is smaller than π_a. When $\tau > 0$, the $\varepsilon(\tilde{x})$ is decreasing in p so that there exist a threshold p_a such that individuals insure if and only if the probability of dependence exceeds p_a. When $\tau = 0$, the insurance regime does not depend on p. Finally, the derivative of $\varepsilon(\tilde{x})$ with respect to τ has the sign of $\underline{\tau} - \tau$ with $\underline{\tau} = \pi - [p\sigma(1-\gamma(1+\xi))]^{1/(1-\sigma)}$. If $\pi < [p\sigma(1-\gamma(1+\xi))]^{1/(1-\sigma)}$, then $\varepsilon(\tilde{x})$ always decreases in τ, and there exists a threshold τ_b such that individuals insure if and only if the tax rate exceeds τ_b. If $\pi \geq [p\sigma(1-\gamma(1+\xi))]^{\sigma-1}$, there exist two thresholds, τ_c and τ_d, such that insurance occurs if and only if the tax rate is smaller than τ_c or greater than τ_d.

It is worth noting that the impact the impact of the parameters of interest on the insurance behavior, described above, is independent qualitatively of the level of γ. Thus, without loss of generality and for the sake of tractability, we follow the Section 4.2 by illustrating the comparative statics on capital accumulation when $\gamma = 0$.

Intensity of intergenerational imitation. We now study the impact of π, the intensity of imitation, on capital accumulation (see Appendix D.2 and Figure 4). As π increases from τ to 1, $\varepsilon(\tilde{x})$ increases from τ/p to $\tau/p + (1-\tau)^\sigma$. Then, the steady state capital stock $\widetilde{k}$ is $\widetilde{k}^i$ (and individuals insure) whenever $\pi < \pi_a = \tau + \max\{0, \min\{(\delta\xi - \tau/p)^{1/\sigma}, 1-\tau\}\}$, and $\widetilde{k}^n$ (and individuals do not insure) if $\pi \geq \pi_a$. Since $\widetilde{k}^i$ and $\widetilde{k}^n$, the steady state capital stock $\widetilde{k}$ is always decreasing in $\pi \in (\tau, 1]$.

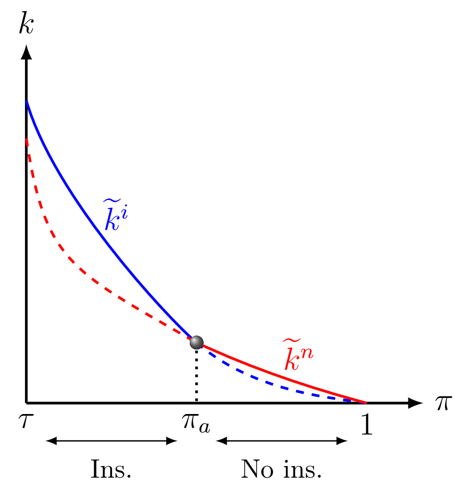


Figure 4: The steady state capital stock $\widetilde{k}$ as a function of π.

Intuitively, if π increases, the fraction of income $\widetilde{x}$ devoted to elderly parents increases.[23] On the one hand, this reduces the disposable income in young age. On the other hand, this also increases the transfer that individuals expect from their children. These two effects lead to a reduction in precautionary saving and private LTC insurance and, consequently, in the capital stock.

[23]Note that this is not necessarily the case along the equilibrium path since $\partial x_t/\partial\pi$ is negative when $\pi > (1 + x_{t-1} + \tau)/2$.

Probability of dependence. The impact of p on capital accumulation is somehow more complex (see Appendix D.3). It depends on whether the government intervenes or not in providing social LTC insurance.

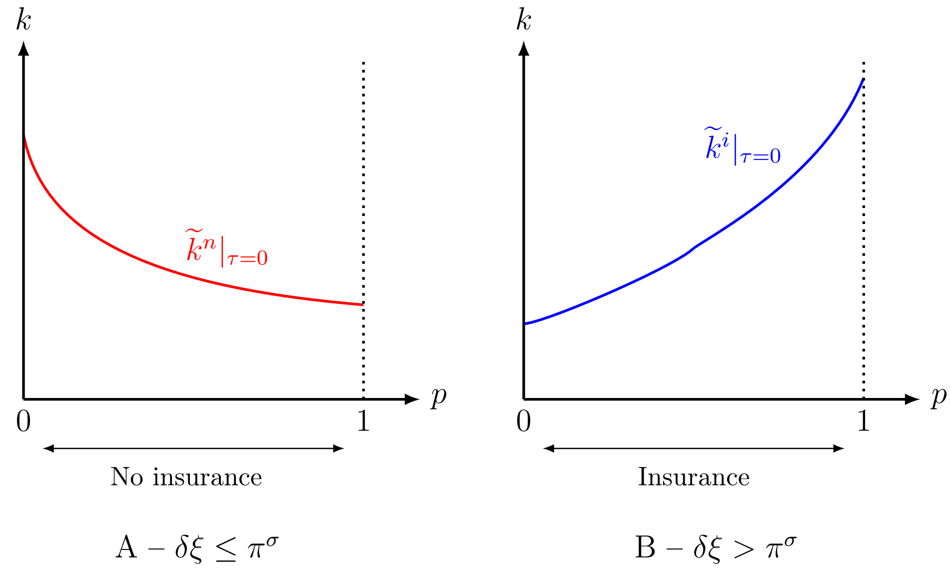


Figure 5: The steady state capital stock $\widetilde{k}$ as a function of p when $\tau = 0$.

When the government does not intervene, $\varepsilon(\tilde{x}) = \pi^\sigma$ does not depend on p and two cases can be identified depending on the intensity of intergenerational imitation. When π is sufficiently high (i.e., $\delta\xi \leq \pi^\sigma$) the family help is so high that individuals decide not to insure. Note also that the steady state capital stock $\widetilde{k}^n|_{\tau=0}$ decreases when p increases (see Figure 5A). Intuitively, as the probability of dependence increases, it becomes less interesting to transfer consumption to the non-dependent state, while the LTC needs will be met by family help. When π is sufficiently low (i.e., $\delta\xi > \pi^\sigma$), individuals decide to insure and the steady state capital stock $\widetilde{k}^i|_{\tau=0}$ is increasing in p (see Figure 5B). Intuitively, the higher the probability of dependence, the more individuals insure for old age, so that the capital stock increases.

Consider now the case where $\tau > 0$. As p increases from 0 to 1, the threshold $\varepsilon(\tilde{x})$ decreases from $+\infty$ to $\tau + (\pi - \tau)^\sigma$. Consequently, when $\delta\xi$ is sufficiently low (i.e., $\delta\xi \leq \tau + (\pi - \tau)^\sigma$) individuals decide not to insure and the steady state capital stock $\widetilde{k}^n$ decreases (resp: increases) when p is lower (resp: larger) than a threshold $\underline{p} \in (0, 1]$ defined Appendix D.3

(see Figure 6A). Thus, we find that the relationship between the probability of dependence and capital accumulation can be non monotonic. Intuitively, when p is sufficiently low, individuals fully rely on the social LTC insurance and family help. As the probability of dependence increases, it becomes less interesting to transfer consumption to the non-dependent state, while the government and the family help cover the dependent state. Thus, the capital stock $\widetilde{k}^n$ decreases. Conversely, when the probability p is high, the return of social insurance, τ/p, is very low, and individuals increase their own savings, so that the capital stock $\widetilde{k}^n$ increases.

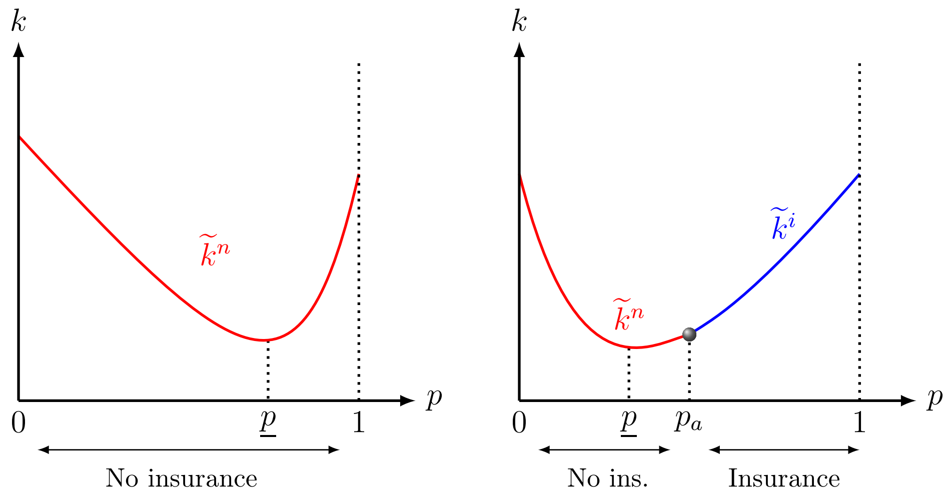


Figure 6: The steady state capital stock $\widetilde{k}$ as a function of p when $\tau > 0$.

When $\delta\xi$ is sufficiently high (i.e., $\delta\xi > \tau + (\pi - \tau)^\sigma$) individuals decide not to insure when $p \leq p_a = \tau/[\delta\xi - (\pi - \tau)^\sigma]$ and insure when $p > p_a$. As $0 < \underline{p} < p_a < 1$, the steady state capital stock, $\widetilde{k}^n$, decreases when $0 < p < \underline{p}$ and increases when $\underline{p} < p < p_a$. Finally, the steady state capital stock, $\widetilde{k}^i$, increases when $p_a < p < 1$ (see Figure 6B). Remark that the threshold p_a increases in π: as the imitation becomes more intense, individuals insure for a smaller range of probabilities of dependence. This is a standard case of crowding out.

The social LTC insurance crowds out private LTC insurance, so that individuals insure only if τ is small enough. The size of τ also affects the impact of the probability of dependence on the insurance decision. To show this, let us compare the case where $\tau = 0$ and

$\tau > 0$, limiting the analysis to the case where π is small enough (Figures 5B and 6B): individuals always insure with no public intervention, while in presence of social LTC insurance, individuals insure as long as $p > p_a$.

Tax rate. In order to inform policy makers about the optimal social LTC insurance, it is important to assess the impact of the tax rate on capital accumulation (see Appendix D.4). We can distinguish two cases depending on the value of π, reminding that the derivative of $\varepsilon(\tilde{x})$ with respect to τ has the sign of $\underline{\tau} - \tau$. As $\gamma = 0$, we have $\underline{\tau} = \pi - (p\sigma)^{1/(1-\sigma)}$.

When $\pi \leq (p\sigma)^{1/(1-\sigma)}$, as τ increases from 0 to π, the threshold $\varepsilon(\tilde{x})$ decreases from π^σ to π/p. Then, the steady state capital stock $\widetilde{k}$ is $\widetilde{k}^i$ (and individuals insure) if $\tau > \tau_b$, while it is $\widetilde{k}^n$ (and individuals do not insure) when $\tau \leq \tau_b$ with:

$$\tau_b = \begin{cases} 0 & \text{if } \delta\xi \leq \dfrac{\pi}{p} \\ \tau_b^\star \in (0, \pi) & \text{if } \dfrac{\pi}{p} < \delta\xi < \pi^\sigma \\ \pi & \text{if } \delta\xi \geq \pi^\sigma \end{cases}$$

where $\tau_b^\star \in (0, \pi)$ is the unique root of the function $\Lambda(\tau) = \delta\xi - \varepsilon(\tilde{x})$. Furthermore, an increase in the tax rate has always a positive impact on the steady state capital stock (see Figure 7A).

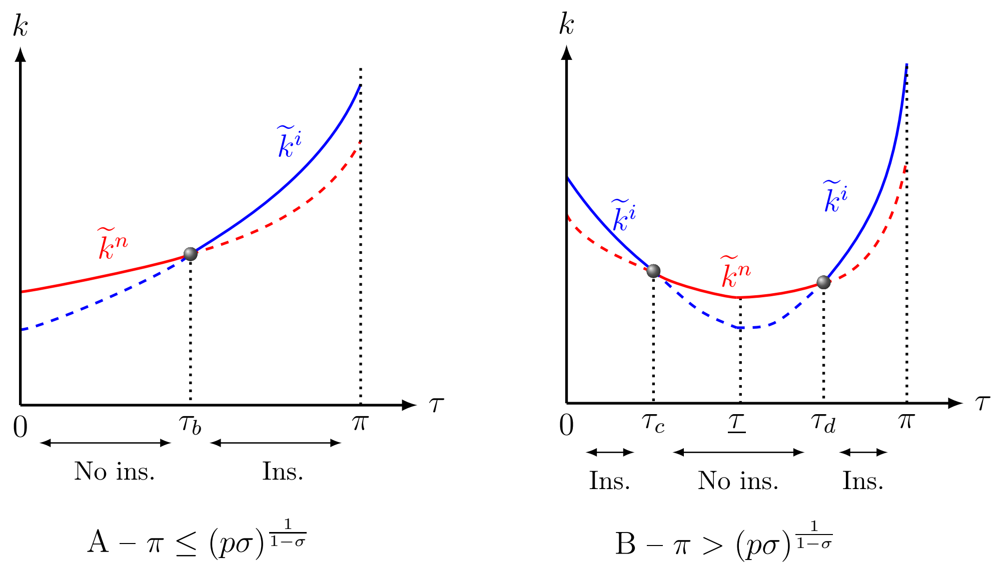


Figure 7: The steady state capital stock $\widetilde{k}$ as a function of τ.

When $\pi > (p\sigma)^{1/(1-\sigma)}$, $\varepsilon(\tilde{x})$ increases in τ if $\tau < \underline{\tau}$, and decreases if $\tau > \underline{\tau}$. Then, the steady state capital stock $\widetilde{k}$ is $\widetilde{k}^i$ (and individuals insure) if $\tau \in [0, \tau_c) \cup (\tau_d, \pi]$, while it is $\widetilde{k}^n$ (and individuals do not insure) if $\tau \in [\tau_c, \tau_d]$ with:

$$\tau_c = \begin{cases} 0 & \text{if } \delta\xi \leq \pi^\sigma \\ \tau_c^\star & \text{if } \pi^\sigma < \delta\xi < \dfrac{\pi}{p} + (1-\sigma)(p\sigma)^{\frac{\sigma}{1-\sigma}} \\ \underline{\tau} & \text{if } \delta\xi \geq \dfrac{\pi}{p} + (1-\sigma)(p\sigma)^{\frac{\sigma}{1-\sigma}} \end{cases} \quad \text{and } \tau_d = \begin{cases} \pi & \text{if } \delta\xi \leq \dfrac{\pi}{p} \\ \tau_d^\star & \text{if } \dfrac{\pi}{p} < \delta\xi < \dfrac{\pi}{p} + (1-\sigma)(p\sigma)^{\frac{\sigma}{1-\sigma}} \\ \underline{\tau} & \text{if } \delta\xi \geq \dfrac{\pi}{p} + (1-\sigma)(p\sigma)^{\frac{\sigma}{1-\sigma}} \end{cases}$$

where $\tau_c^\star \in (0, \underline{\tau})$ and $\tau_d^\star \in (\underline{\tau}, \pi)$ are the roots of function $\Lambda(\tau)$.

Since $\widetilde{k}^i$ and $\widetilde{k}^n$, the steady state capital stock $\widetilde{k}$ is always decreasing up to $\underline{\tau}$ and increasing afterwards. However, Figure 7B encompasses five parameters configurations in terms of insurance behaviors. In two configurations the steady state insurance regime does not change as τ varies. Indeed, individuals always (resp: never) insure when $\tau_c = \tau_d = \underline{\tau}$ (resp: $\tau_c = 0$ and $\tau_d = \pi$). In two other configurations, as τ increases, only one change in the reference regime is possible. This is the case where $0 = \tau_c < \tau_d < \pi$ or $0 < \tau_c < \tau_d = \pi$. Finally, two changes exist when $0 < \tau_c < \underline{\tau} < \tau_d < \pi$.

The comparative statics with respect to τ are surprising. In the absence of family help (see Figure 1), the effect of the tax rate on the capital stock is negative. With family help, the intuition for Figure 7 is to be found in the relative costs and returns of the family norm and of social LTC schemes. On the one hand, at the steady state, $x + \tau = \pi$. This implies that on the contribution side the two schemes are perfect substitutes. On the other hand, the return of the social LTC contribution τw is constant and equal to $1/p$, while the return of xw decreases with x. As a consequence, when τ is big, x is small and yields a return that can be higher than $1/p$. Thus, an increase in τ causes a decrease in x, which in turn implies a decline in LTC expenditures. To compensate for such a decline the individuals increasingly turn to market sources of LTC financing, fostering capital accumulation. Of course, if π is relatively small x is also relatively small, and might have a higher return than $1/p$ for any level of τ. In this case, the steady state capital stock always increases if the tax rate increases. This intuition also explains our finding that social and private LTC insurance can be complements: for sufficiently high levels of the tax, private LTC insurance may emerge as the tax level increases. This message is counterintuitive but important: in presence of family support individuals choose private LTC insurance if the pay-as-you-go social LTC insurance is generous enough; and the more generous the latter, the higher the economic growth. Thus, the fact that an aging population leads the State to establish generous unfunded social LTC insurance may in some circumstances encourage individuals to ensure themselves privately and is therefore beneficial for growth.

5 Conclusion

In this paper we have considered that LTC can be financed by four different channels: savings, private insurance, family help based on a norm, and an unfunded public scheme. Using a simple OLG model we have obtained a number of interesting results along the equilibrium path and on the stationary equilibrium.

In the benchmark case, namely without any family norm, the stock of capital evolves monotonically, either upward or downward. Individuals resort to private insurance when the loading factor is not too high. With the family norm, the evolution is not monotonic anymore. Finally there are plausible cases in which, along the equilibrium path, people switch their insurance behavior: they buy private insurance up to a certain period, then they stop doing it, or vice versa.

Turning to the steady states, we study the effects of three key parameters: the tax rate, the intensity of intergenerational imitation, and the probability of dependence. The relation between the payroll tax and the capital stock is expected to be negative. However, it may be positive when the family help is sizeably more productive that the other LTC financing sources. Since social insurance crowds out family help, individuals may compensate by increasing savings and private insurance. Not surprisingly, the intensity of intergenerational imitation has a depressive effect on capital accumulation. The probability of dependence has an effect on capital accumulation that depends on the prevalence of insurance. With private insurance, it is always positive; without private insurance, its sign is ambiguous. Private insurance arises for a range of intermediate values of p. The introduction of a family norm crowds out private insurance and reduces this range.

Even though our paper is basically positive, it has some interesting policy implications. In particular, it indicates that the intervention of the State in LTC financing may not discourage but foster capital accumulation through saving and private insurance purchase. This being said, we should be extremely cautions in deriving policy recommendations. The optimal allocation will depend on the social rate of discount but also on the resource allocation at each period of time. If family assistance is clearly more effective than private insurance, it might be desirable to have less capital accumulation and better LTC. Dealing with this normative issue is beyond the scope of this paper and is clearly on our research agenda.

In this paper we take the family norm as given without any normative judgment. We also assume identical individuals. If this were not the case and if people were to differ in the extent of the filial norm they are subject to, we would end up with an unfair situation in which only those with children willing and able to take care of them would receive the care they need. In that case, there would be an additional role for the public sector (see on this Stuifbergen and Van Delden (2011)).

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Appendices

Appendix A – Capital accumulation and insurance behavior.

An agent born in t chooses s_t and i_t to maximize $\mathcal{W}_t$ under the constraints $s_t \geq 0$ and $i_t \geq 0$. After computations, the first order condition with respect to s_t is given by (4), and, when $i_t > 0$, the first order condition with respect to i_t is equivalent to (5).

A.1 – Insurance behavior depending on $\delta\xi$ and $\varepsilon(x_t)$.

Merging (4) and (5) to eliminate their first term leads to the following equation: $i_t/p = \xi s_t - \varepsilon(x_t)w_{t+1}/R_{t+1}$. As $\delta w_{t+1} = k_{t+1}R_{t+1}$ and $k_{t+1} = s_t + i_t$, the equation can be rewritten as $[\delta/p + \varepsilon(x_t)]i_t = [\delta\xi - \varepsilon(x_t)]s_t$. Then, agents purchase LTC insurance if and only if $\delta\xi > \varepsilon(x_t)$ and insurance behaviors are described by (6).

A.2 – Capital accumulation when $i_t = 0$.

As $\delta w_{t+1} = k_{t+1}R_{t+1}$ and $k_{t+1} = s_t$, (4) is equivalent to $-1/[(1-\tau-x_t\mathbb{1})w_t - k_{t+1}] + \delta\beta p(1+\xi)/\{[\delta+\tau/p+\psi^\sigma(x_t)\mathbb{1}]k_{t+1}\} + \delta\beta(1-p)/\{[\delta+\gamma\psi^\sigma(x_t)\mathbb{1}]k_{t+1}\} = 0$. As $w_t = A(1-\alpha)k_t^\alpha$, we obtain $\{[\delta+\tau/p+\psi^\sigma(x_t)\mathbb{1}][\delta+\gamma\psi^\sigma(x_t)\mathbb{1}] + \delta\beta p(1+\xi)[\delta+\gamma\psi^\sigma(x_t)\mathbb{1}] + \delta\beta(1-p)[\delta+\tau/p+\psi^\sigma(x_t)\mathbb{1})]\}k_{t+1} = A(1-\alpha)(1-\tau-x_t\mathbb{1})\{\delta\beta p(1+\xi)[\delta+\gamma\psi^\sigma(x_t)\mathbb{1}] + \delta\beta(1-p)[\delta+\tau/p+\psi^\sigma(x_t)]\mathbb{1})]\}k_t^\alpha$. Then, according to appendix A.1, $k_{t+1} = \eta_d k_t^\alpha$ when $\pi = 0$ and $\delta\xi \leq \varepsilon(x_t)$, whereas $k_{t+1} = \eta(x_t)k_t^\alpha$ when $\gamma = 0$, $\pi > \tau \geq 0$ and $\delta\xi \leq \varepsilon(x_t)$.

A.3 – Capital accumulation when $i_t > 0$.

As $i_t > 0$, we obtain from (6) that $[\delta/p + \varepsilon(x_t)]i_t = [\delta\xi - \varepsilon(x_t)]s_t$. Using (3), we then get $i_t = p[\delta\xi - \varepsilon(x_t)]k_{t+1}/[\delta(1+p\xi)]$ and $s_t = [\delta + p\varepsilon(x_t)]k_{t+1}/[\delta(1+p\xi)]$. Using these equations, we obtain $\delta(1+p\xi)\{s_t + i_t/p + [\tau/p + \psi^\sigma(x_t)\mathbb{1}]w_{t+1}/R_{t+1}\} = (1+\xi)k_{t+1}\{\delta + \tau + [p+(1-p)\gamma]\psi^\sigma(x_t)\mathbb{1}\}$. Using (2) and (5) we get $\beta\delta A(1-\alpha)(1+p\xi)(1-\tau-x_t\mathbb{1})k_t^\alpha = \{\delta[1+\beta(1+p\xi)] + \tau + [p+(1-p)\gamma]\psi^\sigma(x_t)\mathbb{1}\}k_{t+1}$. Then, according to Appendix A.1, $k_{t+1} = \mu_d k_t^\alpha$ when $\pi = 0$ and $\delta\xi > \varepsilon(x_t)$, whereas $k_{t+1} = \mu(x_t)k_t^\alpha$ when $\gamma = 0$, $\pi > \tau \geq 0$ and $\delta\xi > \varepsilon(x_t)$.

Appendix B – Results of Section 3 ($\pi = 0$).

B.1 – Capital accumulation when $\pi = 0$.

According to Appendices A.2 and A.3, the dynamics are described by (8). Since the sign of $p\delta\xi - \tau$ is time-independent, no switch in the insurance behavior is possible. As k_{t+1} is an increasing and concave function of k_t, the capital stock k_t converges monotonically to the unique positive steady state k^d. When $\tau = 0$, since $\eta_d = \mu_d = \zeta_p$ and $p\delta\xi > \tau$, individuals insure and the dynamics of capital accumulation $k_{t+1} = \zeta_p k_t^\alpha$ converge to $k_{d|_{\tau=0}}$.

B.2 – Comparative statics with respect to p when $\pi = \tau = 0$).

As $\partial\zeta_p/\partial p = A(1-\alpha)\beta\xi/[1+(1+p\xi)\beta]^2$ is positive, $k_{d|_{\tau=0}} = \zeta_p^{\frac{1}{1-\alpha}}$ increases in p.

B.3 – Comparative statics with respect to τ when $\pi = 0$.

As $\partial\eta_d/\partial\tau = -A\alpha\beta(1-\alpha)^2p^2(1+\xi)(1-\tau)/\{\alpha p[1+(1+p\xi)\beta]+(1-\alpha)[1+(1-p)\beta]\tau\}^2 - \eta_d/(1-\tau)$ is negative, $k_d^n = \eta_d^{\frac{1}{1-\alpha}}$ decreases in τ. As the nominator of μ_d decreases in τ while the nominator increases, $k_d^i = \mu_d^{\frac{1}{1-\alpha}}$ decreases in τ. Then, the capital stocks k_d^i and k_d^n are both decreasing functions of τ. Using Appendix A.1, it is straightforward to show that individuals insure if and only if $\tau < \tau_a = \min\{p\delta\xi, 1\}$. We thus obtain Figure 1.

Appendix C – Results of Section 4 ($\pi > \tau \geq 0$).

According to Appendices A.2 and A.3, we obtain the two dimensional dynamical system described by (1) and (9). Then, the existence and the uniqueness of the positive steady state, denoted $(\widetilde{k}, \widetilde{x})$, are straightforward.

C.1 – Dynamics of family help.

The dynamics of x_t, described by (1) and represented Figure 2, are straightforward and independent of k. Then, the locus $x_{t+1} = x_t$ expressed as a function of k is a vertical line with abscissa $\widetilde{x}$ in the plan (x, k). To the left of this line, $x_{t+1} - x_t > 0$ and, for any $k > 0$, x_t converges towards $\widetilde{x}$. To the right of this line, $x_{t+1} - x_t < 0$ and, for any $k > 0$, x_t converges towards $\widetilde{x}$.

C.2 – Local dynamics with no insurance.

Assume that from a date $\kappa \geq 0$, agents do not insure. The locus $k_{t+1}-k_t = 0$ as a function of x can be written as $g(x) = \eta(x)^{\frac{1}{1-\alpha}}$. Let us define $a(x) = \alpha[1+(1+p\xi)\beta]+(1-\alpha)[1+(1-p)\beta]\varepsilon(x)$. After computations we get $\eta'(x) = -\eta(x)/(1-\tau-x) - \alpha Ap\beta(1-\alpha)^2(1+\xi)(1-\tau-x)\varepsilon'(x)/a(x)^2$. Since $1-\tau-x > 0$, $\eta(x) > 0$, $a(x) > 0$, and $\varepsilon'(x) = \sigma\pi\psi(x)^{\sigma-1} > 0$, it is straightforward to show that $\eta'(x) < 0$ and $(1-\tau-x)\eta'(x)+\eta(x) < 0$. After computations we also get $\eta''(x) = -[(1-\tau-x)\eta'(x)+\eta(x)]/(1-\tau-x)^2 - \alpha Ap\beta(1-\alpha)^2(1+\xi)\{[\varepsilon''(x)(1-\tau-x)-\varepsilon'(x)]a(x)-2a'(x)\varepsilon'(x)(1-\tau-x)\}/a(x)^3$. Since $1-\tau-x > 0$, $a(x) > 0$, $\varepsilon'(x) > 0$, $a'(x) > 0$, $(1-\tau-x)\eta'(x)+\eta(x) < 0$, and $\varepsilon''(x) = -\sigma(1-\sigma)\pi^2\psi(x)^{\sigma-2} < 0$ it is possible to show that $\eta''(x) > 0$.

As $\eta(x)$ is a decreasing and convex function of x, $g(x)$ is also a decreasing and convex function of x. The equation $k_{t+1} = g(x)^{1-\alpha}k_t^{\alpha}$ can be rewritten as $k_{t+1}-k_t = [(g(x)/k_t)^{1-\alpha}-1]k_t$. Thus, below the curve $k_{t+1} = k_t$, for any $x \in (0, 1-\tau)$, k_t converges towards $g(x)$. Above the curve $k_{t+1}-k_t < 0$, for any $x \in (0, 1-\tau)$, k_t converges towards $g(x)$. Then, using Appendix C.1, the dynamics in the neighborhood of $(\widetilde{k}^n, \widetilde{x})$ are described in Figure 3A.

C.3 – Local dynamics with insurance.

Assume that from a date $\kappa \geq 0$, agents insure. The locus $k_{t+1}-k_t = 0$ as a function of x can be written as $h(x) = \mu(x)^{\frac{1}{1-\alpha}}$. Let us define $b(x) = \alpha[1+(1+p\xi)\beta]+(1-\alpha)p\varepsilon(x_t)$. After computations, we get $\mu'(x) = -[1/(1-\tau-x)+b'(x)/b(x)]\mu(x)$. Since $\mu(x) > 0$, $b(x) > 0$, and $b'(x) = (1-\alpha)p\sigma\pi\psi(x)^{\sigma-1} > 0$, it is straightforward to show that $\mu'(x) < 0$ and $\mu'(x)+\mu(x)/(1-\tau-x) < 0$. After computations we also get $\mu''(x) = -[\mu'(x)+\mu(x)/(1-\tau-x)]/(1-\tau-x) - \mu'(x)b'(x)/b(x) - [b(x)b''(x)-b'(x)^2]\mu(x)/b(x)^2$. Since $1-\tau-x > 0$, $b(x) > 0$, $b'(x) > 0$, $b''(x) = -(1-\alpha)p\sigma(1-\sigma)\pi^2\psi(x)^{\sigma-2} < 0$, $\mu(x) > 0$, $\mu'(x) < 0$, and $\mu'(x)+\mu(x)/(1-\tau-x) < 0$, it is possible to show that $\mu''(x) > 0$.

Since $\mu(x)$ is a decreasing and convex function of x, $h(x)$, which represents $k_{t+1}-k_t = 0$, is also a decreasing and convex function of x. The equation $k_{t+1} = h(x)^{1-\alpha}k_t^{\alpha}$ can be rewritten as $k_{t+1}-k_t = [(h(x)/k_t)^{1-\alpha}-1]k_t$. Below the curve $k_{t+1} = k_t$, for any $x \in (0, 1-\tau)$, k_t

converges towards $h(x)$. Above the curve, for any $x \in (0, 1-\tau)$, k_t converges towards $h(x)$. Then, using Appendix C.1, the dynamics in the neighborhood of $(\widetilde{k}^i, \widetilde{x})$ are described in Figure 3B.

C.4 – Global dynamics: the different regimes.

First, remark that $\mu(x_t) = \eta(x_t)$ if and only if $\delta\xi = \varepsilon(x_t)$. Then, since $\eta(x_t)$ is decreasing in ξ while $\mu(x_t)$ increases in ξ, $\mu(x_t) \gtreqless \eta(x_t)$ if and only if $\delta\xi \gtreqless \varepsilon(x_t)$. Since $\varepsilon(x)$ increases in x, $g(x)$ and $h(x)$ cannot cross in more than one point: the point $\hat{x}$ such that $\varepsilon(\hat{x}) = \delta\xi$. Consequently, individuals insure for any $x_t < \hat{x}$ and do not insure for any $x_t > \hat{x}$. Since the dynamics of x_t are monotonic (increasing if $x_0 < \widetilde{x}$ and decreasing if $x_0 > \widetilde{x}$) and independent of k_t, and using the fact that $k_{t+1} = \max\{g(x_t)^{1-\alpha}, h(x_t)^{1-\alpha}\}k_t^{\alpha}$, we can distinguish four types of dynamics. Regime I occurs when $\delta\xi \leq \min\{\varepsilon(x_0), \varepsilon(\tilde{x})\}$. As $g(x) \geq h(x)$, agents do not insure and, according to Appendix C.2, we obtain the dynamics of Figure 3A. Regime II occurs when $\delta\xi > \max\{\varepsilon(x_0), \varepsilon(\tilde{x})\}$. As $h(x) > g(x)$, agents insure and, according to Appendix C.3, we obtain the dynamics of Figure 3B. Regime III occurs when $\varepsilon(\tilde{x}) < \delta\xi \leq \varepsilon(x_0)$. As long as $t \leq T = E\left[\ln\left\{\pi - \tau - \hat{x}/(\pi - \tau - x_0)\right\}/\ln(\pi - \tau)\right] + 1$, $x_t > \hat{x}$ decreases and agents do not insure because $h(x) \leq g(x)$. When $t > T$, $x_t < \hat{x}$, $h(x) > g(x)$ and individuals insure. Then, according to Appendices C.1 and C.2, we obtain the dynamics of Figure 3C. Regime IV occurs when $\varepsilon(x_0) < \delta\xi \leq \varepsilon(\tilde{x})$. As long as $t < T' = E\left[\ln\left\{\hat{x} - \pi + \tau/(x_0 - \pi + \tau)\right\}/\ln(\pi - \tau)\right] + 1$, $x_t < \hat{x}$ increases and agents insure because $h(x) > g(x)$. When $t \geq T'$, $x_t > \hat{x}$, $h(x) \leq g(x)$ and agents do not insure. Then, according to Appendices C.1 and C.3, we obtain the dynamics of Figure 3D.

Appendix D – Comparative statics ($\pi > \tau \geq 0$).

D.1 – Insurance behavior according to π, p and τ.

By definition we have $\varepsilon(\tilde{x}) \equiv \tau/p + [1 - \gamma(1+\xi)](\pi - \tau)^{\sigma}$. As $\partial\varepsilon(\tilde{x})/\partial\pi = [1 - \gamma(1+\xi)]\sigma(\pi - \tau)^{\sigma-1}$, $\varepsilon(\tilde{x})$ is increasing in π. As $\partial\varepsilon(\tilde{x})/\partial p = -\tau/p^2$, $\varepsilon(\tilde{x})$ is independent of p when $\tau = 0$ and decreasing in p when $\tau > 0$. As $\partial\varepsilon(\tilde{x})/\partial\tau = 1/p - [1 - \gamma(1+\xi)]\sigma(\pi - \tau)^{\sigma-1}$, $\partial\varepsilon(\tilde{x})/\partial\tau$ has the sign of $\underline{\tau} - \tau$ with $\underline{\tau} = \pi - [p\sigma(1 - \gamma(1+\xi))]^{1/(1-\sigma)}$. Then, $\varepsilon(\tilde{x})$ always decreases in τ when $\pi < [p\sigma(1-\gamma(1+\xi))]^{1/(1-\sigma)}$ and is always decreasing up to $\underline{\tau}$ and increasing afterwards when $\pi \geq [p\sigma(1-\gamma(1+\xi))]^{\sigma-1}$. According to these variations and since private LTC insurance is positive if and only if $\delta\xi > \varepsilon(\tilde{x})$, it is straightforward to prove the existence of the thresholds π_a, p_a, τ_b, τ_c, and τ_d and, consequently, to obtain the results of the paragraph "Insurance behavior" in Section 4.3.

D.2 – Comparative statics with respect to π when $\pi > \tau \geq 0$.

Since $\partial\eta(\tilde{x})/\partial\pi = -\alpha A(1-\alpha)^2(1-\pi)\beta p(1+\xi)\sigma(\pi-\tau)^{\sigma-1}/\{\alpha[1+(1+p\xi)\beta]+(1-\alpha)[1+(1-p)\beta]\varepsilon(\tilde{x})\}^2 - \eta(\tilde{x})/(1-\pi)$, then $\partial\eta(\tilde{x})/\partial\pi < 0$. Since the nominator of $\mu(\tilde{x})$ decreases in π while the nominator increases, then $\partial\mu(\tilde{x})/\partial\pi < 0$. As $\eta(\tilde{x})$ and $\mu(\tilde{x})$ are decreasing functions of π, $\widetilde{k}^n = \eta(\tilde{x})^{\frac{1}{1-\alpha}}$ and $\widetilde{k}^i = \mu(\tilde{x})^{\frac{1}{1-\alpha}}$ are also decreasing functions of π. According to Appendix D.1, we thus obtain Figure 4.

D.3 – Comparative statics with respect to p when $\pi > \tau \geq 0$.

After computations, $\partial\eta(\tilde{x})/\partial p$ has the sign of $[\delta\xi - \varepsilon(\tilde{x})][\alpha + (1-\alpha)\varepsilon(\tilde{x})] + \alpha(1+\xi)\tau/p$, and $\partial\mu(\tilde{x})/\partial p$ has the sign of $[\delta\xi - \varepsilon(\tilde{x})] + (1+p\xi)\tau/p$.

Consider the subcase where $\tau = 0$. If $\delta\xi \leq \varepsilon(\tilde{x}) = \pi^\sigma$, then individuals do not insure and $\partial\eta(\tilde{x})/\partial p < 0$. Then, $\widetilde{k}^n = \eta(\tilde{x})^{\frac{1}{1-\alpha}}$ is decreasing in p. If $\delta\xi > \pi^\sigma$, individuals insure and $\partial\mu(\tilde{x})/\partial p > 0$. Then, $\widetilde{k}^i = \mu(\tilde{x})^{\frac{1}{1-\alpha}}$ is increasing in p. We thus obtain Figure 5.

Consider the subcase where $\tau > 0$. As p increases from 0 to 1, the threshold $\varepsilon(\tilde{x})$ decreases from $+\infty$ to $\tau + (\pi-\tau)^\sigma$. Consequently, when $\delta\xi \leq \tau + (\pi-\tau)^\sigma$ individuals decide not to insure and the steady state capital stock is $\widetilde{k}^n$. As $\varepsilon(\tilde{x}) = \tau/p + (\pi-\tau)^\sigma$, the sign of $\partial\eta(\tilde{x})/\partial p$ is, after computations, the one of $\lambda(p) \equiv [\delta\xi-(\pi-\tau)^\sigma][\delta+(\pi-\tau)^\sigma]p^2 + 2[\delta\xi-(\pi-\tau)^\sigma]\tau p - \tau^2$. When $\delta\xi \leq (\pi-\tau)^\sigma$, it is straightforward that $\partial\eta(\tilde{x})/\partial p < 0$. Then $\widetilde{k}^n = \eta(\tilde{x})^{\frac{1}{1-\alpha}}$ is always decreasing in p. When $(\pi-\tau)^\sigma < \delta\xi \leq \tau + (\pi-\tau)^\sigma$, $\lambda(p)$ is increasing in p and is negative in $p = 0$. Then, there exists a (unique) threshold $\underline{p}$ such that $\eta(\tilde{x})$ (and also $\widetilde{k}^n = \eta(\tilde{x})^{\frac{1}{1-\alpha}}$) is decreasing up to $\underline{p}$ and increasing afterwards. We thus obtain Figure 6A.

When $\delta\xi > \tau + (\pi-\tau)^\sigma$, $p_a = \tau/[\delta\xi - (\pi-\tau)^\sigma] \in (0,1)$ and individuals decide to insure if and only if $p > p_a$. When $p \leq p_a$ the steady state capital stock is $\widetilde{k}^n$. According to the previous paragraph, $\widetilde{k}^n$ is decreasing up to $\underline{p}$ and increasing afterwards.[24] When $p > p_a$, the steady state capital stock is $\widetilde{k}^i$. Since $\delta\xi > \varepsilon$, we have $\partial\mu(\tilde{x})/\partial p > 0$. Then, $\widetilde{k}^i = \mu(\tilde{x})^{\frac{1}{1-\alpha}}$ is increasing in p. We thus obtain Figure 6B.

D.4 – Comparative statics with respect to τ when $\pi > \tau \geq 0$.

It is straightforward that both $\partial\eta(\tilde{x})/\partial\tau$ and $\partial\mu(\tilde{x})/\partial\tau$ have the opposite sign of $\partial\varepsilon(\tilde{x})/\partial\tau$. Consider the subcase where $0 < \pi < (p\sigma)^{1/(1-\sigma)}$. As $\partial\varepsilon(\tilde{x})/\partial\tau < 0$, $\widetilde{k}^n = \eta(\tilde{x})^{\frac{1}{1-\alpha}}$ and $\widetilde{k}^i = \mu(\tilde{x})^{\frac{1}{1-\alpha}}$ increase in τ. According to Appendix D.1, we thus obtain Figure 7A. Consider now the subcase where $\pi > (p\sigma)^{1/(1-\sigma)}$. As $\partial\varepsilon(\tilde{x})/\partial\tau$ has the sign of $\underline{\tau} - \tau$ with $\underline{\tau} = \pi - (p\sigma)^{1/(1-\sigma)}$, $\widetilde{k}^n = \eta(\tilde{x})^{\frac{1}{1-\alpha}}$ and $\widetilde{k}^i = \mu(\tilde{x})^{\frac{1}{1-\alpha}}$ increase (resp: decrease) in τ if τ is greater (resp: lower) than $\underline{\tau}$. According to Appendix D.1, we thus obtain Figure 7B.

[24] Note that $\lambda(p_a) = \delta(1+\xi)p_a\tau$, which, together with $\lambda'(p) > 0$, implies that $p_a > \underline{p}$.