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Behavioral Biases and Strategies of Insurance Market Players.

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Résumé

Cette thèse a pour objectif d'analyser les interactions entre les agents économiques opérant sur le marché de l'assurance de détail. D'un côté, les assurés souhaitant se couvrir contre un risque de perte doivent explorer le marché afin de souscrire un contrat en ligne avec leur perception du risque. D'un autre côté, les assureurs se font concurrence sur un marché régulé, leur imposant un certain niveau de capital afin de garantir leur solvabilité dans un contexte d'incertitude sur les risques souscrits. D'autre part, des intermédiaires proposent leurs services afin de faciliter l'interaction entre les consommateurs, averses aux risques, et les firmes, preneuses de risques. C'est donc dans ce contexte que nous analysons les comportements des acteurs de l'assurance à travers différentes perspectives. Les Chapitre 1 et 2 de cette thèse résultent d'expérimentations en laboratoire, effectuées à l'aide d'une interface web conçue spécifiquement pour ces études. Les résultats du Chapitre 3, quant à eux, sont basés sur un modèle théorique et des simulations numériques.

Le Chapitre 1 se concentre sur la relation entre l'honnêteté et les croyances en l'honnêteté des agents économiques. À l'aide des données collectées en laboratoire, nous montrons comment l'incertitude et le sentiment de se trouver dans des conditions plus ou moins avantageuses impactent à la fois le niveau d'honnêteté mais aussi la croyance en l'honnêteté envers les autres. En règle générale, les consommateurs surestiment l'honnêteté des intermédiaires. Ainsi, ce résultat justifie leur présence sur le marché de l'assurance. D'autre part, nous montrons aussi que les incitations financières proposées aux intermédiaires sont sources de distorsion des croyances en l'honnêteté. Plus le niveau d'incitation est faible, plus les consommateurs anticipent un comportement malhonnête.

Dans le Chapitre 2, nous mettons en évidence le dilemme dont fait face le consommateur sur un marché comprenant une multitude de canaux de distribution. Doit-il explorer par lui-même et choisir parmi un large ensemble de contrats ou bien déléguer une partie de sa décision à un intermédiaire plus ou moins honnête ? En utilisant une expérimentation en laboratoire, comportant des coûts de recherche, nous montrons que l'obfuscation liée à une importante quantité d'information et les croyances en l'honnêteté des intermédiaires sont les principaux déterminants des décisions de recherche et d'achat. Nous montrons également que l'obfuscation et l'attitude des intermédiaires sont sources d'inefficience dans les prises de décisions, en particulier vis-à-vis des caractéristiques des contrats d'assurance souscrits par les consommateurs. Dans ce sens, l'identification d'un effet de focalisation appuie l'importance du niveau des prix dans les prises de décision au détriment de l'environnement de risque et du niveau de couverture. L'introduction des coûts de recherche dans le processus d'exploration, ainsi que l'hétérogénéité des croyances en l'honnêteté justifient les stratégies de distribution multicanal adoptées par les assureurs.

Une analyse d'un jeu non coopératif répété est exposée dans le Chapitre 3 de cette thèse où les pertes et le comportement des consommateurs sont stochastiques et les assureurs se font une concurrence en prix. Afin d'intégrer les contraintes des régulateurs, nous déterminons les équilibres de Nash sous contrainte de solvabilité. Nous analysons également la sensibilité des primes d'équilibre en fonction des paramètres du jeu, en particulier lorsque les firmes ne bénéficient pas des mêmes avantages comparatifs (i.e. réputation conduisant à différents niveaux de rétention des clients, ancienneté des assureurs conduisant à différents stocks en capital).

Mots-clés: expérimentation, économie comportementale, intermédiation, canaux de distribution, honnêteté, obfuscation, théorie des jeux.

Abstract

This thesis aims at explaining interactions among economic agents operating in the retail insurance market. On the one hand, the policyholder is willing to be covered against a risk. To do so, they have to explore the insurance market to purchase a contract in line with their risk perception. On the other hand, insurers compete in a regulated market which imposes capital constraints for shock loss absorption purposes. In between, intermediaries may provide services in order to facilitate interaction between risk-adverse consumers and risk-taker firms. In this context, we analyze economic behaviors of insurance actors through different perspectives. Chapter 1 and 2 both result from original laboratory experiments, conducted through a web-interface especially designed for these studies. Results in Chapter 3 rely on a theoretical model and numerical simulations.

Chapter 1 emphasizes on the relationship between honesty and beliefs about honesty of economic agents. According to laboratory results, we show how the uncertainty and the perception of advantageous conditions impact the level of honesty and beliefs about honesty. In general, consumers estimate that intermediaries are more honest than they really are, hence supporting their physical presence in the insurance market. However, intermediary financial incentives are a source of distortion of honesty beliefs: the weaker the level of the incentive, the stronger the deviation anticipations.

In Chapter 2, we shed light on the dilemma faced by insurance purchasers under a multichannel distribution. Should the consumer, themselves, choose from a large set of insurance policies, or rather delegate a part of their decision to a more or less honest intermediary? Using experimental approaches, including exogenous search costs, we show that obfuscation and beliefs about intermediary honesty are the main determinants of individual choices. We also find that obfuscation and intermediaries' deviation are the main sources of inefficiency in decision-making, especially regarding the features of the insurance contracts chosen by consumers. Our identification of the focal point effect supports the importance of the price level on purchasing decisions rather than the risk environment or the coverage level. The introduction of search costs in the exploration process, as well as the heterogeneity of beliefs about honesty, justify multichannel distribution strategies adopted by insurers.

An analysis of insurer price competition with a repeated one-period non-cooperative game is conducted in Chapter 3, where both insurer losses and consumer behaviors are stochastic. Because of regulatory obligations, we consider a solvency constraint when computing Nash-Equilibrium. We determine the sensitivity of the premium equilibrium with respect to the parameters, especially when firms do not benefit from same competitive advantages (i.e. reputation effect leading to differences in consumers inertia or market seniority leading to differences in capital stock). We also study insurers' market share in response to the entry of new insurer undercutting prices but dealing with binding solvency constraints.

Keywords: experimentation, customer behaviors, intermediation, distribution channel, honesty, obfuscation, game theory.

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Introduction générale

Introduction

"You can't simply choose what you want, you have to be chosen." A. Roth

1 Introduction Générale

Bien souvent le tout premier réflexe en économie, consiste à raisonner en termes de marché. L'assurance n'échappe à cette règle, à tel point que la notion de marché de l'assurance est aussi souvent évoquée qu'elle reste en revanche peu définie. A priori ce besoin de définition n'apparaît pas primordial : la possibilité de survenance d'un événement indésirable pourrait a priori constituer un objet d'échange sur le marché. C'est ce que nous appellerons plus communément le risque. Bien que le risque peut aussi être défini par un aléa sur un évènement favorable, c'est bien dans le domaine des pertes que le concept d'assurance prend tout son sens. En effet, d'un côté certains agents économiques seraient désireux de se débarrasser d'un risque, motivés entre autres par leur aversion pour le risque, tandis que d'autres seraient prêts à supporter ce risque moyennant une rémunération, appelée prime d'assurance. Dès lors, ce raisonnement pourrait laisser penser que le risque représente un bien comme un autre. Cependant, l'aspect d'incertitude qui l'entoure complexifie le processus d'échange. En effet, le transfert de risque d'un agent à un autre est conditionné par la perception qu'ils en ont.

Ce phénomène n'est pas nouveau, dès l'antiquité les sociétés voient apparaître les prémices de ce qui sera communément appelé "assurance". En effet, les expéditions maritimes, coûteuses et risquées, furent les premières à justifier et illustrer la nécessité d'un marché de l'assurance, fondée sur l'échange de risque. D'un côté, les marchands effectuent un prêt afin de financer la marchandise transportée et paient une somme supplémentaire aux prêteurs correspondant à une partie de la vente de marchandise. Cet échange est cependant conditionné par le bon déroulement de l'expédition. En effet le prêt n'a pas à être remboursé si la marchandise est volée ou perdue suite à un naufrage. D'un autre côté, les prêteurs ou investisseurs sont prêts à supporter le risque de perte d'un navire, parmi une flotte, en échange des montants supplémentaires reçus pour l'ensemble des expéditions se déroulant sans aucune perte (référer à Martin (1876), pour une revue de l'histoire de l'assurance maritime). C'est le principe de mutualisation du risque, pilier du marché de l'assurance. Il faudra cependant attendre le XVII ème siècle, et notamment le grand incendie de Londres de 1666, pour voir apparaître un cadre légal et une généralisation de la souscription de contrat d'assurance dite "moderne" .

La littérature moderne en économie de l'assurance est cependant là pour attester que cette représentation marchande est pour le moins sommaire. Deux éléments fondamentaux sont à prendre en compte, qui rend le marché de l'assurance singulier.

D'une part, le risque qui serait donc l'objet de l'échange est rarement exogène, dans le sens où bien souvent il dépend du comportement (ex ante ou ex post) de l'agent économique. Qui plus est, la nature de cette dépendance reste le plus souvent incertaine. Ainsi, l'asymétrie d'information entre l'assureur et l'assuré est l'une des principales explications des biais comportementaux inhérents au marché de l'assurance. Elle est, de manière plus ou moins consciente, un phénomène connu des praticiens de l'assurance depuis ses origines. Son étude par les économistes est plus récente et a permis de réelles contributions en matière de compréhension des mécanismes des marchés d'assurance. Cette asymétrie d'information donne lieu à deux phénomènes bien connus en économie comportementale : l'anti sélection et l'aléa moral.

L'un des premiers économistes à avoir mis en évidence l'importance et les conséquences de l'asymétrie d'information en matière d'assurance est le Prix Nobel K. J. Arrow (1963). En prenant en compte l'aversion aux risques, aspect élémentaire du comportement humain dans la prise de décision dans un domaine incertain, il apporte une première contribution fondamentale à la compréhension du partage de risque optimal entre l'assureur et l'assuré. Il souligne aussi les problèmes rencontrés par les assureurs faisant face à de l'aléa moral, basés sur une modification des comportements conduisant à l'augmentation du risque a postériori (i.e. après la souscription d'un contrat d'assurance). Ses travaux sur l'anti sélection sont moins connus mais il est le premier à déclarer que "*l'assurance nécessite, pour avoir un effet social pleinement bénéfique, une discrimination des risques qui soit la plus grande possible*". Une deuxième contribution fondamentale à la compréhension des mécanismes de marché et de rencontre de l'offre et de la demande en assurance est celle apportée par Rothschild et Stiglitz (1976), notamment grâce à leur modélisation de l'anti sélection. Ainsi, ils montrent qu'il est possible de mettre en place des mécanismes de sélection des risques même si l'on ne connaît pas, a priori, le degré de risque de chaque individu.

D'autre part, traiter du marché de l'assurance suppose que la rencontre entre les deux côtés du marché se fait spontanément et sans coût. Or les défauts d'information subis par l'offreur d'assurance, le pousse à offrir un service bien plus complexe qu'un simple transfert de risque. En effet, tout vendeur de risques (le consommateur dans la version de base) n'est pas prêt à rencontrer tout acheteur de risque (l'assureur). Inversement, tout assureur cherche à sélectionner, trier, "screener" (Spence (1973)) les consommateurs. A tout cela s'ajoute le fait que l'offre se constitue sous le régime concurrentiel, qui conditionne à son tour les stratégies de prix et d'intermédiation des assureurs.

Ainsi, et de manière générale, le fonctionnement d'un marché concurrentiel ne se caractérise pas nécessairement par le fait que les consommateurs ont la possibilité d'acheter une quantité voulue à un prix donné. Les assureurs vont à la fois spécifier une quantité et un prix afin de procéder à la sélection du risque. En effet, les contrats qui ne discriminent pas suffisamment sont susceptibles d'attirer principalement les mauvais risques présents sur le marché. De fait, ce mécanisme de sélection conduisant à des offres d'assurance hétérogènes rend, au passage, la compréhension des choix d'assurances plus complexes pour le consommateur. Par conséquent, avant même le partage du risque en tant que tel, le marché de l'assurance et l'ensemble des stratégies opérées par ses acteurs sont avant tout conditionnés par la problématique de rencontre et le design de ce processus de rencontre, "matching". Comme souligné par Roth (2015), "*il ne suffit pas de choisir ce que l'on veut, il faut aussi être choisi*".

Le marché de l'assurance se révèle donc comme un terrain d'étude des comportements à la fois complexe et varié. L'incertitude omniprésente sur le marché de l'assurance, conduisant indéniablement à l'asymétrie d'information, rend les mécanismes de choix et les stratégies des agents économiques bien plus complexes. D'un côté, les consommateurs souhaitent maximiser leur utilité, celle-ci conditionnée en partie par leur aversion au risque mais aussi par leurs capacités et possibilités de choix. De l'autre, les assureurs cherchent à maximiser leur profit dans un marché compétitif et fortement contraint par les régulateurs, de par la nature même de leurs produits basés sur : le risque. La complexité de la rencontre entre l'offre et la demande fait naturellement émerger une multitude d'intermédiaires facilitant le transfert de risque et, comportant des caractéristiques diverses elles-mêmes révélatrices d'informations et de biais comportementaux. La démocratisation de l'utilisation d'internet dans le processus et les stratégies d'achat et de vente sont des facteurs supplémentaires de multiplicité et de complexification des comportements. Cela bouleverse l'accès à l'information pour les consommateurs mais aussi les stratégies de ventes et la compétition entre assureurs, notamment grâce à l'émergence de nouveaux intermédiaires d'assurance dans un marché toujours plus concurrentiel (Brown et Goolsbee (2002)).

La singularité du marché de l'assurance a essentiellement été étudiée selon la première dimension au travers bien évidemment de la modélisation de la sélection contraire et de l'aléa moral. En revanche la prise en compte de la deuxième dimension est bien plus récente et fait l'objet précisément des essais contenus dans cette thèse.

Cette thèse se propose donc d'étudier les biais comportementaux et des stratégies des acteurs de l'assurance non-vie. L'objectif de mes recherches consiste en la compréhension des stratégies des assureurs souhaitant acquérir de l'information sur les consommateurs, tout en répondant aux exigences des régulateurs, afin de maximiser leur profit. Je m'intéresse également au processus de décision des consommateurs, dans un environnement incertain, ou l'accès à l'information tend à s'accroître mais aussi à se complexifier. Ainsi, et comme mentionné précédemment, le marché de l'assurance est avant tout un " matching market" où la rencontre entre les assureurs et les futurs assurés se fait à l'aide d'intermédiation. Je consacre donc une partie de cette thèse à l'étude des choix des intermédiaires, en prenant notamment en compte l'émergence de nouveaux acteurs tel que les comparateurs de prix sur internet. D'autre part, les intermédiaires ont également leurs propres intérêts économiques et financiers dans ce jeu de "matching". Ces intérêts structurent le marché de l'assurance, tant l'offre que la demande, et révèlent de nouveaux biais comportementaux.

2 Incitations Financières et Honnêteté

Dans le premier chapitre de cette thèse "Sommes-nous plus honnêtes que l'on croit ? " ("Are we more honest than others think we are ?"), je me suis donc intéressée au comportement d'honnêteté mais aussi aux croyances dans l'honnêteté des autres. Cette étude des comportements d'honnêteté est basée sur une approche expérimentale et l'analyse est effectuée sur les données collectées en laboratoire. L'objectif est de mesurer l'adéquation entre l'honnêteté des uns et la croyance en l'honnêteté des autres.

En effet, comme mentionné précédemment, le processus d'achat en assurance repose sur un large réseau d'intermédiaire. Ces mêmes intermédiaires ont leurs propres incitations financières pouvant les amener à ne pas proposer le meilleur contrat mais le plus rentable. Avant même d'analyser les choix de délégation en assurance, il est donc important de disposer d'une mesure permettant de comprendre le niveau de confiance des consommateurs, et si celle-ci est en adéquation avec le comportement d'honnêteté observé. En effet, les économistes sont d'accord pour dire que les comportements malhonnêtes sont source d'inefficience dans les échanges entre agents économiques. Cependant, tout ne repose pas sur l'honnêteté des uns. Même dans un monde des plus honnêtes, il est primordial que les agents croient en cette honnêteté pour effectivement réduire les coûts de transactions.

De nombreuses recherches se sont déjà intéressées au sujet, notamment en analysant de manière agrégée les impacts des caractéristiques sociales des individus et de leur relation sur leur niveau de confiance et d'honnêteté. Berg *et al.* (1995) furent les premiers à mettre en évidence l'importance du contexte social sur la réciprocité de l'honnêteté. En effet, ils montrent que la présence d'interaction entre les sujets à tendance a accroître le niveau de confiance et d'honnêteté. Dans le même sens, Glaeser *et al.* (2000) montrent que les individus ont tendance à avoir plus confiance aux personnes ayant les mêmes caractéristiques sociales. Plus récemment, Ermisch *et al.* (2009), analysent plus précisément les caractéristiques des populations, et mettent en évidence, par exemple, que les individus ayant des situations financières "confortables" ont tendance à plus faire confiance. Quant à Hugh-Jones (2016), il démontre une réelle hétérogénéité des croyances en l'honnêteté à travers les pays.

Ces études sont des apports essentiels à la compréhension des croyances et de l'honnêteté mais aussi à la compréhension des interactions, plus ou moins facile, entre des groupes d'individus. En effet, l'étude des caractéristiques sociales permet aux économistes de mieux comprendre comment les échanges, basés sur la confiance, se forment et qui sont les plus enclins à y participer. Cependant, dans cette thèse, je me suis concentrée sur l'aspect financier des incitations à l'honnêteté. Il est vrai que déléguer une partie de sa décision peut aussi être étroitement liée à la relation sociale que le consommateur a avec ses intermédiaires. Par exemple, un individu demandera plus facilement un conseil au courtier l'ayant déjà conseillé, et ce, à confiance égale. Or, le but n'est pas d'analyser les effets de réputation et de vécu sur le choix de délégation. En effet, je souhaite ici comprendre à quel point les incitations financières à la malhonnêteté affectent le niveau de confiance et donc le choix de délégation. Ainsi, l'étude des incitations financières sur l'honnêteté dans un contexte anonyme paraît la plus adéquate pour répondre à cette question.

C'est donc dans la lignée de Mazar *et al.* (2008), Fischbacher et Follmi-Heusi (2013), Houser *et al.* (2012), et Galeotti *et al.* (2017) que s'inscrivent les études menées dans ce premier chapitre. Par exemple, Mazar *et al.* (2008), Fischbacher et Follmi-Heusi (2013), Abeler *et al.* (2016) montrent que l'accroissement des incitations financières n'impacte pas significativement le niveau d'honnêteté. Néanmoins, Kajackaite et Gneezy (2017) ne trouvent pas les mêmes résultats en considérant un "jeu de déception" où la possibilité de mentir est explicitement exposée aux sujets. Le fait de faire du mensonge une règle explicite du jeu semble conduire les sujets à apporter plus d'importance au risque d'être pris entrain de mentir. Cela les conduit donc à une plus grande analyse du compromis coûts-bénéfices. Pour appuyer ce résultat, Yaniv et Siniver (2016) montrent que dans un environnement sans danger d'être pris en train de tricher, les individus trichent bien plus. Ainsi, ces résultats semblent démontrer que l'honnêteté des individus est largement affectée par les conséquences de leurs actes sur leur éthique sociale. D'autre part, Houser *et al.* (2012) ainsi que Galeotti *et al.* (2017) mettent en évidence l'importance de la perception de l'environnement sur le comportement d'honnêteté. En effet, les sujets sont moins honnêtes dans des conditions injustes. L'inégalité des situtations diminue la considération de l'éthique sociale dans la prise de décision. Toutefois, à ma connaissance, aucune étude n'a confronté, d'une part le comportement d'honnêteté et d'autre part les croyances en l'honnêteté, étant donné les incitations financières et l'environnement plus ou moins injuste du sujet, tenté à adopter un comportement malhonnête.

J'ai donc élaboré une nouvelle méthode expérimentale afin de créer à la fois, une mesure d'honnêteté et une mesure de croyance en l'honnêteté. Afin que ces deux mesures soient parfaitement comparables et contrairement au jeu standard de Berg *et al.* (1995), j'ai introduit une règle implicite permettant de mesurer l'écart entre la règle et le comportement observé ou estimé. En effet, le jeu de Berg *et al.* (1995) implique des effets d'aversion à l'inégalité et d'altruisme dus au manque d'une règle objective. À aucun moment une règle spécifie aux sujets un devoir de transfert d'argent ou de récompense du transfert d'argent. Chaque sujet a sa propre interprétation du jeu. Ainsi, il aurait été difficile de comparer l'honnêteté d'un côté et les croyances de l'autre. L'intérêt de mes recherches est de comprendre la délégation de choix en assurance et le comportement des intermédiaires incités financièrement. Il existe bien une règle explicite, déontologique, demandant aux intermédiaires de rendre un service de conseil adéquat, autrement dit de proposer le contrat correspondant le mieux au consommateur et ce, au meilleur prix.

D'autre part, et comme mentionné précédemment, je souhaite comprendre dans quelle mesure le niveau d'incitation financière influe sur le comportement d'honnêteté et des croyances en l'honnêteté. C'est donc avec cet objectif que j'ai introduit dans l'expérimentation des conditions plus ou moins favorables lors de la prise de décision. En effet, tous les sujets font face à la même règle mais celle-ci est plus ou moins sévère. Si ils suivent cette règle, certains sujets se retrouvent très peu rémunérés par rapport à la moyenne tandis que d'autres le sont beaucoup plus, affectant donc leurs décisions d'être honnête mais aussi les croyances en l'honnêteté.

Cette nouvelle expérience a quatre principaux avantages. Tout d'abord, je m'assure que la malhonnêteté ne puisse pas être détectée, évitant ainsi un potentiel effet de demande. De plus, après avoir contrôlé l'anonymat des choix, je collecte les données sur le comportement d'honnêteté au niveau individuel contrairement à Fischbacher et Follmi-Heusi (2013) ou encore Hugh-Jones (2016). Enfin, j'introduis différentes conditions plus ou moins favorables afin de comprendre l'impact des incitations financières sur les croyances et l'honnêteté. Afin, cette méthode est facile à mettre en place en laboratoire et relativement rapide.

Les résultats montrent que les individus sont en moyenne moins honnêtes que ce que l'on croit, en particulier quand les conditions sont favorables. Les résultats m'ont permis de modéliser un effet de distorsion des comportements, à la fois de l'honnêteté et des croyances, en fonction des différentes conditions plus ou moins favorables.

Tout d'abord, les individus sont en moyenne malhonnêtes (i.e. ils ne respectent pas la règle). Cependant, cette malhonnêteté est d'autant plus importante lorsque qu'ils font face à des conditions peu favorables. Ainsi, le sentiment d'injustice semble justifier leurs décisions malhonnêtes. Ce qui est d'autant plus intéressant est le fait que les sujets, estimant le comportement d'honnêteté, anticipent ce phénomène. Ils ont moins confiance face à un individu aux conditions peu favorables, tandis qu'ils surestiment significativement l'honnêteté dans des conditions favorables. Cependant, ils ont une meilleure estimation de la malhonnêteté lorsque les conditions se détériorent.

Les implications pour comprendre les comportements d'achat en assurance sont grandes, d'autant plus que la nouvelle Directive de Délégation en Assurance renforce les contraintes sur la transparence des conditions de rémunération des intermédiaires. Les consommateurs pourront alors connaitre dans quelle mesure et à quel niveau leurs intermédiaires sont rémunérés. Cela implique une grande différence de choix de distribution, étant donné que les "bons" risques peuvent être assimilés à de bonnes conditions. Ainsi et si ils sont conscients de leurs caractéristiques avantageuses, ils auront donc tendance à déléguer d'avantage, malgré une forte déviation des intermédiaires.

Dans un second temps, je me suis intéressée à la corrélation entre la confiance en l'honnêteté et l'aversion au risque. Tandis que la littérature est mitigée sur ce point, les résultats de cette expérimentation ont permis de concilier les différents résultats sur le sujet. En accord avec Corcos *et al.* (2012), je trouve que le niveau moyen de confiance en l'honnêteté n'est pas corrélé à l'aversion au risque. Ainsi, faire confiance n'est pas une décision risquée. Cependant, mes résultats supportent aussi une éventuelle corrélation, comme mentionné par Naef et Schupp (2009). En effet, bien que l'aversion au risque n'explique pas le niveau de confiance des individus, elle explique cependant la distorsion des croyances lorsque les conditions deviennent peu favorables. Ainsi, les sujets averses aux risques réduisent significativement plus leur confiance lorsque la situation est défavorable.

D'autre part, tandis que Grolleau *et al.* (2016) montrent que les hommes ont d'avantage tendance à tricher (i.e. être malhonnête), je montre qu'ils ont aussi moins confiance aux autres. Leur malhonnêteté les incite eux-mêmes à ne pas croire en l'honnêteté des autres.

Le premier chapitre de cette thèse m'a donc permis de mieux comprendre le comportement d'honnêteté et de croyances en l'honnêteté suivant différentes conditions financières. Ainsi, nous voyons bien que malgré une règle explicite, les individus sont en moyenne malhonnêtes. De plus, cette malhonnêteté est globalement sous-estimée par la population de notre analyse. Ces résultats justifient la présence d'intermédiaires plus ou moins honnêtes et montrent une réelle hétérogénéité des croyances en l'honnêteté. C'est donc une première explication des différences de comportements d'achat en assurance et des choix de délégation. Cette expérimentation, relativement simple à mettre en place, m'a également permis de créer une mesure contrôlant les croyances en l'honnêteté. Cette mesure est utilisée dans le deuxième chapitre de cette thèse afin de mieux comprendre l'ensemble du processus d'achat en assurance, de la recherche d'information au choix du contrat.

3 Informations, Intermédiaires et Choix d'Assurance

Après avoir créé et analysé une nouvelle mesure d'honnêteté et de croyance en l'honnêteté, je me suis intéressée dans le deuxième chapitre à l'ensemble du processus d'achat d'un contrat d'assurance. Le chapitre central de cette thèse intitulé "Obfuscation et honnêteté : une étude expérimentale de la demande d'assurance sur un marché sous intermédiation" ("Obfuscation and Honesty : Experimental Evidence on Insurance Demand with Multiple Distribution Channel") se concentre sur la demande d'assurance. Autrement dit, il est consacré à l'étude du processus d'achat composé à la fois de recherche de l'information mais aussi du choix de couverture d'assurance, étant donné les caractéristiques de l'assuré et l'information disponible.

Le but de ce chapitre est de mettre en évidence le dilemme dont fait face le consommateur entre, d'une part, explorer le marché par lui-même au risque de se perdre dans une quantité trop importante d'information, ou bien déléguer une partie du processus d'achat au risque de tomber sur un intermédiaire malhonnête. Ainsi, les analyses menées dans ce chapitre m'ont permis de mettre en évidence les effets de focalisation ou d'ancrage dus à une quantité trop importante d'information, mais aussi les effets de déception conduisant à des changements important de stratégies d'exploration. De plus, j'ai aussi étudié l'optimalité des choix afin d'analyser dans quelles mesures la sur-confiance en l'honnêteté des intermédiaires et l'obfuscation sont sources d'inefficience dans les choix.

Le deuxième chapitre de ma thèse est donc essentiellement tourné vers l'étude de la demande d'assurance, bien qu'il permette de mieux appréhender les différentes stratégies possibles et optimales des assureurs suivant les canaux de distribution, mais aussi le comportement des intermédiaires face aux incitations financières des assureurs. Les résultats sont tirés d'une expérimentation en laboratoire dans laquelle une partie des sujets a dû choisir un contrat d'assurance parmi un choix limité, dépendant de leur stratégie de recherche via plusieurs intermédiaires : un comparateur de prix, un courtier et un agent général d'assurance. Les sujets restants ont joué le rôle d'intermédiaires "physiques" ayant leurs propres incitations financières. J'ai donc utilisé dans ce chapitre la mesure de confiance et d'honnêteté détaillée dans le premier chapitre de ma thèse. Celle-ci permettant de contrôler le rôle de la confiance dans le processus d'achat, notamment dans les décisions de délégation, mais aussi dans l'efficience des choix. D'autre part, comme les choix d'assurance, basés sur la théorie de l'utilité espérée (Rothschild et Stiglitz (1976)), sont fortement liés à l'aversion au risque des agents, il a été nécessaire de la contrôler pour l'ensemble des sujets.

Avant de résumer l'ensemble des résultats du chapitre deux de cette thèse et leur implication, il est important de rappeler les éléments théoriques clés qui ont permis la construction de cette expérimentation. Ces éléments sont basés essentiellement sur la théorie de l'economie de l'assurance, tel que la théorie de l'utilité espérée mais aussi sur la théorie des coûts de recherches. En effet, il est important qu'une experimentation comportementale ne comporte pas de choix strictement optimaux. En d'autres termes, il ne doit pas exister de "bonne réponse". Ainsi, chaque décision est révélatrice des préférences et donc des biais cognitifs des sujets étudiés.

Ainsi, afin de révéler au mieux les stratégies de recherche des consommateurs, j'ai intégré dans l'expérimentation des coûts de recherche conditionnant l'accès à l'information. Ces coûts sont donc la monétisation du temps, effort ou argent dépensé par un consommateur recherchant un produit ou un service. Contrairement à Brynjolfsson et Smith (2000), l'objectif de mes recherches n'est pas de mesurer les coûts de recherche individuels. Inspiré de la méthodologie utilisée par Schram et Sonnemans (2011) , ces coûts de recherche sont donc exogènes. Cependant, et encore une fois dans un souci de neutralité des choix, ils ont été défini sous deux différentes contraintes. La première contrainte consiste à définir pour chaque canal de distribution un coût d'accès à l'information, tel que, en espérance le coût pour atteindre le contrat optimal du marché soit identique. En effet, supposons qu'il soit moins couteux de découvrir l'ensemble des contrats en visitant un a un chaque assureur que de demander un conseil à un courtier, dans ce cas, le choix du sujets ne révèle pas ses préférences de canaux de distribution mais uniquement le fait qu'il minimise bien ses coûts de recherches. La deuxième contrainte m'a permis de définir le montant appelé "crédit de recherche" qui permet au sujet d'explorer le marché. Chaque coût de recherche correspondant à une révélation et, est débité sur ce crédit. Ainsi, le montant non consommé par le sujet lui est payé à la fin de l'expérimentation. Ce crédit d'information est donc défini de manière à ce que le sujet puisse être pleinement informé, c'est à dire puisse explorer à l'aide de l'ensemble des canaux de distribution. Le choix du canal n'est donc pas unique.

Du côté de l'offre, elle est également définie de façon exogène et ne tient pas compte des effets des coûts de recherche sur les équilibres de marché. Cependant, il est vrai que les coûts de recherches sont essentiels à la compréhension des équilibres de marché. Le premier à avoir étudié de manière théorique l'impact des coûts de recherches sur les primes d'équilibres est Diamond (1971). Dans son papier de référence, il montre pourquoi le consommateur est prêt à payer plus pour éviter d'engendrer un nouveau coût de recherche, expliquant la relation croissante entre coûts de recherche et prime de marché. Plus tard, Brown et Goolsbee (2002) démontreront de nouveau cette relation sur le marché de l'assurance vie. En utilisant des données empiriques ainsi que des données sur l'utilisation d'internet dans le processus d'achat, ils montrent que l'apparition des cyber-canaux au cours du XXème siècle à significativement diminuée les prix des assurances vie. D'autre part, Baye *et al.* (2004) montrent une forte corrélation entre le nombre de produit listé sur un site de comparaison et le niveau moyen des prix. Ici encore, les cyber-canaux ont pour effet d'augmenter la competition en diminuant les coûts de recherches. Autrement dit, en facilitant l'accès à l'information. Ainsi, leur résultats supportent la loi du prix unique. Enoncée par Bertrand et donnant lieu au paradoxe de Bertrand, elle supporte le fait que, sur un marché totalement compétitif, le prix d'équilibre est unique et est égal au coût marginal du produit vendu. Plus récemment, Branco *et al.* (2012) analysent les prix d'équilibres tout en tenant compte du signal reçu à chaque nouvelle information dévoilée. Bien que les résultats aillent dans le même sens que ceux présentés précédemment, de faible coûts de recherche dans un contexte de recherche séquentielle (i.e. visiter un à un les assureurs par exemple) peuvent augmenter les prix d'équilibres. En effet, les consommateurs utilisent toute information additionnelle comme un signal permettant d'ajuster leurs croyances sur les informations non révélées. Ainsi, les firmes peuvent à l'équilibre proposer une prime plus élevée afin de décourager la recherche, pourtant peu coûteuse.

Ainsi, bien qu'il soit vrai que la stratégie de distribution conditionne également la stratégie de prix et donc que chaque canal ne propose pas nécessairement les mêmes type de contrat, le but de ce chapitre est de révéler les biais comportementaux conditionnant la demande d'assurance. Afin de faciliter l'analyse des choix et leur efficience je me suis donc largement inspirée de Schlesinger (2013) dans la construction des contrats proposés, qui sont donc les mêmes pour chaque canal de distribution. Ces contrats se composent donc de deux éléments : une prime commerciale CP et une franchise D tel que,

$$CP = p \times (R - D) \times (1 + \lambda),$$

où p est la probabilité de perdre le montant R, et λ est le taux de chargement de l'assureur. La définition d'un seul et même contrat à l'aide de deux éléments distincts m'a permis d'inclure dans l'expérience l'aspect d'hétérogénéité et de complexité des contrats d'assurance.

Au total, le "marché" d'assurance proposé dans cette expérimentation comporte quatre assureurs. Puisque dans la réalité les assureurs ne proposent jamais un contrat unique, chaque assureur propose donc deux contrats. Ainsi, l'offre d'assurance disponible se résume à huit contrats. Le sujet "consommateurs" peut donc explorer le marché par lui même en dévoilant un à un les "menus" de contrat de chaque assureur et en demandant éventuellement un conseil à un agent général, au sein même d'un assureur. Il peut également explorer le marché à l'aide du comparateur de prix. Dans ce cas, trois assureurs sur quatre sont présents sur le comparateur. Ce choix est justifié par le fait que, en réalité, la présence des assureurs sur un comparateur et le fruit d'une négociation. Ainsi, tout les assureurs n'y sont pas présents. Le raisonnement est similaire lorsque le sujet choisit de demander un conseil à un courtier (i.e. un sujet jouant le rôle d'intermédiaire). Une information privée est donnée au courtier sous la forme d'un classement représentant l'optimalité des contrats (nous revenons ultérieurement sur le calcul de l'optimalité des contrats). Six contrats (correspondant à trois assureurs) sont affichés et le courtier renvoie un classement de seulement trois de ces six contrats.

Un autre élément, indispensable à l'étude sur l'efficience et l'optimalité des choix, est basé sur la définition de l'aversion au risque permettant le classement des contrats en termes d'optimalité. En effet, bien que les contrats n'ont pas le même taux de chargement, ou autrement dit marge ; cette marge n'est pas le seul élément permettant de définir l'optimalité d'un contrat. Etant donné l'aspect de risque inhérent au marché de l'assurance, est en ligne avec la théorie de l'utilité espérée, il est important de tenir compte de l'aversion au risque des individus. Ainsi, un contrat i est strictement optimal par rapport à un contrat j pour l'individu k si et seulement si

$$(1-p)\times U(W_k-CP_i, r_k)+p\times U(W_k-CP_i-D_i, r_k) > (1-p)\times U(W_k-Cj, r_k)+p\times U(W_k-CPj-D_j, r_k),$$

où W_k est la richesse de l'individu k, $U(.)$ est la fonction d'utilité des individus et r_k est le paramètre d'aversion au risque pour l'individu k, utilisé dans la fonction d'utilité.

De nombreux économistes de Rothschild et Stiglitz (1976) en passant par Schlesinger (2013), pour ne citer qu'eux, se sont intéressés aux choix de couverture des consommateurs d'assurance en fonction de leur aversion aux risque. Ainsi, la théorie économique de l'assurance montre que plus un agent est averse au risque plus il est prêt à payer pour se débarrasser de ce même risque, autrement dit de l'aléa inhérent à ce risque.

Chaque individu k souhaite maximiser son espérance utilité tel que

$$\max_{(CP,D)\in M} (1-p) \times U(W_k - CP, r_k) + p \times U(W_k - CP - D, r_k),$$

où (CP, D) sont les éléments d'un seul contrat appartenant à l'ensemble M de police d'assurance disponible sur le marché. De plus, chaque assureur propose un prime commerciale tel que

$$CP(\alpha) = E(X) \times (1 + \lambda) \times \alpha,$$

où α est la couverture (i.e. $\frac{R-D}{R}$), $E(X)$ l'espérance du coût (i.e. $p \times R$) et λ le facteur de chargement. Etant donné que la probabilité et le montant de la perte sont de connaissance commune, les individus sont capables de déterminer le facteur de chargement pour chaque contrat. Ainsi, la police d'assurance optimale peut-être définie comme

$$(CP(\alpha_k^*), D(\alpha_k^*)) = (p \times R \times (1 + \lambda) \times \alpha_k^*, (1 - \alpha_k^*) \times R),$$

avec α_k^* donné par

$$\arg\max_{\alpha_k}(1-p)\times U(W_k-p\times R\times(1+\lambda)\times\alpha_k, r_k)+p\times U(W_k-p\times R\times(1+\lambda)\times\alpha_k-R\times(1-\alpha_k), r_k).$$

Ainsi, à l'aide des conditions de premier et second ordres suivantes

$$CPO = \frac{\partial E(U(., \alpha_k, r_k))}{\partial \alpha_k} = p(-pR(1 + \lambda) + R)U'(w_k^+, r_k) + (1 - p)(-pR(1 + \lambda))U'(w_k^-, r_k),$$

et

$$CSO = \frac{\partial^2 E(U(., \alpha_k, r_k))}{\partial \alpha_k^2} = p(-pR(1+\lambda)+R)^2 U''(w_k^+, r_k)+(1-p)(-pR(1+\lambda))^2 U''(w_k^-, r_k),$$

où $w_k^+ = W_k - pR(1 + \lambda)\alpha_k - R(1 - \alpha_k)$ et $w_k^- = W_k - pR(1 + \lambda)\alpha_k$. il est facile de montrer que pour un agent averse au risque (i.e. $U'' < 0$) , le programme de maximisation admet une unique solution avec $\alpha_k^* > 0$. Pour un agent aimant le risque ou neutre vis à vis du risque, le contrat optimal ne comportera aucune couverture.

Partant de ces résultats théoriques, je définis un paramètre d'aversion au risque r_k pour chaque sujet k de l'expérimentation. Il est calculé pour chaque individu au début de l'expérimentation à l'aide de la méthode de liste multiple (MPL) de Holt et Laury (2002). Cela consiste à proposer au sujet une liste de choix entre deux loteries plus ou moins risquées. L'espérance mathématique de la loterie la plus risquée des deux augmente petit à petit par rapport à celle la moins risquée. Le sujet va donc, à un moment basculer de la loterie la moins risquée à la loterie la plus risquée. Le niveau de changement du sujet permet donc de mesurer son aversion au risque et donc son paramètre d'aversion, utilisé dans la fonction d'utilité espérée.

Ainsi, ces éléments me permettent de classer les contrats proposés pour chaque individu. Le classement des contrats est réellement important puisque cette information est donnée aux sujets jouant les intermédiaires "physiques" d'assurance , qui sont potentiellement amenés à donner des conseils. La construction du classement des contrats pour chaque individu en accord avec la théorie de l'utilité espérée est aussi importante afin de mesurer l'efficience des choix des sujets tout en contrôlant l'impact de la quantité d'information M (i.e. le nombre de contrat disponible lors du choix), celle-ci largement dépendante de la stratégie d'exploration.

D'autre part, la mesure construite dans le premier chapitre de cette thèse m'a permis de montrer l'importance des croyances en l'honnêteté dans le choix des canaux de distribution. Les individus croyant à l'honnêteté des intermédiaires demandent plus de conseils, en particulier lorsque le risque encouru est important. Cependant, la malhonnêteté des intermédiaires conduit les sujets à changer de canaux, en particulier lorsque leurs attentes sont importantes, mettant en évidence un effet de déception.

Les observations indiquent des stratégies d'exploration très différentes d'un individu à l'autre. Certains sujets économisent l'ensemble de leurs coûts de recherche en allant directement sur le comparateur de prix et en prenant le contrat le moins cher en termes de prime commerciale, ce qui ne signifie par nécessairement celui ayant la marge la plus élevée pour l'assureur. D'autres, quant à eux, préfèrent dépenser en coûts de recherche afin de bénéficier, au moment du choix, de l'information la plus complète possible. Les sujets les plus averses au risque privilégient la seconde stratégie, en particulier lorsque que la probabilité de perte augmente.

Concernant le choix du contrat d'assurance, seule la probabilité de perte influe sur le niveau de couverture choisi. Cependant, les résultats mettent en évidence des effets de focalisation et d'ancrage liés à une importante quantité d'information. L'obfuscation conduit les sujets à choisir leur contrat uniquement sur une caractéristique limitée : la prime commerciale. Cet effet est semblable à celui identifié par Schram et Sonnemans (2011). Cependant, les sujets tiennent aussi compte de la différence de prix entre les contrats disponibles, autrement dit la distance entre chaque contrat, mettant en évidence ici un second effet, celui d'ancrage.

D'autre part, l'effet de focalisation sur les prix, dû à l'obfuscation est une source d'inefficience des choix. Une autre source d'inefficience est liée à la sur-confiance portée aux intermédiaires "physiques". La complexité des contrats et les coûts de recherche conduisent certains sujets, confiants, à déléguer une partie de leur choix à un intermédiaire. Cependant, celui-ci s'avère dans une majorité des cas peu honnête et fortement influencé par ses propres incitations financières. En outre, l'expérimentation m'a permis de montrer que la décision de déviation des intermédiaires est considérée comme une décision risquée. En effet, les intermédiaires les plus averses au risque sont aussi les plus honnêtes.

Ainsi, je montre dans ce chapitre que bien que les intermédiaires ne proposent pas les contrats les plus optimaux, l'obfuscation justifie leur présence sur le marché de l'assurance. Une partie des consommateurs, assez confiants, continue de valoriser les courtiers et agents d'assurance, en particulier lorsque le risque encouru est important. La complexité de l'offre d'assurance, comportant différents niveaux de couverture, de prime et de franchises donne lieu à l'obfuscation. Celle-ci apparaît donc comme un potentiel moyen marketing et est source d'inefficience.

L'ensemble des stratégies est donc source d'inefficience dans les choix de contrat. D'une part l'exploration par soi-même conduit à l'obfuscation et d'autre part la délégation montre une sur-confiance dans les intermédiaires. Ainsi, le développement de stratégies multicanaux de la part des assureurs est essentiel pour leur garantir une bonne couverture du marché. Cependant, cela implique de tenir compte des spécificités de la demande rencontrée sur chaque canal d'offre.

En effet, certains consommateurs ont suffisamment confiance en l'honnêteté des intermédiaires pour déléguer une partie de leur décision afin d'éviter l'obfuscation, tandis que d'autres préfèrent explorer par eux même, conduisant à une quantité et qualité différente de l'information disponible. Tandis que la théorie économique classique en assurance met l'accent sur l'importance de l'aversion au risque et de la nature du risque dans les prises de décision, je montre dans ce chapitre que la quantité et la qualité de l'information sont tout aussi importantes dans le processus de décision.

Les biais identifiés dans ce deuxième chapitre justifient aussi la différence d'inertie des consommateurs, autrement dit d'élasticité-prix. Tandis que certains consommateurs comparent peu ou délèguent la recherche à un intermédiaire, d'autres préfèrent explorer, conduisant à l'obfuscation et justifiant une importante compétition en prix pour les assureurs, notamment à cause des effets d'ancrage et de focalisation.

La stratégie de distribution choisie par les assureurs va donc conditionner leur stratégie d'offre de contrat, puisqu'ils ne feront pas face aux mêmes consommateurs. En effet, le choix du canal de distribution est révélateur des caractéristiques des consommateurs, notamment en terme d'élasticité-prix. Ces résultats m'ont donc poussé à analyser, dans le troisième et dernier chapitre de cette thèse, comment se définissent les prix d'équilibre des assureurs dans un marché concurrentiel ou chaque assureur ne bénéficie pas des mêmes avantages comparatifs, notamment lorsque l'inertie des clients est différente d'un assureur à un autre.

4 Stratégies des Assureurs, Avantages Comparatifs et Contraintes

Le troisième et dernier chapitre de cette thèse, intitulé : "Analyses des stratégies de prix des assureurs dans un jeu non-coopératif " (" A game-theoretic analysis of insurers pricing strategies"), constitue une analyse détaillée des équilibres de marché émergeants d'une competition, modélisée à l'aide de la théorie des jeux. Ainsi, après s'être intéressé aux comportements des intermédiaires et à celui des consommateurs, je propose ici une approche plus théorique afin de comprendre comment se forme et se comporte l'offre d'assurance. En effet, même dans un marché fortement compétitif comme celui de l'assurance non-vie, l'inertie des consommateurs (différente d'un assureur à l'autre) et les contraintes de solvabilité spécifique au marché de l'assurance, assurent des primes au delà du coût marginal (en l'occurence de l'espérance des coûts lorsque l'on parle d'évènement aléatoire).

Concernant l'inertie des consommateurs, celle-ci peut s'expliquer par les nombreux biais cognitifs exposés précédemment. Tout d'abord, les coûts de recherche tendent à décourager les consommateurs à changer d'assurance. Dans ce sens, les assureurs observent un taux de résiliation décroissant en fonction de l'ancienneté du client. Ceci est particulièrement vrai en France, où le consommateur bénéficie d'une reconduction tacite. D'autre part, cette inertie n'est pas la même pour tous les assureurs. En effet, comme vu précédemment, le choix du canal de distribution joue un rôle déterminant sur le niveau d'élasticité-prix des consommateurs ciblés. Les agrégateurs de prix ont tendance à renforcer la compétition. Les assureurs distribuant donc leur offre d'assurance sur ce canal de distribution feront face à des consommateurs plus élastiques que la moyenne. Au contraire, les assureurs distribuant leurs produits via un réseau d'agents généraux ou de courtiers pourront bénéficier de cette intermédiation "physique" pour proposer des primes plus élevées, s'appuyant sur la confiance que peut apporter les consommateurs aux intermédiaires.

Cependant, le dénominateur commun à tout acte d'achat reste le niveau de prix. Il paraît en effet naturel que l'assuré soit tenté de résilier son contrat d'assurance s'il peut trouver moins cher chez un autre assureur. C'est donc pour cette raison que le modèle de compétition proposé dans ce chapitre se concentre sur le niveau de prix proposé par chaque assureur. Le prix est donc la variable de décision de chaque assureur. Bien que le comportement du consommateur ne s'explique pas seulement par le niveau de prix, la detection d'un effet de focalisation et d'ancrage sur les prix me rassure quant à la pertinence de l'utiliser comme le premier déterminant du choix de l'assureur. Toutefois, afin de prendre en compte les autres éléments et biais cognitifs expliquant également la demande d'assurance, le jeu proposé intègre des paramètres de sensibilité de prix. Ces paramètres sont différents d'un assureur à l'autre et permettent d'analyser leur impact sur les decisions des assureurs ainsi que la répartition des consommateurs sur le marché, autrement dit les parts de marché de chaque assureur.

De ce fait, la demande d'assurance est intégrée à l'aide d'un modèle stochastique défini à l'aide du modèle logistique multinomial suivant :

$$p_{j\to k}(\boldsymbol{x}) = \begin{cases} \frac{1}{1+\sum\limits_{l\neq j} e^{f_j(x_j,x_l)}} & \text{if } j=k, \\ \frac{e^{f_j(x_j,x_k)}}{1+\sum\limits_{l\neq j} e^{f_j(x_j,x_l)}} & \text{if } j\neq k \end{cases}.$$

$p_{j\to k}(\boldsymbol{x})$ correspond à la probabilité qu'un consommateur étant assuré chez l'assureur j, change d'assurance au profit de l'assureur k. $\boldsymbol{x}$ est le vecteur des prix proposés par l'ensemble des assureurs sur le marché. La sensibilité-prix des consommateurs est modélisée à l'aide de la fonction f_j définie par

$$\bar{f}_j(x,y) = \mu_j + \alpha_j \frac{x}{y} \text{ ou } \tilde{f}_j(x,y) = \tilde{\mu}_j + \tilde{\alpha}_j(x-y).$$

Dans la littérature économique, $p_{j\to k}$ est considéré comme un modèle de choix discret. Ici, nous considérons la probabilité de survenance de l'évènement, celle-ci comprise entre 0 et 1. McFadden (1981) ou encore Anderson *et al.* (1989) proposent un modèle de choix logistique ou probit multinomial. Tout comme dans Dutang *et al.* (2013), je considère ici uniquement le cas logistique. Une fonction lien probit n'améliore pas nécessairement la modélisation des choix malgré sa complexité additionnelle. De plus, l'objectif central de ce chapitre est de comprendre quels sont les effets de la sensibilité des consommateurs sur les prix d'équilibres. C'est donc la raison pour laquelle, j'intègre également deux types de fonctions de sensibilité, l'une en ratio et l'autre en différence.

Il faut noter tout de même que, bien que la prime d'assurance soit calculée en fonction des conditions de marché, elle dépend tout autant des caractéristiques propres à l'assuré. Ici, et pour raison de simplicité nous considérons tous les assurés identiques. La probabilité de survenance d'un sinistre ainsi que le montant de ce sinistre est donc la même pour chaque assuré (je présente plus loin dans cette section le modèle de perte considéré). De plus, la sensibilité au prix est également la même pour chaque assuré et dépend uniquement de l'assureur chez lequel il se trouve. Ainsi, cela permet une analyse plus global des comportements du marché sachant les différences d'inertie entre les assureurs.

Bien entendu, il est aussi intéressant d'inclure dans l'analyse d'équilibre les deux éléments présentés ci-dessus. C'est ainsi que Albrecher et Daily-Amir (2017) reprennent le modèle de Dutang *et al.* (2013), en y incluant différents types de consommateurs ainsi que de l'asymétrie d'information. Cela permet de prendre en compte les problèmes de selection adverse rencontrés par les assureurs sur un marché où les informations ne sont pas de connaissances communes. Leurs résultats montrent que les assurés de "bon" type (i.e. ayant le moins de sinistres) ont intérêt à se révéler afin de diminuer cette asymétrie d'information. En effet, la théorie économique montre que l'asymétrie d'information a pour conséquence d'augmenter les prix d'équilibres (Rothschild et Stiglitz (1976)). D'un autre coté, Barsotti *et al.* (2016) considèrent quant à eux une éventuelle contagion des résiliations à travers le temps. Leur modélisation permet de prendre en compte d'autres effets comme la mise en vigueur d'une nouvelle loi ou encore la perte de réputation d'un assureur amplifiée par un phénomène de bouche à oreille par exemple.

Ainsi, la modélisation stochastique de la demande considérée dans le chapitre trois de cette thèse me permet d'intégrer sans trop de complexité les différences de sensibilité aux prix des consommateurs d'un assureur à un autre. L'objectif est de se concentrer sur l'offre d'assurance en tenant compte des éléments essentiels, non négligeables et spécifiques au marché assurantiel.

C'est ainsi que j'en viens au second élément caractéristique du marché de l'assurance : l'aléa sur les coûts, conduisant à une forte régulation sous forme de contrainte de solvabilité. En effet, en assurance non-vie, l'individu se prémunit contre les conséquences financières d'un risque envers un de ses biens ou sa personne en achetant une police d'assurance. En contre-partie de cette assurance, l'assuré paie une prime d'assurance au début de la période de couverture, par exemple un an en assurance automobile. L'assureur s'engage donc, en cas de survenance d'une perte, à indemniser l'assuré. Cependant, au moment de la souscription, l'assureur ne connait pas le montant total réel qu'il aura à rembourser : c'est ce que l'on appelle communément le cycle de production inversé.

Par conséquent, chaque assureur se doit de modéliser les éventuels sinistres qu'il aura à rembourser afin d'ajuster au mieux la prime d'assurance. Cette modélisation donne lieu à la prime technique. Elle est égale au montant moyen espéré des coûts lié aux sinistres, chargée éventuellement des coûts de gestion. Cette modélisation est donc essentielle pour chaque assureur afin de se garantir des profits et d'être dans la capacité de répondre à ses engagements vis-à-vis de ses assurés. Elle se base sur les observations passées et peut également intégrer des données externes (données météo, données de géolocalisation,...). L'usage des données externes est d'ailleurs de plus en plus utilisé afin d'affiner la mesure du risque. En effet, grâce aux nouvelles technologies "big data", de nombreuses et nouvelles données sont facilement accessibles.

La méthode la plus communément admise pour la modélisation de cette prime technique est appelée la modélisation fréquence-sévérité (Bowers *et al.* (1997)). Celle-ci est utilisée dans le chapitre 3 de cette thèse dans le but de tenir compte de l'aspect stochastique des éventuelles pertes encourues par les assureurs.

Cette méthode consiste tout d'abord à modéliser le nombre de sinistres espérés pour chaque assuré i au temps t. Dans ce chapitre et comme mentionné précédemment je ne considère pas de sélection adverse. Autrement dit, le nombre de sinistres moyen par assuré est une variable aléatoire indépendante et identiquement distribuée. Cette variable, $M_{i,t}$, suit une loi de Poisson $M_{i,t} \sim \mathcal{P}(\lambda)$ ou bien une loi binomiale-négative $M_{i,t} \sim \mathcal{NB}(r,p)$. Elle représente la fréquence des sinistres.

D'un autre côté, l'assureur doit aussi estimer le coût moyen de chaque sinistre. C'est ce que l'on appelle la sévérité. Je considère également le coût moyen des sinistres indépendant et identiquement distribué entre les assurés et indépendant de la fréquence de sinistres.

Ainsi, $Z_{i,l,t}$, le montant du sinistre l de l'assuré i à la période t suit une loi Gamma $Z_{i,l,t} \overset{\text{i.i.d.}}{\sim} \mathcal{G}(\mu_1, \sigma_1)$ ou une loi lognormal $Z_{i,l,t} \overset{\text{i.i.d.}}{\sim} \mathcal{LN}(\mu_2, \sigma_2^2)$.

Les hypothèses i.i.d me permettent de simplifier le processus de simulation. En effet, le montant total de sinistres pour l'assureur j à la période t, $S_{j,t}$, est égal à :

$$S_{j,t}(\boldsymbol{x}_t) = \sum_{i=1}^{N_{j,t}(\boldsymbol{x}_t)} Y_{i,t}, \text{ où } Y_{i,t} = \sum_{l=1}^{M_{i,t}} Z_{i,l,t},$$

et $N_{j,t}(\boldsymbol{x}_t)$ est la taille du portefeuille de l'assureur j à la période t étant donnée le vecteur de prix $\boldsymbol{x}_t$, résultant des stratégies prix des assureurs.

C'est donc dans ce contexte d'aléa sur les coûts que les assureurs ont à définir au mieux les prix proposés sur le marché. Pour maximiser son profit, l'assureur va donc essayer d'estimer au mieux la sinistralité tout en tenant compte de l'élasticité-prix des consommateurs afin de s'assurer un volume suffisamment important. Au delà du montant total de primes encaissé, l'augmentation du volume de business, grâce à un prix plus faible que ses concurrent, permet à un assureur de diminuer la volatilité de ses pertes, et donc de réduire le niveau de la contrainte. C'est la loi des grands nombres et le principe de mutualisation inhérent au marché de l'assurance. En effet, à l'aide la théorie des ordres stochastiques, je démontre dans ce chapitre qu'il est possible d'ordonner stochastiquement la distribution des pertes encourues par les assureurs, sachant leurs stratégies de prix.

Pour rappel, le but des notions d'ordres stochastiques est de comparer des variables aléatoires. Les ordres se regroupent en deux grandes classes : i) la dominance stochastique d'ordre 1, permettant notamment d'ordonner l'espérance et, ii) la dominance stochastique d'ordre 2 (dont fait partie l'ordre convexe $\leq_{cx}$ utilisé dans les résultats de cette thèse) permettant en particulier d'ordonner la variance. L'ordre convexe a la particularité d'ordonner les variances des lois ayant une espérance égale. Ainsi, on définit la relation de préférence suivante $F \leq_{\text{cx}} G$ lorsque pour toute fonction convexe $u : \mathbb{R} \mapsto \mathbb{R}$, on a

$$\int_0^{+\infty} u(x)dF(x) \leq \int_0^{+\infty} u(x)dG(x).$$

Notons X et Y deux variables aléatoires ayant les fonctions de répartition F et G. Si $X \leq_{\text{cx}} Y$

alors cela implique $E(X) = E(Y)$, $Var(X) \leq Var(Y)$ et $E((X-a)_+) \leq E((Y-a)_+)$.

Ce dernier point est d'autant plus important puisque la régulation en assurance est très contraignante. En effet, pour garantir la solvabilité des assureurs et que ceux-ci soient bien en mesure de rembourser tout sinistre, même en cas de chocs, le régulateur impose un capital minimum. Ce capital est fonction de la volatilité des risques encourus. C'est la réglementation Solvency II qui définit ce montant en capital appelé le SCR : Solvency Capital Requierement. Cette contrainte est donc aussi impactante sur les équilibres de marché. En effet, un assureur subissant un choc sur les pertes va devoir diminuer son capital. Bien qu'il souhaite être compétitif et proposer une prime relativement basse afin de se garantir un volume de business, il sera contraint de proposer une prime plus haute que souhaité afin de garantir le niveau de capital demandé par le régulateur.

Dans le troisième chapitre de cette thèse, j'inclus donc dans mon analyse d'équilibre cette forte contrainte qui prend la forme suivante :

$$g_{j,t}(x_{j,t}) = \frac{K_{j,t-1} + n_{j,t-1}(x_{j,t} - \pi_{j,t})}{SCR(n_{j,t-1})} - 1,$$

où $K_{j,t-1}$ est le capital de l'assureur j en début de période, $\pi_{j,t}$ est coût moyen de chaque police, intégrant à la fois la sinistralité mais aussi les coûts de gestion. $n_{j,t-1}(x_{j,t} - \pi_{j,t})$ correspond donc au futur capital estimé par l'assureur j étant donné sa stratégie de prix $x_{j,t}$ et $SCR(n_{j,t-1}) = k_{995}\sigma(Y)\sqrt{n_{j,t-1}}$ et correspond au montant nécéssaire dont doit disposer un assureur pour faire face à un choc bicentenaire. Autrement dit, c'est une approximation du quantile à 99,5% de la distribution du montant total des sinistres.

Notons tout de même que la contrainte de solvabilité est évaluée en $n_{j,t-1}$ ce qui correspond au nombre de contrat déjà en portefeuille au début de la période. Cette simplification permet de s'affranchir des problèmes de concavité lors de la définition de la fonction objectif maximisée par chaque assureur j. D'autre part, les modèles internes des assureurs permettant de répondre à la contrainte de solvabilité sont calculés en début d'année et donc sur les polices déjà présentes en portefeuille.

Les éléments détaillés ci-dessus sont donc partie intégrante du modèle de théorie des jeux présenté dans le chapitre trois. D'une part, je tiens compte de l'inertie plus ou moins importante des consommateurs face aux prix proposés par chaque assureur. Cette différence de sensibilité peut constituer un avantage comparatif considérable pour les assureurs. Elle est donc déterminante dans la définition du niveau d'équilibre et des parts de marché des assureurs sur le court-terme comme sur le long-terme. D'autre part, l'aléa sur les coûts lié au cycle de production inversé et la régulation en résultant ne peuvent pas être ignoré dans la modélisation d'un marché assurantiel. De plus, chaque assureur se fait concurrence en tenant compte de la stratégie des autres assureurs. Ainsi, chaque assureur a également conscience que sa propre stratégie va influencer celle des concurrents. C'est la raison pour laquelle la théorie des jeux est pertinente pour modéliser des prix d'équilibre d'un marché concurrentiel.

Elle se définit comme l'étude des interactions entre plusieurs agents (hommes, entreprises, animaux, etc...) et regroupe l'ensemble des outils mathématiques nécessaires à la compréhension du phénomène de prise de décision pour un problème donné. Le principe fondamental sous-jacent à la théorie des jeux est que les joueurs tiennent compte, d'une manière ou d'une autre, des comportements des autres joueurs dans leur prise de décision, à l'opposé d'une vision individualiste de la théorie du contrôle optimal.

Elle prend ses racines dans les études économiques d'oligopoles réalisées par Cournot (1838), Edgeworth (1881) et Bertrand (1883). Elle a été popularisée et est devenue une discipline à part entière grâce au livre de von Neumann et Morgenstern (1944), qui pose les bases des jeux à somme nulle à plusieurs joueurs, non coopératifs et coopératifs. Quelques années plus tard, Nash (1950a,b, 1951, 1953) a transformé la théorie des jeux en proposant un nouveau concept d'équilibre et étudié l'existence de tels équilibres. Il a d'ailleurs reçu le prix Nobel d'économie en 1994 pour ces travaux.

C'est sur la base des travaux de Nash que je determine dans ce chapitre les équilibres de marché. Les études de la compétition, à l'aide de la théorie des jeux, s'intéressent généralement à deux principaux types d'équilibres : l'équilibre de Nash et l'équilibre de Stackelberg. L'équilibre de Nash suppose que les joueurs prennent leurs décisions simultanément tandis que l'équilibre de Stackelberg suppose que les actions sont prises séquentiellement (voir Fudenberg et Tirole (1991), Osborne et Rubinstein (2006) pour une description plus complète).

Dans ce chapitre, je considère uniquement l'équilibre de Nash. En effet, sur le marché de l'assurance non-vie, en particulier pour le marché de masse, il est difficile de considérer qu'un assureur prend sa décision en premier. Les stratégies de prix sont généralement définies au début de l'année en fonction des observations de l'année passée telles que les pertes, les parts de marchés, les caractéristiques financières (inflation, valeur temporelle,...) ainsi que des attentes des actionnaires pour l'année suivante.

L'équilibre de Nash $\boldsymbol{x}^\star = (x_1^\star, \ldots, x_J^\star)$ est défini tel que, dans un jeu avec J joueurs, une fonction de gain O_j et une contrainte g_j, tout $j = 1, \ldots, J$ résout le problème suivant :

$$\max_{x_j} O_j(x_j, \boldsymbol{x}_{-j}^\star) \text{ s.t. } g_j(x_j) \geq 0,$$

où x_j est l'action du joueur j et $\boldsymbol{x}_{-j}$ les actions des autres joueurs. De ce fait, quand le marché atteint un équilibre de Nash, aucun joueur, ici les assureurs, ne peut augmenter son profit en modifiant sa stratégie, étant donnée les stratégies optimales des autres joueurs.

L'utilisation de théorie des jeux dans la science actuarielle n'est pas nouvelle. Les premiers travaux remontent à Borch (1962, 1974), Bühlmann (1980, 1984), et Lemaire (1984, 1991), qui ont appliqué des modèles de jeux coopératifs afin de comprendre le transfert de risque entre les assureurs et les réassureurs (voir aussi Aase (1993), Brockett et Xia (1997), Tsanakas et Christofides (2006), Boonen (2016)). Concernant les jeux non-coopératifs, deux types de modèles ont été considérés sur le marché de l'assurance non-vie : a) l'oligople de Cournot, où les assureurs basent leurs stratégies sur le choix d'un volume de contrat (voir Powers et Shubik (1998), Powers *et al.* (1998)) et b) l'oligopole de Bertrand, où les assureurs choisissent le prix des contrats d'assurance (voir Polborn (1998), Rees *et al.* (1999) et Dutang *et al.* (2013)).

Comme mentionné précédemment, le chapitre trois de cette thèse est une extension du modèle proposé par Dutang *et al.* (2013). Ce modèle semble le plus approprié à l'analyse de la sensibilité-prix et des contraintes de solvabilité sur les prix d'équilibres. Il inclut un modèle de résiliation, un modèle de perte ainsi que des contraintes de solvabilité. De plus, je m'intéresse également à des trajectoires de long-terme à l'aide de simulation numérique. Afin de modéliser ces trajectoires de long-terme, je répète le jeu d'une période et réajuste les paramètres au début de chaque période. En effet, bien que ce chapitre n'a pas pour objectif d'analyser les cycles de marché en assurance, de nombreuses recherches ont démontré la présence d'un tel cycle (Rantala (1988), Malinovskii (2010), Emms (2012), Boonen *et al.* (2018)).

Concernant l'analyse théorique temporelle présentée dans ce chapitre, j'ai utilisé la théorie des chaînes de Markov afin de définir les matrices de transitions des assurés d'un assureur à un autre. De plus, dans le cadre d'un marché fortement régulé, les chaînes de Markov me permettent de définir des mesures explicites de convergence. Cela permet d'analyser les probabilités de distribution des assurés au sein d'un marché, autrement dit les parts de marché des assureurs, et les conséquences des prix et de la sensibilité au prix sur cette distribution.

En effet, pour toute période t le jeu présenté dans ce chapitre ne dépend que de l'information disponible au début de la période. Pour rappel, une chaîne de Markov est un processus stochastique où l'information utile pour la prédiction du futur est entièrement contenue dans l'état présent du processus et n'est pas dépendante des états antérieurs.

Autrement dit, un processus aléatoire $(X_n)_n$ à temps discret est une chaine de Markov homogène si pour tout $n \in \mathbb{N}$, pour tout $i_0, \dots, i_{n-2}, i, j, \in E$, on a :

$$P(X_n = j | X_{n-1} = i, X_{n-2} = i_{n-2}, \dots, X_0 = i_0) = P(X_n = j | X_{n-1} = i) = p_{ij}.$$

La probabilité p_{ij} représente la probabilité de transition du processus vers l'état j sachant qu'il est dans l'état i.

Ainsi, la théorie des chaînes de Markov permet de calculer relativement simplement les probabilités de transition des assurés d'un assureur à un autre (Figure 1a). De plus, sur un marché fortement régulé, où les prix sont définis par le régulateur, la chaîne de Markov, illustrant les transitions des assurés, devient homogène dans le temps. Dans ce cas, la théorie Markovienne permet également de définir la mesure invariante, autrement dit, pour un assuré, sa probabilité d'appartenir à chaque assureur (Figure 1b).

La mesure invariante μ est définie telle que $\forall n$, on note $\mu_n = (P(X_n = x))_{x \in E}$ la loi de X_n, où $(X_n)_{n \geq 0}$ est une chaîne de Markov homogène. On a :

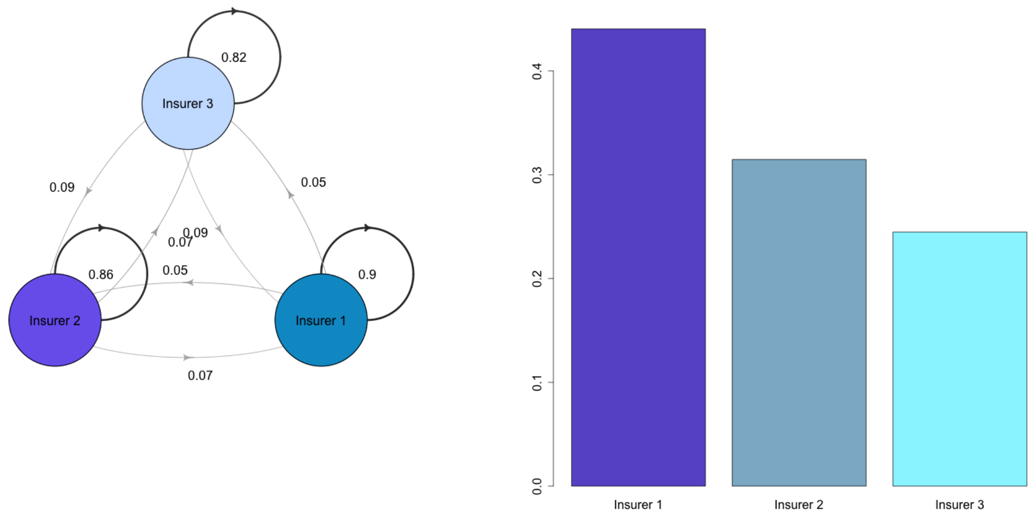
$$\mu_{n+1} = \mu_n P.$$

Ainsi si X_n admet une limite quand n est grand, alors $\mu_n \to \mu$ et μ vérifie :

$$\mu = \mu P.$$

Si de plus $\sum_{x \in E} \mu_x = 1$, alors μ est une mesure de probabilité invariante.

Je considère, par exemple, dans la Figure 1, un marché fortement régulé (i.e. la chaîne de Markov est homogène) de trois assureurs où l'Assureur 1 bénéficie d'un paramètre plus faible



(a) Matrice de transition d'un assuré

(b) Mesure invariante

FIGURE 1 – Chaine de Markov d'un assuré

de sensibilité aux prix par rapport à ses concurrents. En considérant le modèle de résiliation présenté précédemment, un consommateur présent chez l'Assureur 1 a une probabilité de 90% de rester chez ce même assureur, tandis qu'il a une probabilité de 5% d'aller à la période suivante chez l'Assureur 2 ou 3 (Figure 1a). Je définis également la mesure invariante. Ainsi, sur le long-terme un assuré a donc une probabilité de 40% d'être chez l'Assureur 1 (Figure 1b).

La définition de mesures explicites permet de mieux comprendre comment les assureurs définissent leur prix en fonction des avantages comparatifs qu'ils peuvent avoir. D'une part, certains assureurs bénéficient d'une plus grande inertie de leurs consommateurs, pouvant s'expliquer par leur choix de distribution ou encore leur réputation. Ils ont donc la capacité de proposer des prix plus élevés tout en conservant, sur le long-terme, une place de leader. D'autre part, et étant données les contraintes de solvabilité imposées par les régulateurs, les assureurs ayant un stock de capital plus important, seront moins impactés par la contrainte, leur permettant ainsi d'être plus compétitif. Par exemple, les assureurs historiques sur un marché ont tendance à avoir un niveau de capital bien supérieur au SCR demandé. Ce stock de capital, accumulé au fils des années, leur permet de rester compétitif en cas de choc sur la sinistralité. D'autres en revanche seront contraints d'augmenter leur prix au détriment du volume, essentiel pour la mutualisation des coûts mais aussi pour s'assurer un profit de long-terme.

Contributions et principaux résultats

Cette thèse est composée de trois chapitres indépendants et a pour objectif d'analyser les comportements des différents acteurs du marché de l'assurance non-vie. Chaque chapitre est tiré d'articles, chacun abordant une composante du marché : la demande, l'offre et l'intermédiation.

Les deux premiers chapitres reposent sur une approche expérimentale. J'ai construit et développé une interface web à l'aide des languages HTML et JavaScript. Concernant la collecte des données via mon interface, l'architecture a été développée en Java et PostgreSQL. J'ai également organisé et encadré les sessions expérimentales en laboratoire dans les locaux de l'ISFA (l'Institut de Science Fiancière et d'Assurances), à Lyon. Les données collectées ont été analysées à l'aide du logiciel R. Pour une totale transparence, ces données ainsi que le code R sont disponibles sur demande.

Le dernier chapitre de cette thèse inclut une approche plus théorique, reposant sur les outils mathématiques tel que l'existence d'équilibres en théorie des jeux, les principes d'optimisation ainsi que les processus Markovien. J'ai aussi réalisé des simulations numériques, également à l'aide du logiciel R et de paquets R spécifiques, developpés avec Christophe Dutang.

J'ai réalisé cette thèse lorsque je travaillais chez AXA Global France, une compagnie d'assurance. Ainsi, certaines hypothèses et choix de modélisation ont été pris en adéquation avec mon expérience de terrain et mes recherches.

Chapitre 1 : Sommes-nous plus honnêtes que l'on croit ?

Ce chapitre a pour objectif d'analyser les comportements d'honnêteté ainsi que les croyances en l'honnêteté des autres. J'ai construit une nouvelle expérimentation dans le but de mesurer les deux. Cette expérimentation permet de comparer l'adequation entre les croyances et l'honnêteté réellement observée. J'ai inclus différents environnements, plus ou moins avantageux, dans le but d'analyser les distortions de comportement en fonction des incitations financières. J'ai pour cela créé une nouvelle métrique, utilisée dans le Chapitre 2, afin de controler l'effet des croyances sur les décisions de délégation ainsi que l'impact du niveau global d'honnêteté sur la pertinence des conseils des intermédiaires d'assurance.

Globalement, j'observe que nous sommes moins honnêtes que ce que l'on croit. Cependant, la relation entre l'honnêteté et les croyances en l'honnêteté est fortement dépendante des incitations financières et du sentiment d'injustice. Quand l'environnement devient moins favorable, les sujets sont moins honnêtes. De plus, ce comportement est anticipé par les autres, en particulier lorsqu'ils sont averses aux risques. J'observe également que les individus surestiment significativement l'honnêteté lorsque les conditions sont favorables. Cependant, lorsque celles-ci se détériorent (i.e. augmentation des incitations financières à être malhonnête), les sujets donnent une meilleure estimation du niveau d'honnêteté.

Chapitre 2 : Obfuscation et honnêteté : une étude expérimentale de la demande d'assurance sur un marché sous intermédiation.

Dans ce chapitre, je construis une expérimentation reflétant les différents canaux de distribution présents sur le marché de l'assurance non-vie. Le but est de révéler les déterminants du processus d'achat d'assurance, de la recherche d'information au choix du contrat. J'analyse d'abord les stratégies d'exploration des consommateurs, qui peuvent, soit déléguer une part de leur décision à un agent général ou un courtier, soit rechercher par eux-même au risque de faire face à de l'obfuscation. J'étudie ensuite l'impact de la qualité et la quantité d'information sur le choix final du contrat d'assurance. J'explique également, dans quelles mesures, les stratégies de recherche peuvent conduire à des choix sous optimaux, selon la théorie de l'utilité espérée. Pour finir, j'analyse le comportement des intermédiaires qui sont amenés à conseiller les consommateurs, étant données leurs propres incitations financières.

Tout d'abord, j'observe que les croyances en l'honnêteté (mesurées à l'aide de l'expérimentation présentée dans le Chapitre 1) sont déterminantes dans les décisions de délégation. Cependant, les sujets réalisant que les intermédiaires suivent principalement leurs propres incitations changent définitivement de stratégie d'exploration : c'est l'effet de déception. Dans un second temps, je démontre un effet d'obfuscation. J'observe notamment des effets de focalisation et d'ancrage. Lorsque la quantité d'information augmente, les consommateurs basent leurs choix sur un critère limité : le prix. Les variables classiques, telles que la probabilité de perte ou encore l'aversion aux risques, jouent dès lors, un rôle secondaire dans le choix final du contrat. Ainsi, je montre que l'obfuscation est une source importante d'inefficience des choix. Une autre source d'inefficience tient au fait que les intermédiaires ne proposent pas les contrats les plus optimaux mais les plus rémunérateurs. Puisque certains consommateurs sont assez confiants envers les intermédiaires, et ceux pour éviter l'obfuscation, les comportements des intermédiaires les conduisent à choisir des contrats sous-optimaux. Pour conclure, ces résultats me permettent de mieux comprendre pourquoi le processus d'achat est une source d'information pour les assureurs. En effet, il révèle leurs propres caractéristiques, telles que, leur élasticité-prix, leur croyance en l'honnêteté et leur niveau d'aversion aux risques.

Chapitre 3 : Analyses des stratégies de prix des assureurs dans un jeu non-coopératif.

Dans ce chapitre, la théorie des jeux est utilisée afin de comprendre comment les assureurs définissent leurs stratégies de prix. Je considère que chaque assureur maximise sa propre fonction objectif (i.e. le profit sous contrainte de solvabilité) tout en tenant compte que leur stratégie affecte la stratégie des autres assureurs. Le modèle considéré est un jeu non-coopératif, intégrant un comportement stochastique de la part des consommateurs et un aléa sur les coûts. L'objectif est de mesurer l'impact de la différence de sensibilité des consommateurs et des chocs sur les coûts sur les équilibres de marché. J'analyse également les trajectoires de long-terme à l'aide des chaînes de Markov et simule numériquement les interactions des assureurs en fonctions des différents paramètres du jeu.

Je montre que la sensibilité des consommateurs aux prix a un effet majeur sur les prix d'équilibres. Un assureur bénéficiant d'un avantage comparatif sur l'inertie de ses clients a une plus forte probabilité d'être le leader du marché sur le long-terme. De plus, je montre également que plus la taille de portefeuille est importante, plus la volatilité des coûts est faible. Ainsi, cela réduit le niveau de la contrainte de solvabilité. Ce dernier point est en ligne avec le premier. En effet, sur un marché régulé tel que celui de l'assurance, un assureur subissant un choc sur ses coûts lors des périodes précédentes pourra être contraint d'augmenter ses prix au détriment de la taille de son portefeuille. Ainsi, un assureur dont les consommateurs sont peu sensibles aux prix, sera moins impacté par un tel choc. Il gardera donc une part de marché relativement importante, lui garantissant des profits de long-terme. Au delà du niveau de prix, l'inertie des consommateurs peut se justifier par les différents biais étudiés dans les chapitres 1 et 2. Par exemple, la stratégie de distribution ou encore la réputation peut expliquer les différences d'élasticité-prix. Ces résultats soulignent donc l'importance, pour les assureurs d'élaborer des stratégies de distribution et de communication, tout en s'assurant de bien mesurer les risques qu'ils souscrivent.

Contributions and main results

This thesis is composed of three independent chapters and aims at analyzing behaviors of the different players of the non-life insurance market. Each chapter is based on a article and addresses specificities of each market component: the supply, the demand and the intermediation.

The two first chapters are based on an experimental approach. I designed experiments and developed a web interface with HTML and JavaScript, the backend with Java and PostgreSQL as the database. I also organized and monitored experimental sessions in laboratory at the premises of ISFA (Graduate School of Actuariat Studies) in Lyon. The collected data have been analyzed with the R software. For full transparency, I make data and R code available on request.

The third and last chapter of this thesis includes a more theoretical approach using mathematical tools such as equilibrium existence properties in game theory, optimization principles and Markov chains. I also made some numerical simulations, using the R software and packages that I developed with Christophe Dutang.

I realized this thesis while I was working at AXA Global France, an insurance compagny. Hence, some hypothesis and choices modelling have been taken according to my field experience.

Chapter 1: Are we more honest than others think we are?

This chapter aims at analyzing honesty behaviors and beliefs about others honesty. I build an original experiment to measure both. This experiment permits me to compare adequacy between the beliefs about honesty and the effective honesty observed. I also include different environments, more or less lucky, in order to analyze behavioral distortion according to financial incentives. I create a new metrics that I use in Chapter 2 to control honesty beliefs impact on delegation decisions and global honesty effect on advice decisions of insurance intermediaries.

I find that, globally, we are less honest than others think we are. However, the relationship between honesty and beliefs is highly dependent on financial incentives and unfair feeling. When environment becomes less favorable, subjects are less honest. In addition, this behavior is expected by the others subjects, in particular those who are risk-averse. I also find that when conditions are favorable, people significantly overestimate honesty. However, when conditions become less favorable (i.e. increase of incentives to be dishonest), subjects give a better estimate of others honesty level.

Chapter 2: Obfuscation and honesty, experimental evidence on insurance demand with multiple distribution channels.

In this chapter, I design an experiment reflecting the different distribution channels present in the non-life insurance market. The goal is to determine what the determinants of insurance purchasing process are, from the research of information to the choice of contract. Thus, I first analyze the exploration strategies of consumers, who can either delegate a part of their decisions by requesting advice to a broker or a tied-agent, or research by themselves and face potential obfuscation. Second, I analyze the impact of the quality and quantity of information on the final insurance contract choices. I also explain to which extent each research strategy could lead to inefficient choices according to the expected utility theory. Last, I analyze intermediaries behaviors, who should advice consumers under their own financial incentives.

At first, I find that the beliefs in others honesty (elicited according to the experiment presented in Chapter 1) is a determinant of delegation decision. However, subjects who realize that intermediaries follow only their own incentives change definitively their exploration strategies: this is the deception effect. Second, I also demonstrate obfuscation effect. This obfuscation effect leads to focal point and anchoring effects. Indeed, when the quantity of information increases, consumers base their decisions on a single criterium: the price. Classical variables, such as the probability of loss or the risk aversion, play therefore a minor role on the final contract decision. Hence, I demonstrate that obfuscation is an important source of inefficiency of choices. Another source of inefficiency emerges from the intermediairies behaviors, who recommend non-optimal but more remunerated contracts. Since some consumers are confident enough and prefer to avoid obfusctation, intermediaries behaviors conduct to the subscription of non-optimal contracts. To conclude, all these results allow me to better understand why purchasing process decision is a source of information for the insurers. Indeed, consumers exploration choice reveals their own characteristics such as price-elasticity, honesty beliefs and risk-aversion level.

Chapter 3: A game-theoretic analysis of insurers pricing strategies.

In this chapter, I use a game-theoretic approach in order to understand how insurers build their pricing strategies. I consider that each insurer maximizes her own objective function (i.e. the profit under solvency constraint) while taking into account that her strategy will affect others insurers' strategies. The game-theoretic model considered is a non-cooperative game which includes stochastic consumers behaviors and stochastic losses. The objective is to analyze the impact of price-sensitivity of consumers, and the impact of loss shocks on market prices equilibrium. I also analyze long-run patterns with Markov chains and numerically simulate market interactions under different settings.

I demonstrate that price-sensitivity of consumers has a major effect on prices equilibrium. An insurer has a higher probability to be a leader in the long-run if she benefits from a comparative advantage on consumers inertia. In addition, I show that a larger portfolio size implies a lower volatility of losses. Therefore, it reduces the level solvency constraint. This result points in the same direction as the previous one. Indeed, in a regulated market, insurers suffering from sever losses on the previous period could be obliged to increase their market premiums to the detriment of portfolio size. Therefore, insurers benefiting from important inertia of their consumers are less impacted by a shock and are able to keep a relatively important size, which guarantees long-run profit. In addition to price level, consumers inertia could also be explained by the different biases studied in Chapters 1 and 2. For instance, the distribution strategies as well as the reputation can explain the difference in consumers price-elasticity. These results underline the importance for insurers to take care of their distribution and communication strategies while insuring an appropriate estimation of their underwriting risks.

Contributions en économie comportementale

Chapter 1

Are we more honest than others think we are?

—

Sommes-nous plus honnêtes que l'on croit?

This chapter is based on an article co-written with Jean-Louis Rullière.

Abstract

While the laws are justified on the basis of the efficiency they provide to society, policy makers and researchers focus on the reasons why people violate the law. Crimes and violations induce directly costs. But there is another indirect costs that is generally ignored: the fact that a person can violate the law (whether it does or not) can reduce trust in one's honesty. Thus, even if the economic agent is honest and respects the law, this loss of confidence, which could be unfounded, is also a source of inefficiency. We introduce in an experiment, a normative rule of "decision" in order to elicit both honesty and beliefs about honesty from subjects in the lab. There is no direct transfer of money between both part to avoid any inequality aversion or altruism aversion. The main question remains how individuals trust in the honesty of an anonymous group. Subjects are split into two groups: those who are subject to the temptation of (unverifiable) dishonesty and those who value the dishonesty of others. We inform each participant that we cannot identify defection. We find an important heterogeneity of trust in honesty through subjects. On average, subjects A suggests that participants B are more honest than they are. Moreover, we identify distortion of effective honesty and beliefs about other honesty when the environment of players A is unfavorable.

Keywords: *behavioral economics, trust measurement, honesty, experiment.*

Résumé

Tandis que les lois sont un vecteur d'efficience pour la société, les responsables politiques et chercheurs se concentrent principalement sur les raisons pour lesquelles les individus transgressent les règles. Il est vrai que les crimes et délits impliquent des coûts directs pour la société, cependant, il existe un autre coût indirect généralement ignoré : le fait qu'un personne puisse transgresser les règles (qu'il le fasse ou non) peut réduire la confiance portée par les autres. Ainsi, même si un agent économique est honnête et respecte la loi, la perte de confiance, même infondée, est aussi une source d'inefficience. Nous introduisons dans une expérimentation en laboratoire une règle de décision normative afin de révéler à la fois l'honnêteté des uns et les croyances en l'honnêteté des autres. Afin d'éviter toute aversion à l'inégalité ou effets d'altruisme, notre expérimentation ne comprend aucun transfert monétaire entre les deux parties. La principale question reste "À quel point les individus font-ils confiance en l'honnêteté d'un groupe d'anonyme ?". Les sujets sont répartis en deux groupes : ceux sujets à la tentation (non vérifiable) d'être malhonnête et ceux estimant la malhonnêteté des autres. Nous les informons que nous ne sommes pas en mesure d'identifier les éventuelles transgressions. Les résultats nous montrent une importante hétérogénéité vis-à-vis des croyances en l'honnêteté. En moyenne, les individus suggèrent une plus grande honneteté que celle observée. De plus, nous identifions une importante distorsion des comportements quand l'environnement des joueurs devient peu favorable.

Mots-clés: *économie comportementale, mesure d'honnêteté, croyances, expérimentation.*

1.1 Introduction

Honesty and beliefs about others honesty are central in many economic and social interactions. Above the fact that dishonesty leads to direct cost, inadequacy of honesty on the one hand and beliefs in others honesty on the other hand, is also a source of indirect cost and inefficiency. For instance, under-estimating people's honesty leads to useless and costly control of population, or self-privation of value added services. Why hire ticket inspector if everyone is traveling with valid ticket? Why do not rent our houses if they remains intact? At contrary, over-estimating people's honesty can lead to the creation of non-efficient exchange, as investment based on dishonest information. It can also support existence of non-efficient actors such insurance broker advising for most profitable instead of most appropriate contracts.

An important number of researches are devoted to the understanding of social background effects on honesty and confidence. Berg et al. (1995) were the first to shed on light importance of social background into trust reciprocity in particular when subjects already get social history. Along the same lines, Glaeser et al. (2000) show how common social characteristics raise trust and trustworthiness among individuals. More recently, Ermisch et al. (2009) analyze population characteristics impact on trust and trustworthiness by showing, for instance, that individual with "comfortable" situation are more likely to trust. In another study, Hugh-Jones (2016) show in what extent honesty and beliefs about others honesty is significantly different across countries.

Previous results demonstrate the lively interest of the scientific community in understanding relationship between honesty behaviors and beliefs about others honesty. Above the analysis of social interaction, background and individuals characteristics effects, a large part of studies are also dedicate to the analysis of incentive and environment effects. For instance, Mazar et al. (2008), Fischbacher and Follmi-Heusi (2013), Abeler et al. (2016) find that the increase of financial incentive does not significantly impact honesty level. However, Kajackaite and Gneezy (2017) show that this result does not hold when we considered "deception game", where lying possibility is explicitly exposed to subjects. The fact that lying is an explicit rule of the game may conduct subjects to put more importance of the possibility to be caught as a liar, and can lead subjects to deeper analyze cost-benefits trade off. It can thus explain increase in cheating when an incentive rises. To support this result, Yaniv and Siniver (2016) show that in a safe environment people would cheat in a large extent. Hence, previous results seem to indicate that people honesty is mainly driven by the consequences of their acts on their social ethics. In addition, Houser et al. (2012) as well as Galeotti et al. (2017) highlight the importance of perception of a given environment on honesty behaviors. Indeed, subjects tend to be less honest in unfair condition. Unequal situations reduce the social ethics consideration of subject in their honesty decision-making.

The objective of this paper is to deeper understand relationship between effective honesty and people beliefs about others honesty, depending on different conditions more or less favorable. We do not address concern about the relationship between social interaction of a group and degree of honesty in relationship, but we rather focus our analyses on honesty in anonymous subjects. Indeed, as previously mentioned, honesty behaviors (i.e. honesty actions and expectations) depend on feeling to belong to a social group. However, many economic relationships and decisions, based on honesty, take place in anonymous environment without social knowledge or history between interested parts.

We thus design an original experiment in laboratory in order to elicit honesty and beliefs about others honesty, including different level of favorable context. While standard trust game of Berg et al. (1995) is mainly used for studying trust relationship, the lack of normative rule for both sides does not allow us to answer our questions. Indeed, in the standard trust game, there is no rule asking people to transfer money or to reward transfer. Hence, game condition implies subjective rule, leading to inequality aversion and altruism effect. However, the objective here, is to reveal honesty and beliefs in others honesty in an exogenous context (i.e. more or less favorable). In other words, by implicitly adding a normative rule to the game we are able to define a similar measure for those who are subject to the temptation of (unverifiable) dishonesty and those who value the dishonesty of others.

Our experiment also differs from others honesty and cheating game in two ways. First, we inspire our game from repeated coin flip experiment (Cohn et al. (2015)), but we introduce diversity on players' environment. Players who are subject to the temptation of rule violation do not face same "severity" conditions. Last and contrary to others honesty games (Fischbacher and Follmi-Heusi (2013), Hugh-Jones (2016)) we collect data at the individual level for both sides, while ensuring anonymity of choices. This last point allows us to measure effective deviation with respect to the rule, allowing us to introduce different dishonesty temptation at the individual level.

We find that people are less honest than others think they are, particularly when rule conditions are favorable. We also catch a distortion effect of honesty behaviors, that is people honesty decreases when they face unfavorable conditions. Interestingly, this distortion effect is anticipated by subjects valuating honesty of an anonymous group. While literature results remain mixed about the correlation between honesty beliefs and risk aversion of subjects, we find that the average level of honesty expectation is not correlated to risk aversion level. In lines with Corcos et al. (2012), we find that believe in others honesty is not assimilated to a risky decision. However, our results can also support the existence of correlation between trust and risk aversion (Naef and Schupp (2009)). Indeed, we show that risk aversion of subjects in the gain domain is a determinant of honesty expectations distortion. When subjects are risk-averse, the beliefs about others honesty decrease more significantly when conditions become unfavorable.

We also investigate impact of subject's characteristics (such as gender and age) on beliefs in others honesty. While Grolleau et al. (2016) show that men are more likely to cheat, we find that they are also more likely not to believe in others honesty. Finally, we find that the age is not a determinant of honesty beliefs level. However, oldest subjects are more likely to change their expectation when conditions become favorable and, they tend to belong to more extreme profile (i.e. those who do not believe at all or those who entirely believe in others honesty).

The remainder of the paper is organized as follows. We set out the experimental design and procedure in Section 2. Section 3 then describes the detailed results, and Section 4 concludes.

1.2 Experiment Design and Procedure

We design an experiment to identify honesty and beliefs about other's honesty. This design has four main advantages. First, dishonest behaviors cannot be detected at the individual level, which reduces potential demand effect. Second, even after controlling anonymity of honesty choices, we collect data at the individual level. Third, we introduce different favorable condition levels to analyze responding deviation. Finally, the method is easy to implement in a laboratory. We also control for subject's attitude towards risk according to MPL (Multiple Price List) method of Holt and Laury (2002).

1.2.1 Elicitation of honesty and beliefs in others' honesty

During this first experimental part, each subject is assigned to a group, either A or B. This part involves a potential exchange of coins between a wallet and a padded envelope.

Before the session starts, a wallet is put on the table of each participant. This wallet either contains 10 coins of 0.50€ and a small card showing the result of 10 independent draws, or nothing. The 10 draws are of a red or green ball without replacement from a bag containing 7 red balls and 7 green balls (so that there are five possible draws, corresponding to 3, 4, 5, 6 or 7 red balls).

The content of wallets defines the group of participants. If the wallet is not empty, subject belongs to group B. We ask them to discretely apply the following rule. For each green ball draw on the small cardboard, they can collect 0.50€ in the wallet and put these 0.50€ in the padded envelope. Coins put on the envelope represent their gain for this part. The remaining euros in the wallet should correspond to 0.50€ times the number of red balls. Nevertheless, we inform them that no one in the room, including the other participants and the experimenter, are able to know if they have applied the rule or not.

To guarantee anonymity as well as to collect data according to more or less favorable situation, we also ask subject B to let into the wallet the small cardboard. Then, we scrutinize indistinguishable wallets after putting them in the same bag at the end of this part.

If the wallet is empty, subject belongs to group A. We display on their screen the different series distributed to subjects B. Series displayed order is random for each participant. They have to indicate in front of each of them how much, in euros, do they think that subject B have let in her wallet. For this part, to calculate the gain of participant A, we randomly select one of these sequences (i.e. to avoid portfolio effect) and participants A receive 5 euros minus their estimation error.

Our honesty measure is defined as the difference between the amount left in the wallet and the amount which should be left according to the rule.

1.2.2 Elicitation of risk aversion

In this part, all gains are expressed in ECU (Experimental Currency Unit). ECUs are converted at the end of the session according to the following rate: 50 ECU = 1€. Risk preferences have been measured for all subjects in the gain and loss domain. Subjects have to make two series of 10 decisions between an alternative "A" and an alternative "B". First 10 questions concerns risk attitude towards gain. For instance, they have to choose between receiving 50 ECU with a probability of 10% and receiving 20 ECU with a probability of 90% (alternative "A") or receiving 85 ECU with a probability of 10% and receiving 5 ECU with a probability of 90% (alternative "B").

The next questions concern risk attitude towards loss. We attribute to each subject and for each question an initial endowment. Then, they have to choose one alternative more or less risky. For instance, they should choose between the two following alternatives: loss 50 ECU over their 100 ECU (i.e. initial endowment) with a probability of 10% and loss 80 ECU with a probability of 90% (alternative "A") or loss 15 ECU over their 100 ECU with a probability of 10% and loss 80 ECU with a probability of 95% (alternative "B").

In addition, to compare risk aversion in loss and gain domain we fit payoff such that the expected payoff for each level of question in the gain and loss are equal. In other words, if the alternative "A" of the first questionnaire is the following: win 10 ECU with a probability of 10% then alternative "A "of the second one is: over 100 ECU losses 90 ECU with a probability of 10%.

We measure risk aversion among participants with respect to the first switch from a safe to a risky option, for both series of questions. A subject who switches at Question 5 is considered to be more risk-averse than a subject switching at the Question 2.

1.2.3 Procedure

An HTML web interface and a server database were designed especially for this experiment. Subjects were students from the University of Lyon 1 – Claude Bernard. 217 subjects participated in the experiment, 27 in average for each session including 5 participants of Type B for a total of 8 different sessions.

The honesty game was played first; all subjects received identical instructions, including comprehension questions. Then, they made their decision for this part. Afterwards, subjects received written instructions for the risk elicitation task and made their choices. Finally, we asked them to answer some general questions about age, gender and education degree. All treatments were framed in a neutral manner. The sessions lasted about 35 minutes. The average payoff was about 8 euros including a show up fee of 3€.

1.3 Results

1.3.1 Descriptive Statistics

We define here the deviation of B-type subjects receiving draw t as

$$dev_{i|t} = RB_t - L_i,$$

where RB_t corresponds to the number of red ball of the draw t times 0.50€ and L_i corresponds to the observe amount left into the wallet by subject i.

Thus, $dev_{i|t} = 0$ if B-type subject i is fully honest and perfectly follow the rule while $dev_{i|t} > 0$ if we observe dishonesty.

We detail in Table 1.1 some general descriptive statistics. The average deviation of B is 0.99€ (std. dev. 1.26). Over the 40 B-type subjects, 19, or 47% respect the rule and 14, or 35%, fully deviate. Others 18% (i.e. 7 subjects) take in average 0.71€.

Table 1.1 – Descriptive Statistics

Participant Type	Deviation	% of subjects who respect the rule	% of subjects who fully deviate	Others	% of men	Average Age	Risk Aversion Level in the gain (loss) domain*
A-type (Expectation)	0.49€	15%	33%	52%	72%	21.1	5.6 (4.9)
B-type	0.99€	47%	35%	18%	70%	20.6	5.5 (4.4)

* *Question number of the first switch from a safe to a risky option*

We also define beliefs in others honesty $HB_{i,t}$(i.e. deviation expectation) of a A-type subject i for draw t as

$$HB_{i,t} = RB_t - A_{i,t}$$

, where $A_{i,t}$ corresponds to the answer of subject i (i.e. "How much do you think that B have left into the wallet?") for draw t.

Thus, $HB_{i,t} = 0$ when A-type subject i believes in others honesty, otherwise beliefs in $HB_{i,t} > 0$ when A-type subjects i do not trust other's honesty.

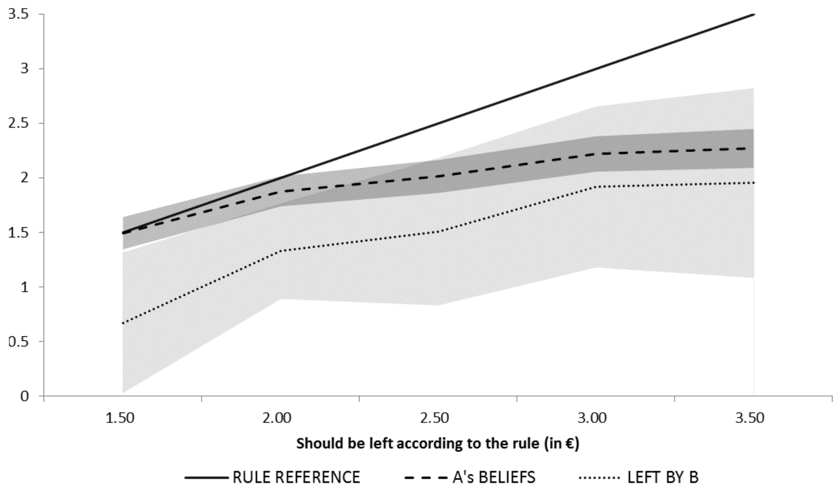
The average deviation expectation of A is 0.49€ (std. dev. 1.29). Over the 177 A subjects 27, or 15%, believe in other's honesty and 104, or 59%, have positive value of deviation expectation. Using a non-parametric signed-rank Wilcoxon test ($p - value < 2.09^{-9}$), we show that subjects are significantly less honest than others think they are.

Table 1.2 – Correlation Analysis under Draw Condition

Draw Condition	Average Expected Deviation Type A	Average Deviation Type B	p-value - H0:Expected H0:Expected Deviation < Deviation (signed ranked Wilcoxon Test)	% of fully Confident Type A	% of fully Honest Type B	% of full Expected Deviation Type A	% of full Deviation Type B
Favorable	0.14	0.73	< 0.001	36.7%	46.5%	15.3%	31.6%
Unfavorable	1.01	1.32	0.17	28.5%	50.3%	10.7%	36.2%
H0:Favorable < Unfavorable	< 0.001	< 0.001	-	-	-	-	-

Interestingly, results seem to be highly dependent on draws' conditions. It reminds us Fehr et al. (1993) and more recently Galeotti et al. (2017) results, where subjects who are paid more are more likely to reciprocate by shirking less. Indeed, in Figure 1.1, the difference between the amount which should be left according to the rule and the amount estimated by A-type subjects increases with unfavorable draw condition. We observe the same effect with respect to the effective amount left by B-type subjects.

Figure 1.1 – Average behavior w.r.t the rule



We thus classify draw condition into 2 types: favorable (i.e. amount $RB_t \leq 2.5$) vs unfavorable (i.e. amount $RB_t > 2.5$). In Table 1.2, we find significant positive difference in deviation and expected deviation under favorable with respect to unfavorable conditions. A-type subjects anticipate the strongest deviation of B-type subjects in front of unfavorable conditions. However, while under favorable conditions A-type subjects over estimate honesty ($p-value < 0.001$), this result does not hold anymore for unfavorable draw ($p-value > 0.17$) . These results suggest that subjects are able to anticipate distortion of honesty with respect to incentive conditions but overestimate honesty when incentives to be dishonest are lowest.

1.3.2 Econometrics Modelling Specification

1.3.2.1 Honesty Beliefs Distortion

We previously see how draw condition affect both honesty and beliefs about others honesty. We thus propose in this section some econometric regressions in order to better understand the distortion effects, and in what extent subjects anticipate dishonest behavior. We focus our analysis on beliefs in others honesty by performing a linear regression.

We specify the following relationship, where $A_{i|t}$ is answer of subject i for draw t:

$$A_{i,t} = f(X_i, X_t) \times RB_t + \epsilon_{i,t},$$

and $f(X_i, X_t)$ is a linear combination of individual (X_i) and draw (X_t) explanatory variables. Hence, $E(f(X_i, X_t)) = 1$ means that subject i fully believes in others honesty. However, when $E(f(X_i, X_t))$ decreases, it means that subject i has lowest expectations with respect to others honesty.

We show in Model 1 of Table 1.3 that, in average, subjects do not believe in others honesty. Indeed, the average level of honesty beliefs is significantly lower than 1. A believe that B are going to take 23% more than according to the rule.

Table 1.3 – Ordinary Least Squares Regression

Explanatory Variable	**A's Expectation**				
Coefficients *Std. error*	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**
Should be left	0.77*** *0.01*	0.84*** *0.05*	1.47*** *0.08*	1.48*** *0.15*	1.49*** *0.15*
Should be left × Risk Aversion (loss)		0.08 *0.08*			
Should be left × Risk Aversion (gain)		−0.02** *0.00*	−0.01** *0.00*	−0.01** *0.00*	−0.01** *0.00*
Should be left × Should be left			−0.21*** *0.02*	−0.21*** *0.02*	−0.20*** *0.02*
Should be left × Age				−0.01 *0.01*	−0.01 *0.01*
Should be left × Gender (Ref. level: Female)				0.04 *0.03*	0.04 *0.03*
Should be left × B's Deviation					−0.02** *0.01*
Nb. Observations	885	885	885	885	885
Nb. Subjects	177	177	177	177	177
R^2	0.721	0.723	0.744	0.744	0.745
Adjusted R^2	0.721	0.722	0.743	0.742	0.744

Signif. codes for p-values: 0.01 '***' 0.05 '**' 0.1 '*'

In Model 2 (Table 1.3), we find that risk-averse subjects in the gain domain tend to have less honesty expectations when the draw conditions of B-type subjects are unfavorable. This result is particularly interesting since we find in Model 6 of Table 1.4 that risk aversion in the loss and gain domain are not significant on the average level of honesty beliefs. Indeed, we model the Average Honesty Beliefs (i.e. deviation expectation) of each subject i, defined as following

$$AHB_i = \frac{1}{T} \sum_{t=1}^{T} HB_{i,t},$$

where t is the different draw distributed during a session to each B-type participants (i.e. $T = 5$). Thus and in lines with Eckel and Wilson (2004), we do not find correlation between the level of trust (in our case Average Honesty Beliefs) and risk aversion. Hence, confidence in others honesty is not a risky decision. While risk aversion level (i.e. in the gain domain) does not explain the base level of subjects honesty beliefs, it explains why some people change their expectations when incentives to be dishonest are highest. Risk-averse participants reduce more their honesty expectations when B-type participants face unfavorable conditions.

Because we are interesting on the shape of honesty beliefs given draw conditions, we add in Model 3 (Table 1.3) the square of the amount who should be left according to the rule. As previously seen, A's expectations significantly depend on B draws: favorable condition reduces rule violation expectation.

Elsewhere, we check in Model 4 of Table 1.3 and Model 6 of Table 1.4 eventual subjects' characteristics effects (i.e. Age and Gender). We want to explain both: the average level of honesty beliefs and the distortion effect with respect to draw conditions. We do not find significant effect of such variables in both models and others results remain true.

However and because of the weak dispersion of subjects age into our sample, we decide to create a categorical variable with few values. Thus, we split our sample into three classes: subjects being the same age as the median (i.e. 21 years old) who represent 20% of our sample and subjects being a higher (resp. lower) age than the median, representing 41% (resp. 39%) of our sample. We find in Model 5 bis of Table 3 bis of Appendix that youngest subjects have lowest distortion beliefs effect. While this method allows us to catch information from tails distribution of age, we find in model 6 bis of Table 4 bis (Appendix) that age does not explain global honesty beliefs level of subjects (in lines with Model 6 of Table 1.4).

Table 1.4 – Ordinary Least Squares Regression

Explanatory Variable	**A's Average Honesty Beliefs**
Coefficients *Std. error*	**Model 6**
Risk Aversion (loss)	−0.02 *0.04*
Risk Aversion (gain)	0.06 *0.03*
Age	0.03 *0.02*
Gender (Ref. level: Female)	0.14 *0.16*
B's Average Deviation	0.13 *0.11*
Constant	−0.04 *0.66*
Nb. Observations	177
Nb. Subjects	177
R^2	0.02
Adjusted R^2	-0.04

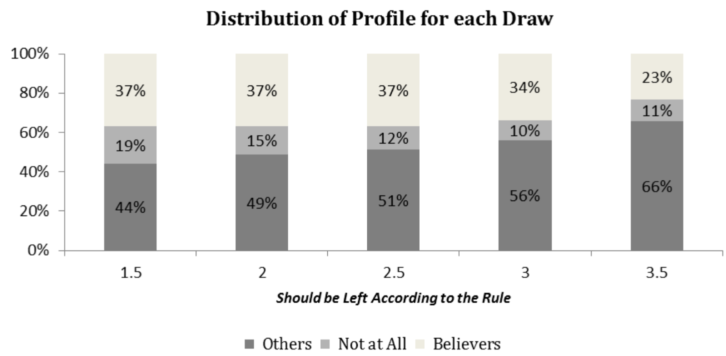
Signif. codes for p-values: 0.01 '***' 0.05 '**' 0.1 '*'

Last, we add in Model 5 (Table 1.3) the effective deviation observed for each draw, in order to better catch anticipation effects. While the rule enforced to B remains the more important effects on expectation, we show that A-type subjects significantly anticipate B's deviation distortion.

1.3.2.2 Honesty Beliefs Profile

After analyzing distortion of honesty beliefs, we now focus our study on more global honesty belief profiles. In Figure 1.2, we split our subjects into three different profiles. First, subjects who do not expect any deviation from B-type subjects, named the "Believers". "Believers" profile represent in average 33% of our sample, and Figure 1.2 displays a decreasing percentage when draw conditions becomes less favorable. Second, we classify subjects with no honesty beliefs into a "Not at All" profile, they represents around 13%. We regroup all others subjects into a last group called "Others", representing in average 53% of subjects.

Figure 1.2



We carry out a multinomial logistic regression including random effects (Table 1.5, Model 7 and 8) with the honesty beliefs group as the dependent variable (i.e. "Believers", "Not at All" and "Others"). Random effects allow us to control for the relationship between errors in our panel data (Croissant (2018)).

Table 1.5 – Multinomial Logistic Regression including Random Effect

Explanatory Variable	**Honesty Beliefs Profile**			
	Model 7		**Model 8**	
Coefficients *Std. error*	"Not at all"	"Others"	"Not at all"	"Others"
Should be left	−0.64** *0.27*	0.65*** *0.16*	−0.62** *0.27*	0.66*** *0.17*
Risk Aversion (gain)			0.16* *0.09*	−0.03 *0.06*
Age			−0.26 *0.18*	−0.08* *0.04*
Gender (Ref. level: Female)			1.90** *0.59*	−0.75** *0.25*
Constant (Ref. level: "Believers")	−3.15*** *0.99*	−0.95*** *0.42*	0.25 *2.35*	1.48 *1.10*
Nb. Observations	885		885	
Nb. Subjects	177		177	
R^2	0.259		0.263	
Adjusted R^2	0.254		0.253	

Signif. codes for p-values: 0.01 '***' 0.05 '**' 0.1 '*'

As previously seen in Figure 1.2, we show in Model 7 (Table 1.5) that the percentage of "Believers" is decreasing when conditions become unfavorable. Under stable coefficients with respect to Model 7, we catch in Model 8 additional information on subjects honesty beliefs profile by adding some subjects variables (i.e. risk aversion level, age and gender).

According to Models 2, 3, 4 and 5 of Table 1.3, subjects more risk-averse in the gain domain get higher probability to belong to "Not at All" profile.

Gender of subjects do not appear as a significant determinant of distortion in honesty beliefs (Models 4 and 5 of Table 1.3). It does not explain global honesty beliefs level either (see Model 6 of Table 1.4). However, by analyzing more global profile in Model 8 (Table 1.5), we find that men have highest probability to belong to extreme profile (i.e. “Believers” or “Not at All” profile). Last, using the extent of coefficient of gender variable, men have significantly lower expectation than women about others honesty.

As a reminder, we find in Model 6 (6 bis) of Table 1.4 (4 bis) that age does not explain global level of honesty beliefs but explains expectation distortion beliefs(Model 5 bis of Table 3 bis). We find here, in Model 8 (8 bis) of Table 1.5 (Table 5 bis) that oldest subjects tend to belong to more extreme profile. However and according to Table 1.4, results do not allow us to determine difference in the honesty beliefs level.

1.4 Conclusion

We detail in this paper the design of a new honesty elicitation game. The objective consists on the measure of adequacy between honesty and honesty beliefs in an anonymous context. We introduce different level more or less favorable and avoid any inequality aversion or altruism effect by defining a normative rule to determine honest behavior. These new metrics defined by $dev_{i|t}$ for B-type subject and $HB_{i,t}$ for A-type subjects is relatively quick and easy to implement.

We distinguish the honesty concept from that of trust, which is widely explored in the behavioral economics. We are interested not only in honesty and its measure, but especially in the question of beliefs about the supposed honesty of others. Economists are agree to say that rise of economic agents honesty lead to an increase to social welfare. For instance, Fetchenhauer and Van der Vegt (2001) argue that honesty lead to reduce transaction cost and thus stimulates economics growth.

However, this approach do not take into account adequacy between honesty and beliefs. Indeed, even in a fully honest world, transaction costs could no be reduced if no one believes in others honesty. Hence, the level of honesty remains important to explain sources of inefficiency of economic relationship, yet an interaction decision is also mainly determined by beliefs about others honesty.

We globally find that people are less honest than other think they are. However, this measure should be used with caution. Indeed, we show an important increasing pattern regarding honesty (i.e. and beliefs) and favorable condition. Peoples defect more when they are in an unfavorable environment and others anticipate. However, in a “safety” context, expectation are lower than effective dishonesty.

Second, we find that beliefs in others honesty is not a risky decision. Indeed, risk aversion (in the gain and loss domain) does not explain average level of honesty beliefs. However and regarding to the distortion of beliefs in unfavorable condition, we show that risk-averse subjects in the gain domain downgrade more significantly their expectations about other honesty.

Last, while individuals characteristics do not allow us to determine average beliefs about others honesty, they explain more general profile. Indeed, we find that oldest subjects are more likely to change their expectations depending on condition and belong to extreme profile. Besides, a larger part of men than women do not believe at all in others honesty and expect a full deviation (i.e. no remaining euros in the wallet).

The relevance of our measure has been verified in Mouminoux et al. (2018) to explain delegation decisions in insurance markets. We find that beliefs in other honesty is a major determinant of intermediaries choices. Indeed, even under an high degree of brokers dishonesty, because of their own financial incentives, (i.e they do not propose the most optimal contract), some consumers prefer to ask for advice. This is a good example of consequences of gap between honesty and beliefs about others honesty.

Elsewhere people seems to allow themselves to do something bad (i.e. dishonest) because of injustice feeling. The fact that expectations are relatively closed to effective behaviors in bad conditions reinforced the idea that its cognitive bias is globally anchored in decision making. This results may have important consequences in fraud behaviors for instance. We can imagine that an insurance policyholder who never declared claims while paying their insurance premium for many years will miss-declared her first claims.

We do not adress here issues about repetitive interactions and consequences of the observation on honesty behaviors. It could be interesting to add a dynamic part into our honesty elicitation gain in order to better understand how individuals adjust their beliefs. We can imagine that dishonesty observation in favorable condition may have significant effect on honesty expectation in unfavorable one.

Another interesting approach, and in lines with Yaniv and Siniver (2016), could consist in adding in our experiment potential punishment of dishonest behaviors. For instance, we can add a probability to be detected and may expect difference in both: honesty behavior and beliefs about honesty. It could also be interesting to analyze, under punishment condition, if distortion of honesty resulting from eventual feelings of injustice keeps hold.

1.5 Appendix

Table 3 bis – Ordinary Least Squares Regression

Explanatory Variable	**A's Expectation**	
Coefficients	**Model 5**	**Model 5 bis**
Std. error		
Should be left	1.49***	1.42***
	0.15	*0.09*
Should be left × Risk Aversion (gain)	−0.01**	−0.02**
	0.00	*0.01*
Should be left × Should be left	−0.20***	−0.20***
	0.02	*0.03*
Should be left × Age	−0.01	
	0.01	
Should be left × Age<Median Age^i		0.09**
		0.04
Should be left × Age>Median Age^i		−0.01
		0.04
Should be left × Gender (Ref. level: Female)	0.04	0.04
	0.03	*0.03*
Should be left × B's Deviation	−0.02**	−0.03**
	0.01	*0.01*
Nb. Observations	885	885
Nb. Subjects	177	177
R^2	0.745	0.748
Adjusted R^2	0.744	0.746

Signif. codes for p-values: 0.01 '***' 0.05 '**' 0.1 '*'
^{i} ref. level: Age = Median Age

Table 4 bis – Ordinary Least Squares Regression

Explanatory Variable	**A's Average Honesty Beliefs**
Coefficients	**Model 6 bis**
Std. error	
Risk Aversion (loss)	−0.02
	0.04
Risk Aversion (gain)	0.06
	0.04
Age<Median Age^i	−0.26
	0.20
Age>Median Age^i	0.03
	0.21
Gender (Ref. level: Female)	0.14
	0.17
B's Average Deviation	0.12
	0.11
Constant	−0.04
	0.66
Nb. Observations	177
Nb. Subjects	177
R^2	0.043
Adjusted R^2	-0.003

Signif. codes for p-values: 0 '***' 0.001 '**' 0.05 '*' 0.1 ' ' 1
^{i} ref. level: Age = Median Age

Table 5 bis – Multinomial Logistic Regression including Random Effect

Explanatory Variable	Honesty Beliefs Profile			
	Model 8		Model 8 bis	
Coefficients *Std. error*	"Not at all"	"Others"	"Not at all"	"Others"
Should be left	−0.62** *0.27*	0.66*** *0.17*	−0.62** *0.27*	0.66*** *0.17*
Risk Aversion (gain)	0.16* *0.09*	−0.03 *0.06*	0.37*** *0.01*	−0.05 *0.07*
Age	−0.26 *0.18*	−0.08* *0.04*		
Age<Median Age^i			−0.52 *0.54*	−0.25 *0.29*
Age>Median Age^i			0.72 *0.54*	−0.74** *0.29*
Gender (Ref. level: Female)	1.90** *0.59*	−0.75** *0.25*	1.35** *0.54*	−0.75** *0.25*
Constant (Ref. level: "Believers")	0.25 *2.35*	1.48 *1.10*	−6.42*** *1.70*	0.22 *0.62*
Nb. Observations	885		885	
Nb. Subjects	177		177	
R^2	0.259		0.263	
Adjusted R^2	0.254		0.253	

Signif. codes for p-values: 0.01 '***' 0.05 '**' 0.1 '*'
^{i} ref. level: Age = Median Age

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Chapter 2

Obfuscation and Honesty: Experimental Evidence on Insurance Demand with Multiple Distribution Channels

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Obfuscation et honnêteté : une étude expérimentale de la demande d'assurance sur un marché sous intermédiation

This chapter is based on an article co-written with Jean-Louis Rullière and Stéphane Loisel.

Abstract

This paper aims to shed light on the dilemma faced by insurance purchasers faced with multiple distribution channels. Should the consumer herself choose from a large set of insurance policies or rather delegate a part her decision to an intermediary who is more or less honest? We consider decisions based on a number of real-world insurance distribution channels with different information frames. Beliefs about intermediary honesty are the main determinants of individual choice. In addition, the obfuscation of information is the main source of inefficiency in decision-making, particularly regarding the characteristics of the insurance contracts chosen by consumers.

Keywords: *behavioral economics, distribution channels, honesty, insurance, intermediation, obfuscation, search costs*

Résumé

Cet article a pour objectif d'étudier le comportement d'achat d'assurance sur un marché comprenant différents types de canaux de distribution. Le consommateur choisit-il par lui-même parmi un large choix de police d'assurance ou délègue-t-il une partie de sa decision à un intermédiaire plus ou moins honnête ? Nous étudions ici le processus d'achat d'assurance dans un cadre inspiré des canaux de distribution d'assurance du marché réel conditionnant l'accès à l'information. La croyance en l'honnêteté des intermédiaires est le principal déterminant des choix de canaux de distribution. De plus, l'obfuscation liée à une quantité importante d'information conduit à des choix d'assurances sous-optimaux.

Mots-clés: *économie comportementale, canaux de distribution, honnêteté, assurance , intermédiation ,obfuscation, coûts de recherche.*

2.1 Introduction

The internet has significantly affected purchasing behavior by changing access to information. No industry has been immuned to this considerable change, including more recently the insurance sector, see e.g. Cappiello (2018). In principle, greater access to market information allowing the better comparison of offers is a good news for consumers. In the past ten years, we have seen the erosion of the traditional distribution channels of brokers and tied-agents, see e.g. technical reports of FFA (French Federation of Insurance). However, these channels continue to co-exist with new channels such as cyber-brokers and online insurance websites. Consumers therefore continue to value physical intermediation.

This is partly explained by the complexity of insurance products, leading to obfuscation. Obfuscation here reflects the limited discernment of agents due to excess information or the introduction of a great deal of irrelevant information. Physical intermediaries reduce this obfuscation by offering comparisons and advice. However, physical intermediaries also have their own financial incentives from insurers and consumers demand for their services. The extent to which intermediaries are honest and how consumers perceive this honesty are therefore central in the delegation decision. Consumers will delegate only if they believe others to be sufficiently honest. The consumer decision is thus based on a trade-off between finding the appropriate information in a context of obfuscation and obtaining good information from more or less honest intermediaries.

The aim of this paper is to understand how consumers choose their insurance contracts in this kind of multichannel environment. Consumer decision-making is complex as, in addition to insurance-contract choice, they have to choose their search strategy. We here provide some insights into how this choice reveals information about the customer profile (price elasticity, risk profile etc.). The results underline the importance for firms of managing their distribution strategy and contract offers among heterogeneous consumers in order to screen between them.

We take into account consumers potential search costs to access information when considering delegation and information-gathering. These costs are defined as the time, effort and money expended by a consumer who searches for a product or service. Our goal here is not to measure these individual search costs, contrary to Brynjolfsson and Smith (2000). Search costs are considered to be neutral and exogenous, such that the expected search cost to find the optimal policy is the same for each distribution channel. The presence of search costs ensures that individual exploration choices are taken conscientiously. We also do not consider the effect of search costs on the market price equilibrium, contrary to Diamond (1971), **?**, Baye et al. (2004) and Branco et al. (2012)), and focus on the demand side of the market with exogenously-defined offers.

Social psychology theory suggests that greater available information and choice should improve the quality of decisions by informing consumers of all of the possibilities. However, S. Iyengar and Lepper (2001) carry out a number of different behavioral experiments and show that this assumption does not necessarily hold: the desire for choice is not unlimited. This conclusion also pertains in the experimental analysis in Schram and Sonnemans (2011) regarding the choice of health insurance. They find that when there are many alternatives, subjects only consider a smallest part of the available information and carry out a process of elimination based on limited characteristics. Hence, search costs and delegation choice affect the quantity of information available for choice, which in turn might lead to focal point and anchoring effects due to obfuscation (see e.g. Gabaix et al. (2006), Ellison and Ellison (2009), and Ke et al. (2016)).

Search costs and obfuscation can lead to delegation by consumers. Insurance intermediaries are meant to follow deontological rules and propose optimal contracts, as they are paid to do so. However, the relationship between the consumer and physical intermediaries underlines the role of honesty. This exchange game affects trust, which might at first lead us to think of trust-game analysis (Berg et al. (1995)). However, their exchange game contains no normative rules (i.e. there is no rule that individuals should transfer money or reward transfers). Thus, this approach does not allow for objective deviation to be controlled for, as each individual will have a subjective view of the behavior to be adopted, which could depend on inequality aversion (Glaeser et al. (2000)).

The insurance market, as mentioned above, has intermediaries who sell advice, implying a normative rule. The presence of this implicit rule therefore leads us to appeal to more recent concepts of honesty (Fischbacher and Föllmi-Heusi (2013)) and beliefs about others honesty (Hugh-Jones (2016)). Although there are normative rules for insurance intermediation, Cummins and Doherty (2006) note the importance of insurers' financial incentives on their advice. Houser et al. (2012) find greater dishonesty when people have been treated unfairly. Inequality observed by the intermediary between different bonus levels can produce feelings of unfairness as from her point of view these inequalities are groundless. This result is confirmed by Galeotti et al. (2017), who show that financial incentives influence intermediaries' honesty. Intermediary behavior is a concrete issue and regulatory concern, as the IDD (Insurance Distribution Directive) illustrates. This new European directive comes into force in October 2018, and aims to monitor insurers financial incentives more strictly.

As honesty is difficult to observe in the field, we here elicit honesty and belief about others honesty in a laboratory experiment. The experimental approach allows us to measure individual risk aversion, which affects decisions in uncertain contexts such as insurance.

All of the above concerns appear in our experiment. We find that honesty beliefs are important in the choice of distribution channel. Subjects who expect honesty ask for more advice, in particular when the probability of loss is higher. Risk-averse individuals do not directly visit insurers and prefer to explore the market via brokers or cyber-brokers. We also see that intermediary dishonesty leads subjects to change channel, in particular when they initially expect honesty.

We identify two opposing search strategies: "saving search (costs)" and "deep search". Risk-averse subjects avoid "saving search", especially as the probability of loss rises. Regarding contract choice, we find that only the probability of loss has a significant positive effect on coverage choice. However, we also find focal-point and anchoring effects with respect to the quantity of information. Obfuscation leads subjects to choose their contract by comparing prices (i.e. the focal-point effect). Subjects also take into account the gaps between contract prices, with the average contract price acting as a benchmark in purchasing decisions (i.e. the anchoring effect).

We also find that obfuscation leading to focal-point effects produces inefficient choice. As the quantity of information rises, so does the probability of choosing the lowest price, which leads to inefficient choices. Intermediary deviation is also a source of inefficient choice, although the detection of deviation leads to more efficient choice. The trade-off between delegation and self-exploration thus essentially depends on beliefs in others' honesty when market exploration is costly and information is not easy to compare.

Last, as intermediaries have their own financial incentives, we find considerable deviation (i.e. they do not propose the best contract), especially when it is profitable to do so. Wealth and the nature of risk do not affect deviation. However, deviation is considered to be risky, as risk-averse intermediaries deviate less. We also find a correlation between honesty and deviation in advice in the insurance market at the session level, where more honest intermediaries are more likely to suggest the optimal contract.

The remainder of the paper is organized as follows. We set out the experimental design in Section 2, and Section 3 briefly summarizes the procedure. Section 4 then describes the detailed results, and Section 5 concludes.

2.2 Experimental design

We designed an experiment to identify the determinants of insurance choice, including intermediaries and search costs (the cost of exploration). After controlling for individual risk attitudes and beliefs about other's honesty, the experiment allows us to investigate the consumer's exploration decision and contract choice. The elicitation of risk aversion is used to parameterize the design of the contract choice. The instructions for each part of the experiment as well as screenshots of the interface can be found in Appendix. Subjects are divided into two groups, A and B, and remain in the same group throughout the experiment.

2.2.1 The elicitation of honesty and beliefs about others' honesty

In this first part, we propose an original experiment in order to elicit individual honesty and beliefs about other's honesty. We do so via the exchange of coins between a wallet and a padded envelope.

Before the session starts, a wallet is put on the table of each participant. This wallet either contains 10 coins of 0.50€ and a small card showing the result of 10 independent draws, or nothing. The 10 draws are of a red or green ball without replacement from a bag containing 7 red balls and 7 green balls (so that there are five possible draws, corresponding to 3, 4, 5, 6 or 7 red balls).

The content of each wallet establishes the participant's group. If the wallet is not empty, the subject is in group B. They are asked to discreetly apply the following rule: for each green ball drawn on the card they can take 0.50€ from the wallet and put it in the padded envelope: the coins in the envelope represent their gains for this part of the experiment. The Euros remaining in the wallet should then correspond to 0.50€ times the number of red balls.

However, participants are informed that no-one in the room, including other participants and the experimenter, will check if they follow this rule. The wallets cannot be identified within individuals, and are all put in the same bag at the end of this part of the experiment.

If the wallet is empty, the subject is in group A. We show on their computer screen the different draws that were distributed to the B subjects. They then indicate for each draw how many Euros they think that the B subjects left in their wallets. The gain of the A participants in this part comes from the random selection of one of these draws, with their earnings being 5 Euros minus the estimation error.

2.2.2 The elicitation of risk aversion

In this part, all gains are expressed in ECU (Experimental Currency Units), which are converted at the end of the session using the following rate: 50 ECU = 1€. Risk preferences are measured for all subjects in the gain and loss domains. We use the Multi Price List (MPL) method suggested by Holt and Laury (2002) and convert dollars into ECU.

Subjects make two series of 10 decisions between alternatives “A” and “B”. The first 10 questions concern risk attitudes towards gains. For example, subjects have to choose between receiving 50 ECU with a probability of 10% and 20 ECU with a probability of 90% (alternative “A”) and 85 ECU with a probability of 10% and 5 ECU with a probability of 90% (alternative “B”).

There are then questions about risk attitudes towards losses. We attribute to each subject and for each question an initial endowment. They then have to choose one more or less risky alternative, for instance between losing 50 ECU of their 100 ECU (the initial endowment) with a probability of 10% and losing 80 ECU with a probability of 90% (alternative “A”) and losing 15 ECU of their 100 ECU with a probability of 10% and losing 80 ECU with a probability of 95% (alternative “B”).

It can be argued that the house-money effect (Thaler and Johnson (1990)) makes participants more willing to take risks. However, Etchart-Vincent and l’Haridon (2011) compare subjects’ risk attitudes in three payment conditions: a real loss condition with a random lottery, “losses-from-an-initial-endowment” and a hypothetical-loss condition. They find no significant difference between these three payment conditions in the loss domain, supporting our approach.

In addition to comparing risk aversion in the gain and loss domains, we choose payoffs such that the expected payoff for each level of the gain and loss question are the same. In other words, if alternative “A” in the first questionnaire is “win 10 ECU with a probability of 10%”, then alternative “A” in the second is “from 100 ECU lose 90 ECU with a probability of 10%”.

We measure participants’ risk aversion by their first switch from a safe to a risky option, for both series of questions. For instance, a subject who switches at Question 5 is considered to be more risk-averse than one who switches at Question 2.

2.2.3 Main game

In the main game, participants retain their type from the first part of the experiment (A or B): A-type participants now play the role of insurance customers while B-type participants are "human" intermediaries. The game is repeated for eight periods (called rounds), including two trial rounds.

A participants select an insurance contract in each round that protects against a known loss with known risk (probability). Each contract includes a fixed premium and a deductible paid in the case of loss. B-type participants can provide advice to A-type participants. The A's can call on human intermediaries to obtain information about the contracts. In this case, one of the B's is chosen randomly to provide advice. We explain below the precise nature of this interaction.

2.2.3.1 The design of the exploration game

The A's choose one contract in each round: they cannot remain uninsured. In each round, they have an initial level of wealth and a probability and amount of loss. These elements can vary by period. There are eight different available contracts grouped into four menus of two contracts each (A, B, C and D). These different menus represent the insurance companies on the market. These contracts are hidden at the beginning of each round, and to obtain information on them A's have to explore the market. To do so, they have an exploration allowance that is equally re-endowed at the beginning of each round. This credit is run down for each exploration action, and the unused exploration credit forms part of A's earnings. A screenshot of the instructions at the beginning of each round appears in Appendix.

Participants can explore the market directly by choosing one of the four "company" menus. This corresponds to individual sequential research, such as visiting insurance vendors or websites one by one. To do so, the A's pay 12 ECU (debited from their exploration credit). Once they access the information about the two contracts on the menu, they can ask for a recommendation for 4 ECU (also deducted from the exploration credit) corresponding to time spent with a tied-agent or a call-center for advice about the products from one unique company. In this case, one of the B's is selected to give advice.

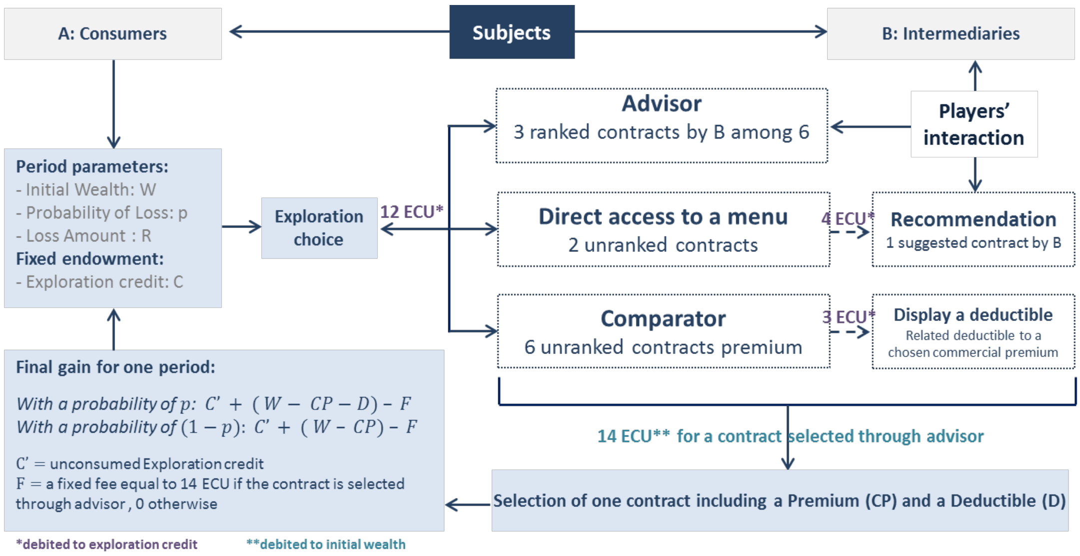
This B-type participant receives additional information about the contracts: their screen shows a ranking of the two contracts depending on the policy features and A's risk attitude. We explain below how this ranking is produced. In addition, there is a bonus for each contract that is paid to B if A chooses this contract via B's intermediation. The bonuses are not paid by A. B decides which contract to suggest according to this information. The A-type participants do not know the bonus information.

Another possibility for A-type subjects is to look for an advisor (i.e. a B-type) corresponding to a broker. To do so, A has to pay 12 ECU (from their exploration credit). One B-type participant is then randomly selected. This B's screen shows the same information as discussed above, but now for six different contracts (from three of the four companies). B here suggests three ranked contracts from the six to participant A. In addition to the potential bonus, B receives a fixed fee of 14 ECU (debited from A's initial wealth). A screenshot of the interaction with B as broker or tied-agent appears in Appendix.

Note that the difference between tied-agents and brokers comes from the nature of their relationship with the insurance companies: brokers can sell insurance from different insurers, while tied-agents are constrained by an exclusivity contract to offer insurance from only one company.

Last, A can explore the market using a comparator, corresponding to a cyber-broker (also called an aggregator). Using the comparator costs 12 ECU (debited from the exploration credit). Six of the eight possible contracts then appear on A's screen (from three of the four companies). Only premium information is shown: to find out the associated deductible of a contract, A has to pay 3 ECU (i.e. it is possible to choose a contract without knowing the corresponding deductible). The comparator does not rank these contracts.

Figure 2.1



Subjects can continue to explore as long as they have exploration credit. We explain below how we define the search costs. To sum up, Figure 2.1 sets out the different decisions and potential interactions between players.

The final gain for this part of the experiment is that from one of the last six rounds, randomly selected (the first two rounds are trials). The loss event comes about according to the probability in this selected round. In addition to the potential gains from player interaction, B's receive a fixed endowment in this part.

2.2.3.2 Theoretical Framework

2.2.3.2.1 The definition of contracts

For each round, we generate eight different contracts, defined by a premium CP and a deductible D. The contracts are designed as in Schlesinger (2013) such that

$$CP = (1 + \lambda)p \times R \times \alpha, \tag{2.1}$$

where λ is the loading factor of the insurer, p the probability of loss, R the amount of the loss and α the coverage rate. In order to offer contracts similar to those in the real world, we define a deductible D from (2.1) such that

$$\alpha = \frac{(R - D)}{R} \Leftrightarrow D = R - \frac{CP}{(1 + \lambda) \times p}. \tag{2.2}$$

We generate for each round a set of contracts G_1 such that

$$\left\{ G_1 = CP_{G_1} \in [0, W]; D_{G_1} = R - \frac{CP_{G_1}}{(1 + \lambda) \times p}, \lambda \in [-0.2, 1] \right\}, \tag{2.3}$$

where W is the initial wealth allocates to subjects. For simplicity, we round each element to the nearest integer.

We then define a new subset $G_2 \subset G_1$ such that all contracts are unique, possible and non-dominated. We define a possible contract k such

$$W - CP_k - D_k - 14 > 0. \tag{2.4}$$

We cannot offer contracts that produce a negative final gain in a given period. As a reminder, 14 ECUs correspond to the fixed fee paid to B when A chooses a contract via B's intermediation: this is the fee paid for broker advice.

A contract i is dominated by contract j if $CP_i > CP_j$ and $D_i > D_j$. We exclude dominated contracts as these could affect purchasing decisions and, in the real world, contracts include so many different elements that it is difficult to say that one contract clearly dominates another. We finally randomly draw eight contracts from G_2 in each period.

2.2.3.2.2 The definition of ranking

In order for intermediaries to provide advice in insurance markets, we give them private information via a ranking of contracts. Contracts are ranked by expected utility based on risk aversion. For simplicity, and because wealth changes from one round to another, we use a CRRA utility function (Constant Relative Risk Aversion). The use of IRRA or DRRA (Increasing/Decreasing relative risk aversion) (Saha (1993)) requires the estimation of two parameters, and therefore multiple applications of the Holt and Laury test with different levels of wealth. We therefore assume that consumers display the same risk-aversion for any risk-wealth ratio:

$$U(x) = \begin{cases} \frac{x^{1-r}}{1-r}, & \text{if } r \neq 1 \\ \ln(x), & \text{otherwise} \end{cases}, \tag{2.5}$$

where r is the aversion parameter with $r = 0$ for risk-neutral subjects, and $r > 0$ (resp. $r < 0$) for the risk-averse (resp. risk lovers). A contract i is optimal with respect to another contract j at round t if

$$p_t \times U(W_t - CP_{i,t} - D_{i,t}) + (1-p_t) \times U(W_t - CP_{i,t}) > p_t \times U(W_t - CP_{j,t} - D_{j,t}) + (1-p_t) \times U(W_t - CP_{j,t}). \tag{2.6}$$

The next step consists in the definition of the subject's risk-aversion parameter from the answers to the MPL (Multiple Price List). As risk aversion is different in the gain and loss domains (Tversky and Kahneman (1981)) and insurance consumers face risks in the loss domain, we use only the answers from the loss domain.

Table 2.1 – Risk Aversion Parameters (Holt and Laury (2002))

Questions of first switch from Safe to Risky Option	**Range of Relative Risk Aversion**	**Risk Preference Classification**	**Parameters Used**
0-1	$r \leq -0.95$	highly risk loving	-0.95
2	$-0.95 < r < -0.49$	very risk loving	-0.72
3	$-0.49 < r < -0.15$	risk loving	-0.32
4	$-0.15 < r < 0.15$	risk neutral	0
5	$0.15 < r < 0.41$	slightly risk averse	0.28
6	$0.41 < r < 0.68$	risk averse	0.55
7	$0.68 < r < 0.97$	very risk averse	0.83
8	$0.97 < r < 1.37$	highly risk averse	1.17
9-10	$1.37 \leq r$	stay in bed	1.37

To do so, we take the mean of the Holt and Laury intervals (see Table 2.1), which gives us a coefficient according to the first switch from a safe to a risky option. It can be argued that our ranking definition is based on a strong hypothesis. As suggested in Kobberling and Wakker (2005), the reference point (here initial wealth) may play an important role in the risky decision. However, parameter estimation at the individual level would require multiple decisions for the same subject, which, for time reasons, is unrealistic in our experiment. Another criticism refers to our definition of risk aversion. A Fechner specification (popularized by Hey and Orme (1994)) or a Luce (1959) specification, which consists in the estimation of parameters by log-likelihood including all the answers to the MPL, could allow us to relax the hypothesis of expected utility. However, it also introduces assumptions about the distribution of the probability of choices. For a complete review of risk aversion in the laboratory, we refer to Harrison and Rutström (2008).

We do not suppose that we perfectly know participants' optimal policies. In reality, intermediaries provide advice with respect to their own knowledge and interpretation of the risk. Potential errors are therefore an integral part of the distribution process. In addition, the ranking does not much depend on risk aversion, with the margin defined as $\lambda \in [0.8, 2]$ playing a major role in the definition of optimality. Thus, rankings are almost the same for different levels of risk aversion. When a contract is optimal, it is so for about 30% of the risk-aversion profile defined as below. This reduces any problems of errors in ranking for individuals based on their risk aversion from the MPL.

2.2.3.2.3 The definition of search costs and the fixed fee

We include search costs defined as the cost of each exploration action. To encourage consumers to reveal their exploration preferences, in other words their distribution-channel preferences, we include a fixed search cost of x ECU to access each kind of exploration.

We add an additional search cost u for asking for a recommendation within a specific menu. This cost represents the time spent with a tied-agent in a traditional insurance context, as well as the time spent on the phone with an insurance advisor. We also add an additional search cost y for the revelation of deductible information by the comparator. In reality, price comparators display a ranking of contracts depending mainly on the premium: consumers have to click on a particular contract to find out the details, which is costly.

Last, consumers pay an additional fixed fee k to choose a contract that is suggested by the advisor. This cost represents the fees paid to a broker in exchange for their services. We do not consider this fee as a search cost, and it is therefore deducted from initial wealth.

These costs mentioned above are constrained in two ways. First, one channel should not dominate another. For instance, imagine that it is more expensive to seek out an advisor and underwrite through her than to consult four menus sequentially and ask for a recommendation for each of them. This will affect our results due to the presence of an optimal exploration strategy whatever the subjects' attributes and the round parameters.

We hence set search costs and the advisor fixed fee so that the expected costs of underwriting the optimal contract in the market are the same for each intermediary. We of course assume at this stage that there is no problem of obfuscation (the identification of optimal contracts) and beliefs about honesty. We thus define the expected matching cost (EMC) for each decision design as follows:

$$EMC(Advisor) = k + \frac{8}{6}, \tag{2.7}$$

$$EMC(Comparator) = \frac{8}{6}(x + y + \frac{5}{6}y + \frac{4}{6}y + \frac{3}{6}y + \frac{2}{6}y + \frac{1}{6}y), \tag{2.8}$$

$$EMC(Menu) = x + \frac{3}{4}x + \frac{2}{4}x + \frac{1}{4}x, \tag{2.9}$$

Using (2.7),(2.8) and (2.9), we have

$$EMC(Advisor) = EMC(Comparator) = EMC(Menu) \Leftrightarrow \tfrac{4}{3}x + k = 4x = \tfrac{4}{3}(x + 6y).$$

We in addition do not want to limit subjects in their exploration and let them explore the entire market. We assume that it is always possible to be fully informed but that consumers do not benefit from any time saved. Thus, $C = 7x + 6y + 4u$, where C is the exploration credit.

2.2.3.2.4 The definition of the bonus

As mentioned below, intermediaries are financially motivated by insurers in order to build up profitable portfolios. We thus introduce bonuses for each contract proposed. These bonuses represent the commission paid by insurers to their "human" intermediaries (brokers or tied agents). In our game, we randomly define bonuses such that

$$B_i = \max((CP_i - p \times (R - D_i)) \times \mathcal{U}[0.2; 0.4], 0). \tag{2.10}$$

The bonus is therefore proportional to the profit generated by the contract. Nevertheless, the bonus rate is defined as a uniform random variable over the interval (0.2, 0.4). The reason for doing so is that the most profitable contract should not be the most incentivized: insurers may wish to incentivize less-profitable contracts for reasons such as loyalty goals or brand image. We do however assume that non-profitable contracts are never incentivized.

2.3 Procedure

A web interface and server database were designed specifically for this experiment. The interface was developed with HTML and JavaScript, the backend with Java and PostgreSQL as the database. The subjects were students from the University of Lyon 1 – Claude Bernard, France. 217 subjects participated in the experiment, 27 in average for each session including 5 participants of type B.

The honesty game was played first. All subjects received identical instructions, including the comprehension questions. Subjects were assigned to a group for the rest of the session. Then, they made their decisions for this part. Afterwards, subjects received written instructions for the risk-elicitation task and made their choices. Finally, they received instructions for the main part of the experiment and a comprehension questionnaire that we corrected with them. Before leaving the room to privately receive their payments, we asked them to answer some general questions about age, gender and education. The different periods of the exploration game were displayed randomly for each session. All treatments were framed in a neutral manner.

We decided to play the honesty game at the beginning of the session to avoid learning effects from potential interactions during the main part of the experiment. The risk-elicitation task was played just before the main game in order to be able to rank contracts for each A subject. The payoffs in the different tasks were revealed at the end of the entire experiment. The sessions lasted about 105 minutes. The average payoff was about 16 Euros including a show up fee of 3 Euros.

For full transparency, we will make available the R code (RCore-Team (2018)) and experimental data used in this paper upon request. Please note that authors should be referenced in the case of the use of these data or this code for further research.

2.4 Results and Analysis

2.4.1 Subject variables: distribution and impact on the main game

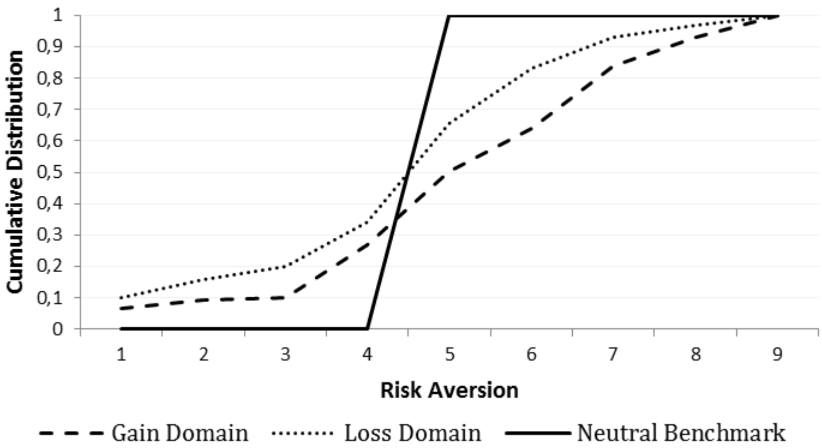
2.4.1.1 Risk Aversion

We first construct, for each individual, two variables representing their risk aversion level in the gain and loss domain resulting from the Holt and Laury (2002) elicitation game. We look at risk aversion in both domains for two reasons. In insurance, there is a probability of wealth loss. However, while searching information, individuals are uncertain about the value of the information gain.

Risk aversion is defined by the number of the question of the first switch from the safe to the risky option. Neutral subjects will switch at question 5, while the risk-averse will switch later. The average question switch number is 5.56 in the gain domain and 4.82 in the loss domain. These figures are significantly different using a non-parametric two-sided Wilcoxon test ($p - value < 1.7e^{-6}$). This result is in line with Chakravarty and Roy (2009), who found significantly less risk-aversion in the loss than in the gain domain (Figure 2.2).

We find that risk-averse individuals avoid search on their own, and prefer to compare offers via a cyber-broker or broker (Table 2.2). Risk aversion in the loss domain has a more significant impact than that in the gain domain for distribution-channel choice. The risk averse in the gain domain also collect more information in the exploration process (Table 2.5). However, contrary to standard expected-utility theory, neither risk aversion in the loss or gain domains affect contract choice (Table

Figure 2.2 – Empirical cumulative distribution functions from 1st switch



2.6). Regarding intermediaries, we find a strong correlation between risk aversion in the loss domain and deviation behaviors (i.e. the suggestion of a non-optimal contract) (Table 2.9). Deviation is therefore considered to be a risky decision.

2.4.1.2 Honesty and beliefs about others' honesty

As we here analyze distribution-channel choice in insurance including human intermediaries, our second key variable is honesty. Honesty beliefs are individual, but we only observe actual honesty at the session level.

As a reminder, we distributed different draws to the five B participants in each session including 3, 4, 5, 6 or 7 red balls. We find interesting patterns of honesty and beliefs about others' honesty in the different draws (Chapter 1: Mouminoux and Rulliere (2018)). The favorable draw produces more honesty and greater honesty expectations, as in Houser et al. (2012) and Galeotti et al. (2017). For simplicity we here only use the answer of A faced with the least-fortunate draw given to the B's in each session (the draw with 7 red balls, where B should leave 3.5€ in the wallet according to the rule). This draw corresponds to the situation where honesty and beliefs about honesty are the most heterogeneous and the lowest. Honesty beliefs for subject i (i.e. HB_i) are defined as follows:

$$HB_i = answer_i - 3.5\text{€}\ .$$

On average, A's think that B's will take 1.23€ more than the rule. Over the 177 A subjects, 41 trust the B's, 8 are over trusting (they think that the B's take less than the rule) and 128 are untrusting (they think that B's take more than allowed by the rule).

We define dishonesty at the session level as the average amount taken above the rule by the five B participants in each session. On average, the B's take 0.97€ more than the rule: 47% respect the rule and 35% fully deviate.

As risk aversion and honesty beliefs will be introduced together, we check the correlation between them: this is insignificant (Kendall, Pearson and Spearman correlation tests), as in the seminal paper of Eckel and Wilson (2004) who find no correlation between trust (in our case honesty beliefs) and risk aversion.

Honesty beliefs help explain insurance purchase. Those with high honesty expectations avoid cyber-brokers (Table 2.2) and are more sensitive to broker deviation, suggesting disappointment (Table 2.4). Regarding intermediary behavior, we find a correlation between honesty and the deviation rate at the session level. The more honest B players are less likely they deviate in their suggestions (Table 2.9).

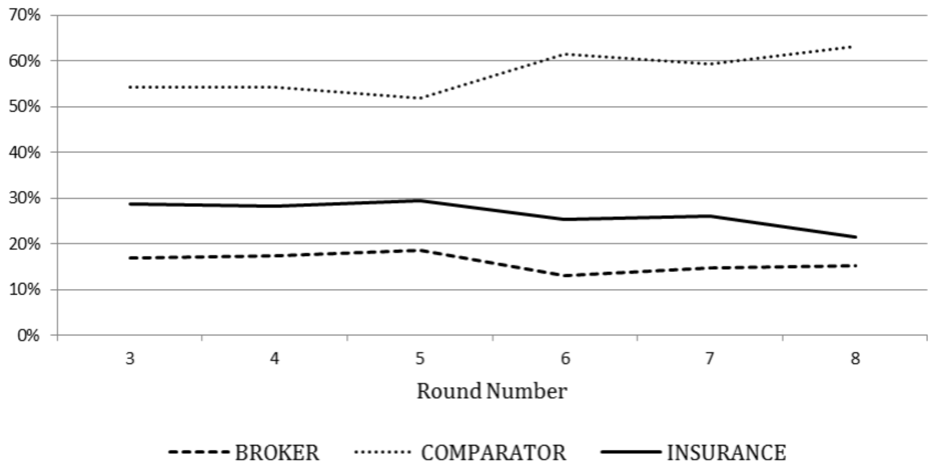
2.4.2 Purchasing Process

As noted above, we break the purchasing process down into two parts: subjects explore the market to uncover alternatives before choosing an insurance contract. We are first interested in the "search strategy", i.e. the determinants of distribution-channel choices. To do so, we distinct two different independent variables called "Rounds' First Choice Channel" and "Rounds' Underwriter Channel" corresponding respectively to the first channel chosen for exploration and the last channel chosen for underwriting. We then investigate extreme search strategies: subjects who adopt a "saving search cost strategy" (i.e. take the lowest price on the comparator without uncovering the deductible) and the "deep search cost strategy" (i.e. subjects who uncover at least one contract from all insurers). Then, given the information gleaned from these search strategies, we consider the final insurance-contract choice.

2.4.2.1 Search Strategies

Figure 2.3 shows final channel decision of type A individuals. Globally over the entire experiment, we note that 57.4% (std. dev. 0.045) of subjects chose their insurance via the comparator, while 16.0% (std. dev.0.029) chose via a broker and 26.6% (std. dev.0.029) directly from an insurer. However, from Figure 2.3, we see that channel choice changes over round, with a rise in the use of comparators over time. We first focus on first channel choices and inter-period learning effects. We then investigate the final underwriting choices and intra-period channel decisions.

Figure 2.3 – Percentage of Choices by Underwritting Distribution Channel



2.4.2.1.1 First Channel Choices and inter-period effects

In the following, we consider a series of three statistical models to analyze first channel choices and inter-period effects, where the reference level is the broker channel. We first carry out a multinomial logistic regression with random effects (Table 2.2, Model 1) with the first channel choices as the dependent variable (i.e. Broker, Comparator or Insurer). Random effects allow us to control for the relationship between errors in our panel data (see e.g. Croissant (2013)). We also consider p-value under Holm-Bonferroni multiple correction in all presented models (?). In the first two regressions, we consider only non-lagged explanatory variables.

Honesty beliefs are a significant determinant of first channel choices. The probability that subjects first choose the comparator falls with honesty beliefs ($p-value < 0.001$). Risk averse individuals avoid insurance as a first choice ($p-value < 0.001$). Regarding the "round" parameters, only the probability of loss has a significant effect on first choices: a higher probability leads subjects to avoid comparator ($p-value < 0.001$).

Model 2 of Table 2.2 does not demonstrate learning effects in the significance of the round number. Previous results on honesty beliefs, risk aversion and probability of loss continue to hold.

Finally, by incorporating dummies of lagged variable (i.e. explanatory variables followed by $t-1$) in Model 3 of Table 2.2 we try to capture potential hysteresis in choice. We add the subject's previous round's first choice and do not find any significant effects. We also add a dummy variable for the subject having asked for advice (though a broker or insurer) in the previous period and having received sub-optimal advice. We find no effect here: subjects do not seem to detect or to take into account deviation in previous rounds.

We also control the effect of the number of revealed contracts in the previous period. Again, this control does not reveal learning effects. The other estimated coefficients in Model 3 are similar to those in Models 1 & 2.

Table 2.2 – Multinomial Logistic Regression with Random Effect

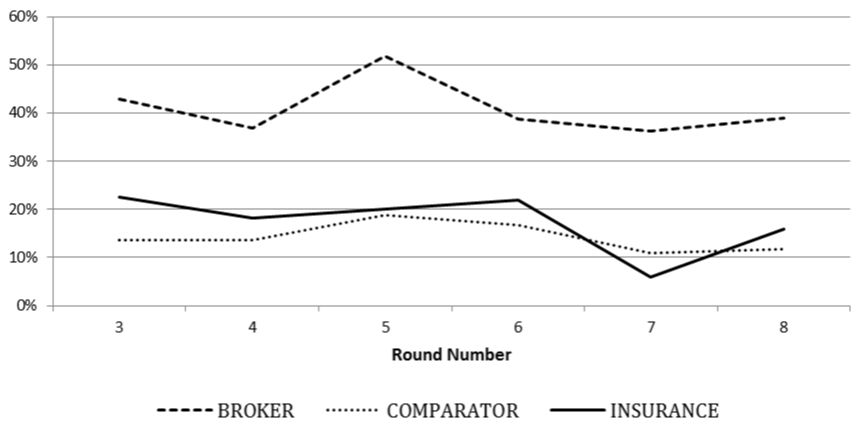
Explanatory Variable	**Rounds' First Choice Channel**					
	Model 1		**Model 2**		**Model 3**	
Coefficients *Std. error*	COMPARATOR	INSURANCE	COMPARATOR	INSURANCE	COMPARATOR	INSURANCE
Honesty Beliefs	−0.67***	0.26	−0.69***	0.29	−0.79***	0.24
	0.12	*0.14*	*0.12*	*1.99*	*0.16*	*0.17*
Risk Aversion (loss)	−0.01	−0.40***	0.00	−0.41***	0.04	−0.35***
	0.07	*0.09*	*0.07*	*0.09*	*0.08*	*0.11*
Initial Wealth	0.00	0.00	0.00	0.00		
	0.00	*0.00*	*0.00*	*0.00*		
Loss Amount	0.00	0.00	0.00	0.00		
	0.00	*0.00*	*0.00*	*0.00*		
Probability of Loss	−4.42***	−3.44	−4.32***	−3.67	−3.78***	−3.03
	1.29	*1.63*	*1.30*	*1.65*	*1.04*	*1.30*
Round number			0.14	−0.22	0.23	−0.26
			0.07	*0.10*	*0.11*	*0.26*
First choice $t-1$: COMPARATOR					−0.23	0.60
					0.46	*0.58*
First choice $t-1$: INSURANCE					0.47	1.20
					0.47	*0.60*
Deviation $t-1$					−0.08	0.31
					0.42	*0.53*
Contract no. discovered $t-1$					−0.18	−0.06
					0.08	*0.10*
Constant (ref. level: BROKER)	2.07***	2.24***	1.25*	3.44***	1.08	2.70
	0.53	*0.65*	*0.67*	*0.81*	*0.98*	*1.08*
Nb. Observations	1062		1062		865	
Nb. Subjects	177		177		177	
R^2	0.336		0.346		0.340	
Adjusted R^2	0.327		0.336		0.325	

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

2.4.2.1.2 Intra-Period switching behavior

Now, we consider a series of four statistical models to analyze intra-period switching behavior, still with the reference level being the broker channel. Over the entire experiment, we note that 21.0% (std. dev. 0.047) of subjects decide to switch channel type at least once during a round. Figure 2.4 shows the percentage of switches with respect to the first channel choice. On average, 41.7% (std. dev. 0.058) of subjects who first chose a broker switch, with the analogous figures for Comparator and Insurance being 14.1% (std. dev. 0.030) and 17.6% (std. dev. 0.061). Among the switchers, 15.2% (std. dev. 0.032) finally return to their first choice.

Figure 2.4 – Percentage of Switch by First Choice Channel



In Model 4 of Table 2.3, we show that these switches are not explained by the round parameters or subject variables. Only first channel choice in the period is significant ($p-value < 0.001$) in explaining the final underwriting channel choice.

We find intra-period inertia in choice and, when subjects switch, they prefer to switch between the comparator and an insurer. Switching might depend on first channel choice and we do not capture this effect in Model 4. We thus separate our sample into three parts conditional on first channel choice, and run for each a logistic regression where the dependent variable is a dummy for the subject switching channel type during the round (Models 5, 6 and 7 of Table 2.4).

Model 5 focuses the switching analysis on subjects who first chose a Broker. The round parameters and risk aversion do not affect switching. We then consider the deviation degree, defined as the average gap between the ranks of the three contracts proposed compared to the optimal ranking that appears on the brokers' screen. This deviation degree is computed for each interaction between A and B. If the broker proposes the ranking that appears on the screen, then the deviation degree is zero.The maximum deviation degree would come from proposing the worst contract (ranked 6) first, then the second worst (ranked 5) second, and the third-worst third: this produces an average gap of $\frac{1}{3}((6-1)+(5-2)+(4-3)) = 3$. The probability of switching rises with deviation degree, in particular for subjects with high honesty expectations.

Contrary to the lack of an inter-period deviation learning effect (Model 3 of Table 2.2), subjects do take into account intra-period deviation. They react more strongly when they are disappointed by their choices (i.e. "Deviation Deg. x Honesty Belief" significant at 10%). Since subjects can choose different channels during a same round, they can collect additional information allowing us to detect important intermediaries' deviation. We also control in Model 5 bis of Table 4 bis in Appendix where

Table 2.3 – Multinomial Logistic Regression with Random Effect

Explanatory Variable	Rounds' Underwriter Channel	
	Model 4	
Coefficients *Std. error*	Comparator	Insurance
Honesty Beliefs	0.00 *0.15*	0.06 *0.15*
Risk Aversion (loss)	0.10 *0.09*	0.06 *0.09*
Initial Wealth	0.00 *0.00*	0.00 *0.00*
Loss Amount	0.00 *0.00*	0.00 *0.00*
Probability of Loss	−0.19 *1.58*	−1.35 *1.58*
Round number	0.08 *0.09*	0.04 *0.10*
First choice $t-1$: Comparator	5.50*** *0.42*	2.44*** *0.43*
First choice $t-1$: Insurance	2.44*** *0.54*	5.45*** *0.49*
Constant (ref. level: Broker)	−2.62** *0.94*	−2.92* *0.90*
Nb. Observations	1062	
Nb. Subjects	177	
R^2	0.505	
Adjusted R^2	0.495	

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

the subjects switch to. The results are in line with our intuition: disappointed subjects choose the comparator channel which does not include "human" intermediaries.

The results are different for subjects who first choose insurers (Model 6 of Table 2.4) here only honesty beliefs significantly predict switching and switchers tend to change to the comparator (Model 6 bis of Table 4 bis in Appendix). Deviation by tied-agents does not affect switching. However, we prefer to avoid any misinterpretation of these results due to a lack of power: recommendations for only one insurer by a tied agent were only requested 10 times.

Finally, Model 7 of Table 2.4 focuses on subjects who first choose the comparator. We here include the number of deductible values uncovered. As expected, switch probability falls with the latter: revealing the deductible value is costly. We also test for linearity here, and find that when the number of deductibles is large enough, the probability of switch increases (i.e. the coefficient on the squared term is positive).

In Model 7 bis of Table 4 bis in Appendix, we find that these effects are significant only for those who switch from the comparator to an insurer. Only 15 subjects switch from the comparator to brokers. This lack of observations prevents us from modelling this specific switch. However, all of the previous results continue to hold for switchers from comparator to insurers. The positive effect of the deductible on switches from the comparator to insurers can be explained by some subjects deciding to adopt a "deep search strategy", by obtaining information on at least one contract from each insurer. As a reminder, the comparator offers information for three of the four insurers in the market. By first using the comparator and revealing at least 4 deductibles, subjects can then obtain complete information about at least one contract from all three of these insurers. They can then select the missing insurer and collect information about at least one contract for all insurers on the market. This leads us to analyze extreme search strategies.

Table 2.4 – Logistic Regression with Random Effect

Explanatory Variable	**Intraround Channel Switch**		
	Model 5	**Model 6**	**Model 7**
Coefficients	Broker	Insurance	Comparator
Std. error			
Honesty Beliefs	−0.98	0.74**	0.39
	0.80	*0.37*	*0.23*
Risk Aversion (loss)	−0.06	−0.06	−0.04
	0.18	*0.19*	*0.13*
Initial Wealth	0.00	0.01	0.00
	0.00	*0.01*	*0.00*
Loss Amount	0.00	0.00	0.00
	0.00	*0.00*	*0.00*
Probability of Loss	3.96	−2.15	4.32
	2.81	*3.26*	*2.14*
Round number	−0.18	−0.18	−0.20
	0.17	*0.23*	*0.15*
Deviation Degree	0.59	0.53	
	0.30	*0.45*	
Deviation Deg. x Honesty Belief	0.58*	−0.19	
	0.24	*0.33*	
Deductible no. discovered			−1.38***
			0.40
Deductible no. discovered^2			0.17**
			0.06
Constant (ref. level: No Switch)	−2.19	−3.73**	−2.01
	1.75	*1.86*	*1.38*
Nb. Observations	237	210	615
Nb. Subjects	79	67	140
R^2	0.272	0.167	0.177
Adjusted R^2	0.239	0.126	0.164

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

2.4.2.1.3 "Saving Search" and "Deep Search" strategies

We here focus on extreme search strategies. We split our observations up into three different profiles (Figure 2.5). We identify two extreme strategies corresponding to extended and restrained search, as opposed to various moderate search strategies. 6.7% of subjects obtain information about at least one contract from all insurers: we call this the "deep search strategy". On the contrary, 13.8% of subjects choose "saving search", selecting the comparator as first channel choice and underwriting the contract with the lowest premium. Strategy choice is not affected by round number, meaning there are no learning or boredom effects.

We carry out multinomial logistic regressions to investigate extreme search strategies. We find in Models 8, 9 and 10 of Table 2.5 that the probability of loss has a significant impact. As this probability rises , the probability of choosing a saving search strategy decreases. In other words, subjects prefer to explore the market as the probability of losing part of their wealth rises. We in addition see that subjects who believe in others' honesty avoid the saving search strategy ($p - value < 0.001$). For the same search cost, these subjects could obtain more information by choosing a broker but could then be confronted with deviation.

Unlike to our other models, risk aversion is only significant in the gain domain here ($p-value < 0.001$). As noted above, information search introduces uncertainty about the value of the information gain. We can thus argue that risk-averse subjects prefer to reveal more in order to reduce this uncertainty. However, we find no significant differences between the standard and deep extreme strategies by risk aversion. Finally, we control in Model 10 of Table 2.5 for any learning or boredom effects by adding the round number. This does not attract a significant coefficient and the other results are unaffected.

Figure 2.5 – Distribution of Search Profile by Round Number

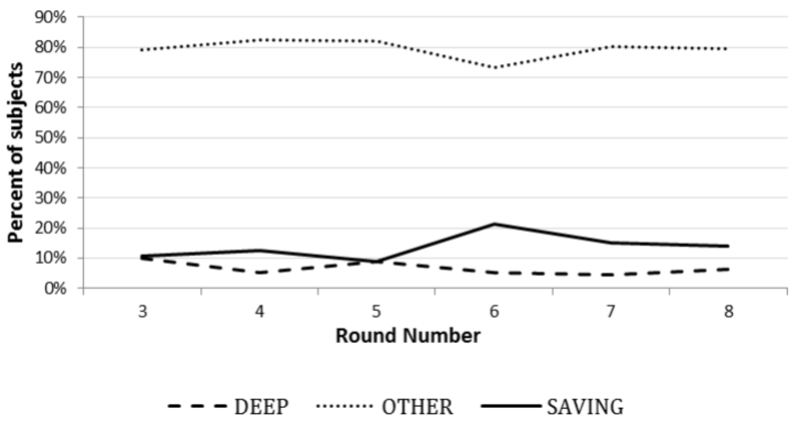


Table 2.5 – Multinomial Logistic Regression with Random Effect

Explanatory Variable	**Rounds' First Choice Channel**					
	Model 8		**Model 9**		**Model 10**	
Coefficients *Std. error*	Saving	Deep	Saving	Deep	Saving	Deep
Honesty Beliefs	−0.46*** *0.11*	0.10 *0.17*	−0.50*** *0.11*	0.16 *0.17*	−0.49*** *0.11*	0.18 *0.17*
Risk Aversion (gain)			−0.22** *0.07*	−0.11 *0.10*	−0.22** *0.07*	−0.11 *0.10*
Risk Aversion (loss)	−0.07 *0.07*	−0.06 *0.10*				
Initial Wealth	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*		
Loss Amount	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*		
Probability of Loss	−4.50*** *1.32*	3.67 *1.79*	−4.52*** *1.31*	3.58 *1.80*	−3.35*** *0.93*	3.34 *1.22*
Round number					0.12 *0.08*	−0.18 *0.11*
Constant (ref. level: Others)	−2.14** *0.66*	−4.85*** *0.94*	−1.30 *0.66*	−4.39*** *0.96*	−1.73 *0.65*	−3.02* *0.89*
Nb. Observations	1062		1062		865	
Nb. Subjects	177		177		177	
R^2	0.176		0.183		0.184	
Adjusted R^2	0.163		0.172		0.174	

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

2.4.2.2 Contract Choice

After determining individuals' search strategies, we now turn to the final contract choices. We first determine the drivers of coverage choice and highlight some focal-point and anchoring effects. We then investigate the consequences of the quantity of information on the efficiency of choices with respect to Expected Utility Theory.

2.4.2.2.1 Coverage choice

This subsection focuses on subject contract choices. We first estimate the relative coverage chosen via a linear regression. As a reminder, coverage α is defined as follows $\alpha = \frac{LossAmount-Deductible}{LossAmount}$. We here model relative coverage to standardize the dependent variable across subjects, as they do not face the same choice set, defined as:

$$\frac{Coverage - Min.\ coverage\ available}{Max.\ coverage\ available - Min.\ coverage\ available}.$$

Relative coverage is 0 if the subject choses the minimal coverage available and 1 if she chooses the maximum coverage. The regressions cover subjects who have at least two choice possibilities: we saw above that some subjects adopt a 'saving search strategy' so that they do not compare contracts.

As expected, coverage rises with the probability of loss ($p - value < 0.1$ in Models 11, 12 and 13 of Table 2.6), but falls with the number of alternatives (Model 12, Table 2.6). As the coverage choice and the number of alternatives are both linked with the search strategy, we control for this in Model 13. We find no significant effect of the latter, but the number of alternatives is not significant anymore. This loss of significativity is due to the Holm-Bonferroni multiple hypothesis correction.

However, this result leads us to investigate in more detail the effect of available information on contract choice. Contrary to what might have been expected, contract choice is not mainly driven by standard parameters (except for the probability of loss). We suspect a focal-point effect due to obfuscation here: subjects with too much information end up making their choices based only on the simplest characteristic: the premium. As we designed the experiment to avoid strictly-dominated alternatives, the lowest coverage here corresponds to the lowest premium.

Table 2.6 – OLS Regression with Random Effect

Explanatory Variable	**Relative Coverage Choice**		
Coefficients *Std. error*	**Model 11**	**Model 12**	**Model 13**
Honesty Beliefs	0.02 *0.02*	0.02 *0.02*	0.02 *0.02*
Risk Aversion (loss)	0.00 *0.01*	0.00 *0.01*	0.00 *0.01*
Initial Wealth	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*
Loss Amount	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*
Probability of Loss	0.33** *0.15*	0.36** *0.15*	0.35* *0.15*
Round number	0.00 *0.01*		
No. available alternative		−0.02* *0.01*	−0.02 *0.01*
First choice t: COMPARATOR			−0.01 *0.05*
First choice t: INSURANCE			0.06 *0.04*
Constant	0.36*** *0.09*	0.41*** *0.08*	0.43*** *0.08*
Nb. Observations	915	915	915
Nb. Subjects	171	171	171
R^2	0.021	0.025	0.028
Adjusted R^2	0.014	0.019	0.020

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

2.4.2.2.2 Focal-point and Anchoring effects on contract choice

In order to control for focal-point effects regarding the premium, we run a logistic regression on a dummy indicating individuals choosing the lowest price. With more alternatives, the probability of choosing the lowest price should fall. However, Models 14, 15 and 16 of Table 2.7 show that the more alternatives subjects have the more likely they are to choose the lowest price. This is consistent with focal points.

Contrary to previous models of coverage choice, we find that the probability of loss increases the probability of choosing the best price. This point encourages us to control for eventual anchoring effects. By construction, the higher the probability of loss the higher the premium, in a given round. We could then argue that the probability of choosing the lowest price rises with the price, independently of the probability of loss.

To avoid the misinterpretation of this coefficient we add in Model 15 (2.7) the difference between the average and minimal premium. We find a significant anchoring effect, and the probability of loss is no longer significant. In other words, adding a coefficient for the difference between the average and the minimal premium reduces significantly the probability of choosing the lowest price.

Table 2.7 – Logistic Regression with Random Effect

Explanatory Variable	**Lowest Price**		
Coefficients *Std. error*	**Model 14**	**Model 15**	**Model 16**
Honesty Beliefs	0.13 *0.06*	0.14 *0.07*	0.13 *0.07*
Risk Aversion (loss)	−0.02 *0.04*	−0.03 *0.04*	−0.04 *0.04*
Initial Wealth	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*
Loss Amount	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*
Probability of Loss	2.79*** *0.76*	0.45 *0.98*	0.35 *0.98*
Round number	−0.01 *0.04*	0.00 *0.05*	0.00 *0.05*
No. available alternative	0.31*** *0.05*	0.49*** *0.06*	0.49*** *0.06*
Average Premium - Minimal Premium		−0.02*** *0.00*	0.02*** *0.00*
First choice t: COMPARATOR			−0.21 *0.19*
First choice t: INSURANCE			−0.31 *0.24*
Constant (ref. level: NO)	−0.77 *0.09*	−0.70 *0.08*	−0.48 *0.08*
Nb. Observations	1067	1067	1067
Nb. Subjects	177	177	177
R^2	0.068	0.095	0.097
Adjusted R^2	0.060	0.086	0.087

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 ‘***’ 0.05 ‘**’ 0.1 ‘*’

To sum up, subjects with “too much” information make their decisions based on the premium: this is the focal-point effect due to obfuscation. However, their decisions are also driven by all available contract information, and as the average premium rises relative to the minimum, the probability of choosing the cheapest contract falls: this is the anchoring effect.

2.4.2.2.3 Efficiency of Choices

In this section, we focus our analysis on the efficiency of subject choice. As described above, the eight available contracts are ranked according to Expected Utility Theory. On average, 44% (std. dev. 4%) of subjects choose the optimal contract through the discovered contracts (i.e. the available alternatives at the moment of choice). The majority of subjects hence do not make their choices according to standard Expected Utility Theory. As previously noted, the quantity and quality of information are significant drivers of the final contract choice.

We thus estimate models in order to determine the role of obfuscation (i.e. the quantity of information) and intermediaries' honesty (i.e. the quality of information) on the efficiency of choice.

In Models 17, 18 and 19 of Table 2.8, the dependent variable is a dummy for the subject choosing the optimal contract according to Expected Utility Theory. We estimate these models only on subjects who have complete information on at least two choices and choose a contract with complete information. This leaves 892 observations from the 1062 collected (i.e. 84%). Some subjects discover more than two contracts but finally decide to select a contract without finding out about its information. The individual hence cannot determine the rank of this contract with respect to other contracts discovered.

We first model the probability of choosing the optimal contract with respect to the round parameters (i.e. probability of loss, loss amount and initial wealth), subject variables (honesty beliefs and risk aversion) and the number of available alternative (Model 17, Table 2.8). We find that the round parameters and subject variables do not significantly explain inefficient choices.

However, as the number of alternatives rises, the probability of making an inefficient choice significantly increases. While one could argue that more-informed consumers should make more efficient choices, we show that too much choice leads to inefficient decisions. As the distribution channels do not have the same levels of obfuscation or honesty, we add in Models 18 (Table 2.8) an estimated coefficient for either the interaction between the number of alternatives and the channel choice or the interaction between the deviation and the channel choice.

Physical intermediaries, could lead to inefficiency since the quality of information depends on the honesty of intermediaries. However, intermediaries have their own financial incentives, leading to deviations (deviation is represented here by a dummy for the intermediaries who do not advise the optimal contract at first). We find in Model 18 of Table 8 that intermediary deviation significantly increases the probability of inefficient choice. In other words, subjects who avoid obfuscation by delegating a part of their decision to a physical intermediary are also exposed to inefficiency in decision-making. Interestingly, by adding variable between deviations and underwriting-channel choice we also find that consumers who faced deviation and finally choose the comparator make more optimal choices. The consumers who detect deviation and decide to switch channel have the highest ability of identifying the most efficient contract.

Last, we include in Model 19 of Table 2.8 a dummy for subjects choosing the lowest price. We also add an interaction between this dummy and the number of available alternatives since we showed in Table 7 of the previous section that obfuscation leads to focal-point effects.

While adding interaction between the number of alternatives and the channel choice in Model 18 leads to loss significativity of the number of alternative (under Holm-Bonferroni correction), Model 19 allow us to find new evidence on obfuscation effect. Indeed, we include a dummy for subjects choosing the lowest price. We also add an interaction between this dummy and the number of available alternatives since we showed in Table 7 of the previous section that obfuscation leads to focal-point effects. While choosing the lowest price tends to increase the probability of optimal choice, we find that when this choice is made because of focal-point effects due to obfuscation, the efficiency of choice falls. We also control for a potential learning effect in all previous models via the variable for the number of rounds. We do not find any evidence of learning effects.

Table 2.8 – Logistic Regression with Random Effect

Explanatory Variable	**Optimal Choice (EUT)**		
Coefficients *Std. error*	**Model 17**	**Model 18**	**Model 19**
Honesty Beliefs	0.07 *0.06*	0.06 *0.07*	0.08 *0.07*
Risk Aversion (loss)	0.06 *0.04*	0.08 *0.04*	0.07 *0.04*
Initial Wealth	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*
Loss Amount	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*
Probability of Loss	−0.90 *0.75*	−1.09 *0.76*	−0.57 *0.78*
No. available alternative	−0.37*** *0.05*	−0.12 *0.16*	−0.03 *0.17*
Underwriting Channel t: COMPARATOR		−0.17 *0.68*	−0.08 *0.70*
Underwriting Channel t: INSURANCE		0.06 *0.69*	−0.25 *0.71*
Deviation		−0.98** *0.37*	−0.88** *0.39*
Lowest Price			2.11*** *0.43*
Underwriting Channel t: COMPARATOR x Nu. available alternative		−0.35* *0.18*	−0.30* *0.18*
Underwriting Channel t: INSURANCE x Nu. available alternative		−0.23 *0.17*	−0.19 *0.18*
Underwriting Channel t: COMPARATOR x Deviation		1.83** *0.57*	1.59** *0.59*
Underwriting Channel t: INSURANCE x Deviation		0.33 *0.56*	0.23 *0.62*
Lowest Price x No. available alternative			−0.23** *0.11*
Round Number	0.03 *0.04*	0.03 *0.05*	0.02 *0.05*
Constant (ref. level: NO)	0.89** *0.44*	0.83 *0.75*	0.08 *0.78*
Nb. Observations	892	892	892
Nb. Subjects	171	171	171
R^2	0.065	0.081	0.129
Adjusted R^2	0.055	0.064	0.111

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

2.4.3 Intermediary strategies

We last examine the behavior of subjects playing "human" intermediaries (the B's). On average, each B subject is called 1.2 times per round. The B-type players have financial incentives via the bonuses: just over 60% of the time they do not propose the best policy to subjects who ask for advice. We run a logistic regression on a dummy indicating for intermediary deviation (not first suggesting the best contract).

The consumer variables (initial wealth, the loss amount and the probability of loss) do not affect deviation (Models 20, 21 and 22 of Table 2.9). However, risk-averse intermediaries deviate less ($p - value < 0.05$). As a reminder, they receive the contract bonus only if the customers choose the policy via their intermediation. Deviation is hence considered as a risky decision.

Table 2.9 – Logistic Regression with Random Effect

Explanatory Variable	**Deviation**		
Coefficients *Std. error*	**Model 20**	**Model 21**	**Model 22**
Risk Aversion (loss)	−0.28** *0.09*	−0.28** *0.09*	−0.28** *0.09*
Initial Wealth	0.01 *0.00*	0.00 *0.00*	0.00 *0.00*
Loss Amount	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*
Probability of Loss	−2.22 *1.66*	−0.67 *1.68*	−0.53 *1.71*
Optimal Contract Bonus	−0.05 *0.02*	0.06 *0.04*	0.06 *0.04*
Status: TIED-AGENT	−1.28* *0.68*	−1.47* *0.64*	−1.46* *0.64*
Dishonesty Session Level	0.59** *0.26*	0.63** *0.26*	0.62** *0.26*
Round Number	−0.01 *0.10*	0.00 *0.10*	0.00 *0.10*
Mean Bonus - Optimal Contract Bonus		0.18** *0.06*	0.18** *0.06*
No. of demand received			0.10 *0.32*
Constant (ref. level: NO)	2.66** *0.89*	3.09** *0.98*	2.88* *1.07*
Nb. Observations	289	289	289
Nb. Subjects	40	40	40
R^2	0.181	0.213	0.210
Adjusted R^2	0.152	0.182	0.176

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

Moreover, the level of the bonus of the optimal contract do not affect deviation(Model 20). However, the difference between the average possible bonus and the optimal-contract bonus is significant in explaining deviation: a smaller gap here produces less deviation (Model 21). Hence, it is not the absolute level of remuneration which affect intermediaries deviation decisions but the difference of remuneration between offers.

We also control for intermediary type and find that when the demand for advice comes from a particular insurer (versus a broker) intermediaries deviate significantly less ($p - value < 0.1$). The reason is simple: participants only receive two different contracts from insurers but six from brokers.

We consider honesty at the session level from the honesty game and the intermediary deviation. After controlling for session effects (Model 20 bis of Table 9 bis in Appendix), we find a strong correlation between deviation in the honesty game (called 'Dishonesty Session Level' here) and intermediary deviation. This result comforts or idea about the existence of close relationship between deontological ethics imposed to insurance physical intermediaries and honesty concept.

Finally, we look for portfolio effects by adding the number of demands previously received during the round (Model 22). If deviation is risky, intermediaries receiving a number of demands for advice in the same round may diversify their risks. However, we do not find any significant evidence of this.

2.5 Conclusion

In this paper, we analyze the purchasing behavior in personal non-life insurance markets. The personal insurance market is a useful context in which to analyze purchasing behavior, as it includes a number of types of intermediaries. Consumers can explore the insurance market via brokers, cyber-brokers or directly visit a specific insurer and may ask tied-agents for advice. Based on the search-cost theory, we here designed an experiment to help understand distribution-channel choices.

The complexity of insurance supply, including different levels of coverage, premium and deductibles, gives rise to potential obfuscation, which appears to be a marketing device and a source of inefficiency in decision-making.

Consumers can decide to explore the market on their own, but have to deal with many comparisons involving incomplete information. Another possibility is then to ask for advice to help choose. Although the deontological rule implies that intermediaries should provide the best advice, they are themselves influenced by incentives such as bonuses or lobbying. We find that this kind of delegation is conditional on honesty beliefs.

Even with considerable intermediary deviation, the complexity of choices in a risky environment and obfuscation drive subjects to delegate part of their decisions. Even with free access to online insurance markets, our results suggest that consumers continue to value broker and tied-agent services, although deviation leads to inefficient choices. However, deviation also explains channel choices, and consumers who detect deviation tend to switch channel and make more efficient contract choices.

We also find that intermediary behaviors depend on their risk aversion and the general honesty level, so that deviation is a risky decision. We present evidence of the importance of honesty and beliefs about others' honesty. Because both search strategies (i.e. delegation and self-exploration) are sources of inefficiency in decision-making, the development of a multi-channel distribution strategy by insurers is required to cover a large part of market. Some consumers have sufficient belief in others' honesty to delegate a part of their decisions and avoid obfuscation.

However, the evaluation of the economic benefits of physical intermediaries remains difficult. While intermediary costs are easy to evaluate (the fees and bonuses), the associated benefit is not. On the one hand, consumers save on search costs and avoid obfuscation, but on the other these savings do not necessarily compensate for the risk of dishonesty. As in Bergstresser et al. (2009) we find that financial incentives for physical intermediaries produce sub-optimal contracts. Higher incentives push intermediaries to be more aggressive and makes them less honest. There is a potential crowding out effect of incentive towards consumers. In the context of multiple distribution channels, the design of physical intermediary incentives could lead to consumer mobility across channels.

In addition, we show that the quantity and quality of information have a considerable impact on final contract choice. While standard economic theory suggests that the main drivers are consumer risk-aversion and the nature of the risk, final choices in our experiment are based on the premium and the probability of loss. Anchoring effects regarding both of these, consistent with obfuscation, can lead to more price competition as insurance companies compete on the premium level.

We here look at compulsory insurance; we may expect some behavioral differences under optional insurance, in particular regarding selection. The risk-averse are more likely to purchase optional insurance (Corcos et al. (2017)). Also, as in Kuksov and Villas-Boas (2010), we can imagine that the effect of the number of alternatives under optional insurance might reduce the focal and anchoring effects, as consumers can decide against purchase.

In order to be able to compare the different distribution channels ceteris paribus, the proposed contracts are the same in each channel. However, real-world insurers do manage the policies that they propose according to the way in which they are distributed. For instance, due to the focal-point effect we can imagine that loss-leaders will appear on the comparator and that insurers will focus their efforts on cross-selling and upgrade options. This is particularly true when we take consumer search costs into consideration Diamond (1971). In addition, brokers are likely better informed about the range of contracts that are available, and may propose contracts that are more difficult to find.

Our paper has focused on first insurance purchase: we have not looked at renewal and customer inertia due to switching costs. It would be of interest to adapt this experiment to a repeated game including renewal and switching costs, as in Schram and Sonnemans (2011), as well as retaining the possibility that consumers explore the market via the different channels.

Last, we uncover evidence of a correlation between customer risk profiles and their acquisition strategy. Risk aversion and honesty both reflect the individual risk profile. The first could explain adverse selection while the second could be related to fraud. The consumer's distribution-channel strategy could then reflect screening. We would now like to collect claims data from different insurers who use different distribution channels in order to analyze, ceteris paribus, the differences in insurers' portfolio loss according to the way in which the insurance contract was acquired.

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2.6 Appendix

INSTRUCTIONS

You are taking part in an experiment in the context of the **SAF** (Sciences Actuarielle et Financières) research program of the **ISFA** (Institut de Science Financière et d'Assurances) of the University of Claude Bernard Lyon 1.

If you read these instructions carefully you can earn a substantial amount of money. Your final earnings will depend on your decision and the decisions of the other participants in this session. In any case, you will receive a fixed remuneration of 3€ for your participation. Please note that average final earnings are well above this amount. Once the experiment is over, we will give you an attestation of payment that you must sign in exchange for your earnings that will be paid directly and privately in cash.

This experimental session consists of **three independent parts**:

- The first part involves a potential exchange of coins between a **wallet** and a **padded envelope (empty)** that are already on your table.
- The second part consists of 20 questions.
- The third part includes 8 successive independent rounds. In each round, you have to choose an insurance contract in order to be protected against a known risk.

Your final earnings in Euros is the sum of 4 amounts:

- 3 Euros for your attendance.
- The sum of your earnings in each of the three parts.

Communication is forbidden during all the experimental session. If you do not respect this rule, we will cancel this session and you will not be paid. For any questions, please raise your hand.

You will receive an instruction sheet and a comprehension questionnaire (if necessary) at the beginning of each part of this experimental session.

A three-letter login was placed on your table. Please enter your login in order to access the experimental interface.

First part:

During this first part, you will be assigned a type, either A or B. You will keep this type for the entire duration of this experiment. This type defines your role in the different parts.

There is a wallet and a padded envelope (empty) on the table of each participant. Please wait for our signal to open the wallet.

The wallet can:

- Either be **empty.**
- Or contain **10 coins of 50 cents** (5 Euros in total) and **a card** showing the result of 10 independent draws. Each draw is carried out using two balls: green and red.

<u>If your wallet is NOT EMPTY you are a participant of type B:</u>

For this part, **we ask you to apply the following rule:**

- **For each green ball shown, you can take 0.50€ from the wallet and put these 0.50€ in the padded envelope.**
- The remaining Euros in the wallet correspond to **0.50€ times the number of red balls**.

However, the experimenter and the other participants cannot know if you apply this rule. You are not monitored and all wallets are put together in the same bag at the end of this part of the experiment.

For this part, your earnings correspond to the amount that you put in the padded envelope.

PLEASE LEAVE THE SMALL CARD IN THE WALLET

Example : If your card is the following : 

By applying the rule, you:

- Take six 0.50€ coins, and
- Leave four 0.50€ coins in the wallet.

<u>If your wallet is EMPTY you are a participant of type A:</u>

In this part of the experiment, the different draws given to the B participants appear on your screen. You have to indicate for each of these **How much, in Euros, do you think that the B participant left in the wallet?**
To calculate your earnings for this part, we randomly select one of these draws and you will receive:

5 € - |your estimation error |

Example: If the draw selected is the following : 

B actually left 1€ (two 0.50€ coins) in the wallet and you had estimated that 2€ (four 0.50€ coins) would be left. You therefore earn:

5€ - |your error of estimation| = 5€ - |2€ - 1€| = 5€ - 1€ = 4€

Second part:

In this second part we are going to present numbers in ECU (*Experimental Currency Units*). This measure is converted into Euros at the end of the session at the following rate: **1 ECU = 0.02€; 1€ = 50 ECU.**

In this second part, **whatever your type,** you have to answer **two series of 10 questions.** For each question, you should **choose one option (A or B).**

For this part, we will randomly select one question and your earnings will be calculated according to the outcome of your corresponding chosen choice.

Example :

1st possible case: **<u>in the case of a Gain</u>**

Please choose between **A** and **B** for the 10 following questions.

For your earnings, we will randomly select one question and your earnings will be calculated according to the outcome and your corresponding choice.

Option A						Option B				
% chance	Gain	and	% chance	Gain		% chance	Gain	and	% chance	Gain
10 %	50 ECU		90 %	20 ECU	○ ○	10 %	85 ECU		90 %	5 ECU

You should indicate if:

You prefer a **1-in-10 chance of winning 50 ECU** and a **9-in-10 chance of winning 20 ECU** (Option A)

or

You prefer a **1-in-10 chance of winning 85 ECU** and a **9-in-10 chance of winning 5 ECU** (Option B)

2nd possible case: **<u>in the case of a Loss</u>**

Please choose between **A** and **B** for the 10 following questions.

For each questions you have 100 ECU, questions are independents.

For your earnings, we will randomly select one question and your earnings will be calculated according to the outcome and your corresponding choice.

Option A						Option B				
% chance	Loss	and	% chance	Loss		% chance	Loss	and	% chance	Loss
10 %	40 ECU		90 %	45 ECU	○ ○	10 %	10 ECU		90 %	80 ECU

You should indicate if, from **100 ECU** :

You prefer a **1-in-10 chance of losing 40 ECU** and a **9-in-10 chance of losing 45 ECU** (Option A)

or

You prefer a **1-in-10 chance of losing 10 ECU** and a **9-in-10 chance of losing 80 ECU** (Option A)

Third part:

This part consists of eight successive independent rounds. The first two rounds are trials and will not affect your earnings. Whatever your type, your earnings in this part correspond to your earnings in one of the six remaining rounds, selected randomly.

You are type A:

In each round, you have an **initial wealth level**, a **probability of loss**, and an **amount of loss**. You have to choose an insurance contract in order to be protected against the loss. **It is compulsory to underwrite a contract**.

Each contract contains:

- A fixed **premium** (this is the price of the contract).
- A **deductible** that you have to pay in the case of loss.

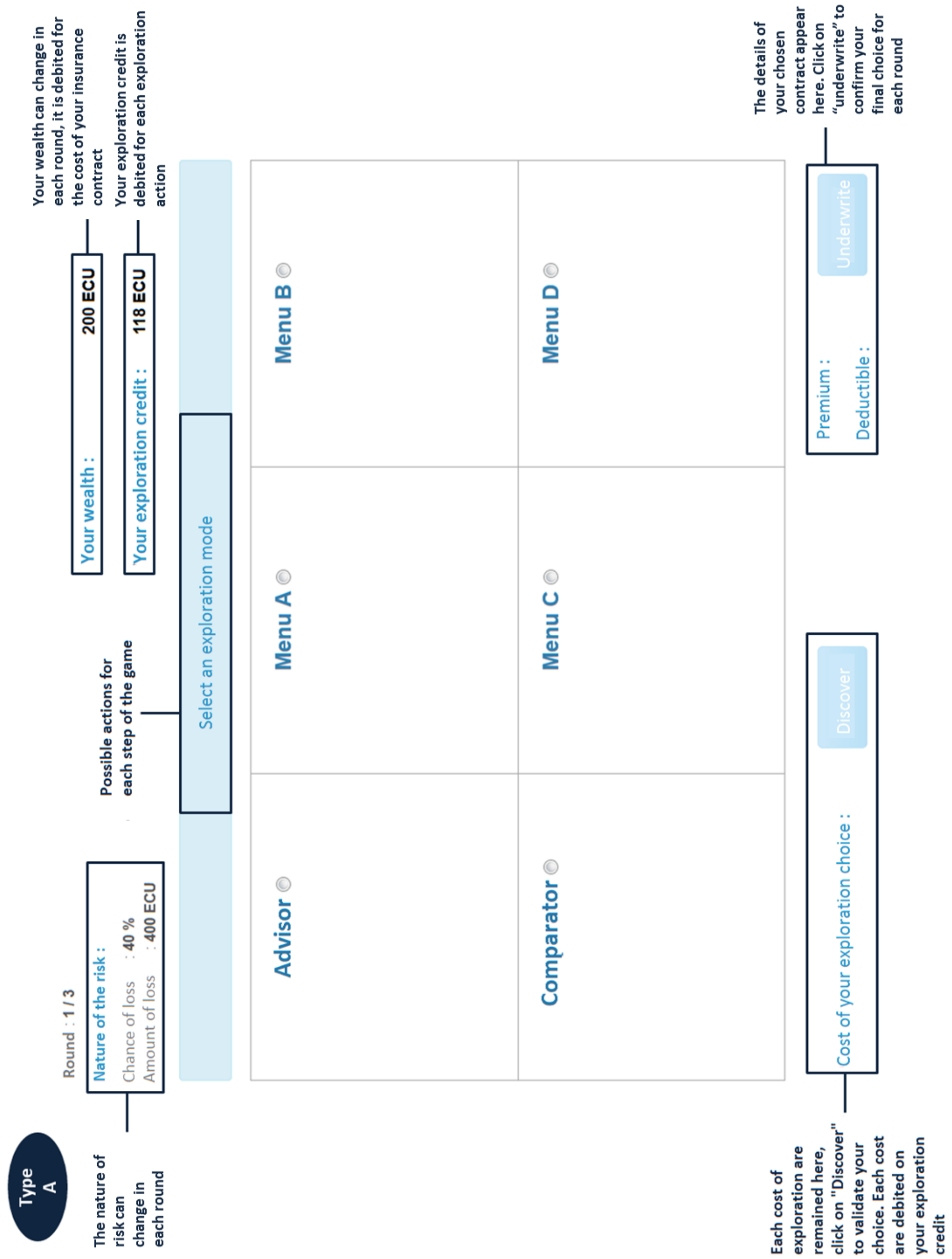
Example: The loss amount is 1000 ECU with a probability of 17% and initial wealth of 180 ECU, If you select an insurance contract with a premium of 70 ECU and a deductible of 25 ECU:

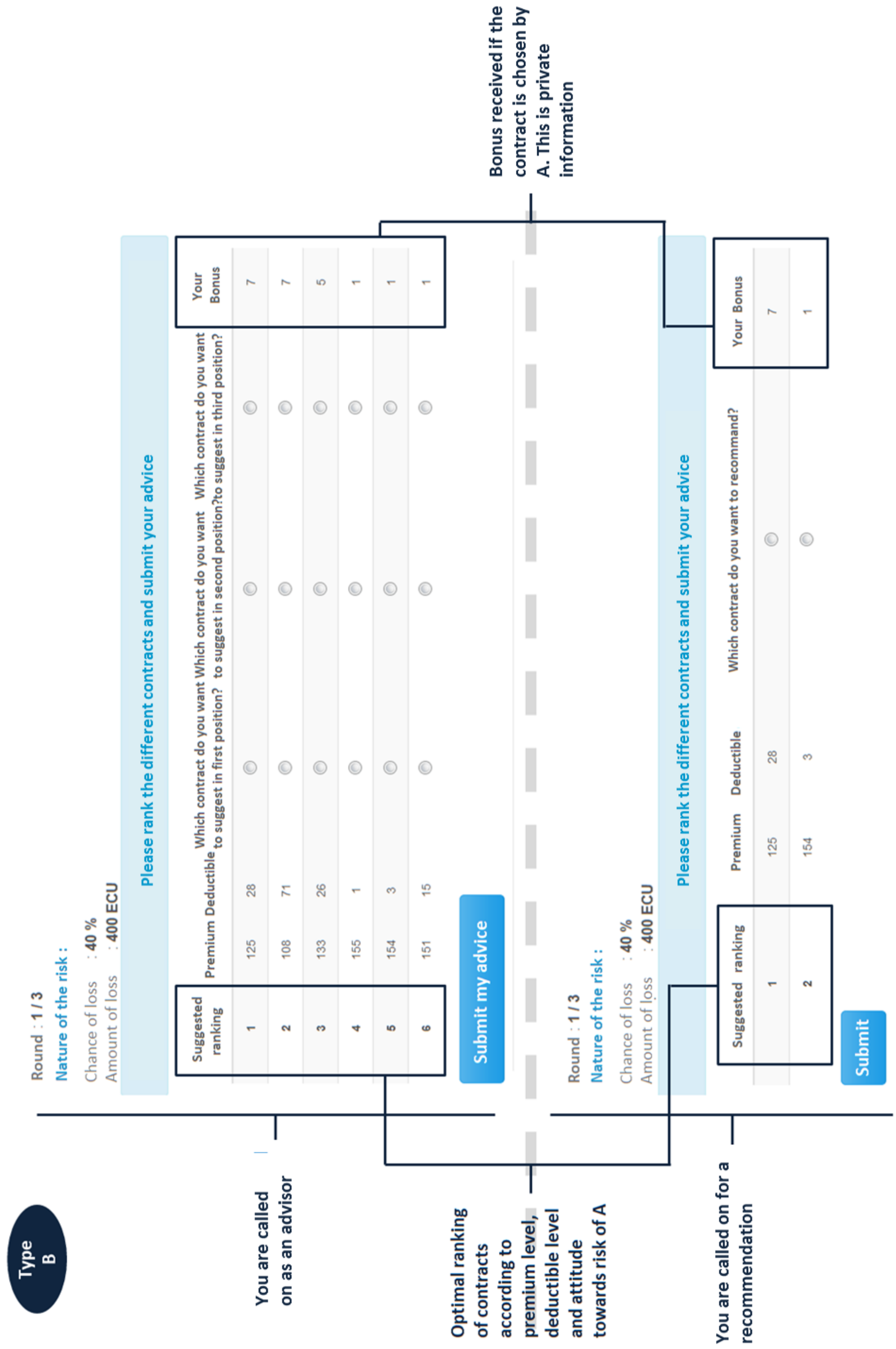
- If the loss occurs (with probability of 17%), you earn: 180 – 70 – 25 = 85 ECU
- If the loss does not occur (with a probability of 83%), you earn: 180 – 70 = 110 ECU

There are **eight different potential contracts**. These contracts are not visible at the beginning of the round, and will partly or fully appear on your screen as you choose different exploration actions. To do so you have an **exploration credit** that is used up according to the different types of exploration you choose. Any unconsumed exploration credit is part of your earnings.

To find out about the potential contracts, you can:

- **Explore one of the four menus (A, B, C and D)**. Each menu consists of 2 contracts. To find out about the contracts in one particular menu you have to pay **12 ECU** (debited from your exploration credit). Once you access this menu, you can ask for a recommendation from a type-B participant. Asking for a recommendation costs **4 ECU** (debited from your exploration credit). Participants of type B are informed of a contract-ranking according to your own attitudes towards risk. In addition, they know the bonus associated with each contract. The B participant receives the bonus associated with the contract if you choose the contract that they recommend for you. The A participants do not know either the contract ranking or the bonuses.
- **Look for an advisor** (i.e. a type-B participant). This costs **12 ECU** (debited from your exploration credit). A type-B participant is randomly selected and will be informed of a contract-ranking covering six different potential contracts (including contracts from three of the four different menus). The ranking is based on your own attitudes towards risk. The B participant knows the bonus associated with each contract. The A participants do not know either the contract ranking or the bonuses. The B participant suggests a ranking of three contracts to participant A. If you choose a contract through the advisor, you will pay **14 ECU** (debited from your initial wealth).
- **Explore via a comparator.** Access to a comparator costs you **12 ECU** (debited from your exploration credit). Six of the eight possible contracts will then appear on your screen (from three of the four menus). However, only the premium information will appear. To find out about the associated deductible you have to pay **3 ECU** per contract. You do not know the ranking of these contracts.


Type A
Round : 1 / 3
Nature of the risk :
Chance of loss : 40 %
Amount of loss : 400 ECU
The nature of risk can change in each round
Possible actions for each step of the game
Your wealth : 200 ECU
Your exploration credit : 118 ECU
Your wealth can change in each round, it is debited for the cost of your insurance contract
Your exploration credit is debited for each exploration action
Select an exploration mode
Advisor
Menu A
Menu B
Comparator
Menu C
Menu D
Cost of your exploration choice :
Discover
Each cost of exploration are remained here, click on "Discover" to validate your choice. Each cost are debited on your exploration credit
Premium :
Deductible :
Underwrite
The details of your chosen contract appear here. Click on "underwrite" to confirm your final choice for each round


Type B
You are called on as an advisor
Optimal ranking of contracts according to premium level, deductible level and attitude towards risk of A
You are called on for a recommendation
Bonus received if the contract is chosen by A. This is private information
Round : 1 / 3
Nature of the risk :
Chance of loss : 40 %
Amount of loss : 400 ECU
Please rank the different contracts and submit your advice
Suggested ranking
Premium
Deductible
Which contract do you want to suggest in first position?
Which contract do you want to suggest in second position?
Which contract do you want to suggest in third position?
Your Bonus
Submit my advice
Round : 1 / 3
Nature of the risk :
Chance of loss : 40 %
Amount of loss : 400 ECU
Please rank the different contracts and submit your advice
Suggested ranking
Premium
Deductible
Which contract do you want to recommand?
Your Bonus
Submit

<u>**You are type B:**</u>

You can be called by a type-A participant in two different ways:

- **If a type A seeks an advisor.** There will appear on the screen of one selected type-B participant a ranking of **six different contracts** (from three of the four menus) and **a bonus associated with each contract**. **This ranking depends on the premium, the deductible and the profile of the type-A participant regarding risk.**

  If you are called here you should **suggest a ranking of three contracts** to type A. If the type-A participant chooses one of the contracts suggested, you earn a **fixed fee of 14 ECU**, paid by type A, and the bonus associated with the chosen contract.

- **If participant A asks for a recommendation**. There will appear on the screen of one selected type-B participant **a ranking of two contracts** and **a bonus associated with each contract.**

  If you are called here you should **recommend only one contract**. If the type-A participant chooses one of these contracts, you earn the associated bonus.

Participant A does not pay the bonus. There is a transfer from A to B only if A chooses a contract through the advisor (a fixed fee of 14 ECU).

For a request from an A participant only one B is randomly selected. It is therefore possible for each B participant to be called once, a number of times or never during a round.

For each round, your **earnings are equal to a fixed remuneration of 120 ECU and the potential additional earnings resulting from interactions with the A participants**. For this part, your final earnings are equal to those in one of the six rounds, randomly chosen.

Thanks for your participation

Before leaving the room to receive your payment, please fill out the final questionnaire. Then, please click on “Validate”.

Before leaving your table, please:

- Take your three-letter login.
- Take all instruction sheets.

Comprehension Questionnaire for Part 1 of the Experiment:

We will mark this questionnaire in a few minutes.

If you are A:

Your wallet contains 5 Euros:	TRUE ☐	FALSE ☐
For each draw, you should indicate the amount in Euros that B left in the wallet:	TRUE ☐	FALSE ☐
For each draw, you should indicate your estimation of the amount that B left in the wallet:	TRUE ☐	FALSE ☐
Your earnings depend only on your estimation:	TRUE ☐	FALSE ☐

If you are B:

Your wallet only contains 5 Euros:	TRUE ☐	FALSE ☐
You have to leave 0.50€ in the wallet for each red ball:	TRUE ☐	FALSE ☐
The experimenter or the A's know the amount that you left in the wallet:	TRUE ☐	FALSE ☐
No-one knows the amount that you left in the wallet:	TRUE ☐	FALSE ☐

Comprehension Questionnaire for Part 3 of the Experiment:

We will mark this questionnaire in a few minutes.

If you are A:

It is possible to return to an exploration design if you have enough exploration credit :	TRUE ☐	FALSE ☐
Your final earnings for this part can depend on the first two rounds:	TRUE ☐	FALSE ☐
The comparator displays the premium and deductible of each contract for free:	TRUE ☐	FALSE ☐
Your initial wealth, the amount of loss, the probability of loss and the exploration credit can change from round to round:	TRUE ☐	FALSE ☐
It is possible to choose a contract without knowing its associated deductible:	TRUE ☐	FALSE ☐
It is possible to choose a contract suggested by the advisor without paying the fixed fee:	TRUE ☐	FALSE ☐
When you choose a contract suggested by the advisor, the fixed fee is debited from your exploration credit:	TRUE ☐	FALSE ☐
If you do not spend your exploration credit, this amount is added to your earnings:	TRUE ☐	FALSE ☐

If you are B:

It is possible to never be called during a round:	TRUE ☐	FALSE ☐
You have to rank contracts in the same order as displayed:	TRUE ☐	FALSE ☐
The ranking displaying on your screen depends only on the bonus:	TRUE ☐	FALSE ☐
The bonus is paid by A:	TRUE ☐	FALSE ☐

Table 4 bis – Multinomial Regression with Random Effect

Explanatory Variable	**Intraround Channel Switch**					
	Model 5 bis		**Model 6 bis**		**Model 7 bis**	
First choice:	Broker		Insurance		Comparator	
Coefficients *Std. error*	Comparator	Insurance	Comparator	Broker	Insurance	Broker
Honesty Beliefs	−2.00 *1.85*	−0.15 *0.74*	0.76** *0.35*	0.52 *2.65*	0.31 *0.28*	0.30 *0.37*
Risk Aversion (loss)	0.05 *0.28*	−0.11 *0.28*	−0.04 *0.21*	−0.05 *0.92*	0.31 *0.16*	−0.53 *0.59*
Initial Wealth	0.00 *0.00*	0.00 *0.00*	0.01 *0.01*	0.01 *0.02*	0.00 *0.00*	−0.01 *0.01*
Loss Amount	0.00 *0.00*	0.00 *0.00*	0.00 *0.00*	0.01 *0.01*	0.00 *0.00*	0.00 *0.00*
Probability of Loss	3.28 *4.68*	4.47 *3.96*	−1.69 *4.19*	6.62 *14.47*	4.14 *2.22*	6.33 *2.22*
Round number	−0.29 *0.25*	−0.05 *0.24*	−0.10 *0.25*	0.74 *2.48*	−0.09 *0.16*	−0.49 *0.32*
Deviation Degree	1.67 *0.99*	0.21 *0.38*	−0.35 *2.45*	2.09 *2.31*		
Deviation Deg. and Honesty Belief	1.41* *0.84*	0.47 *0.29*	−0.41 *1.68*	0.06 *0.94*		
Deductible no. discovered					−1.83*** *0.47*	−0.92 *0.75*
Deductible no. discovered^2					0.21* *0.07*	0.13 *0.14*
Constant (ref. level: No Switch)	−9.12 *5.74*	−4.49 *2.13*	−2.91 *2.04*	−14.8 *22.78*	−5.52* *1.80*	−1.14 *2.48*
Nb. Observations	237		210		615	
Nb. Subjects	79		67		140	
R^2	0.293		0.224		0.281	
Adjusted R^2	0.221		0.132		0.256	

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 ‘***’ 0.05 ‘**’ 0.1 ‘*’

Table 9 bis – Logistic Regression with Random Effect

Explanatory Variable	**Deviation**	
Coefficients *Std. error*	**Model 20**	**Model 20 bis**
Risk Aversion (loss)	−0.28** *0.09*	−0.29** *0.09*
Initial Wealth	0.01 *0.00*	0.01 *0.00*
Loss Amount	0.00 *0.00*	0.00 *0.00*
Probability of Loss	−2.22 *1.66*	−2.22 *1.73*
Optimal Contract Bonus	−0.05 *0.02*	−0.04 *0.02*
Status: TIED-AGENT	−1.28* *0.68*	−1.30* *0.68*
Dishonesty Session Level	0.59** *0.26*	
Session 1		0.34 *0.48*
Session 2		−0.26 *0.66*
Session 3		−0.98 *1.07*
Session 4		−0.17 *0.77*
Session 5		−1.07 *0.73*
Session 6		−0.94 *0.55*
Session 7		−0.80 *0.53*
Round Number	−0.01 *0.10*	−0.01 *0.10*
Constant (ref. level: NO)	2.66** *0.89*	2.59** *0.95*
Nb. Observations	289	289
Nb. Subjects	40	40
R^2	0.181	0.189
Adjusted R^2	0.152	0.141

Signif. codes for p-values under Holm–Bonferroni correction: 0.01 '***' 0.05 '**' 0.1 '*'

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Contributions en sciences actuarielles

Chapter 3

A game-theoretic analysis of insurers pricing strategies

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Analyses des stratégies de prix des assureurs dans un jeu non-coopératif

This chapter is based on an article co-written with Hansjoerg Albrecher, Christophe Dutang and Stéphane Loisel.

Abstract

In this paper, we extent the repeated non-cooperative game of Dutang et al. (2013) to model a non-life insurance market. We provide market premium equilibrium analysis and its sensitivity according to solvency constraint as well as consumers' price sensitivity. We also derive, using Markov chains methodology, convergence measures of long run market shares and properties of a highly regulated market. In a second step, we provide numerical illustrations of the repeated one period game and we analyze market premium equilibrium path according to different price sensitivity parameters, loss history and market size.

Keywords: *game theory, non-cooperative game, consumers' price sensitivity, solvency constraint, stochastic behaviors, Markov chains*

Résumé

Ce papier est une extension du jeu non-coopératif présenté dans Dutang et al. (2013) afin de modéliser le marché de l'assurance non-vie. Nous proposons une analyse des impacts de la sensibilité aux prix des consommateurs et des conséquences de la présence d'une contrainte de solvabilité sur les primes d'équilibres. Utilisant la théorie des chaînes de Markov, nous calculons des mesures convergentes de la distribution, à long-terme, des consommateurs à travers les différents assureurs. Nous montrons également les propriétés particulières d'un marché fortement régulé. Dans un second temps, nous illustrons le jeu répété à l'aide de simulations numériques. Nous analysons les trajectoires de prix de marché et leurs sensibilités vis-à-vis des paramètres d'inertie des consommateurs, des sinistres observés et de la taille du marché.

Mots-clés: *théorie des jeux, jeu non-coopératif, élasticité prix, contrainte de solvabilité, comportements stochastiques, chaînes de Markov.*

3.1 Introduction

In the insurance world, determining an appropriate and attractive premium is always an important issue because of the strong competition among different insurers. This is particularly true for non-life retail insurance mass markets, such auto insurance and household insurance, where an important number of insurance companies and mutuals compete. Policyholders are looking for the best services at a lower price and thus tend to migrate to firms displaying advantageous insurance premium.

Basic economic models suggest that the equilibrium premium is the marginal cost (i.e. actuarial premium) and in a Bertrand competition (i.e. competition in price), any upward deviation from this equilibrium will result in losing all the policies in the next period. However, this theory suggests that consumers are perfectly mobile and their decisions are only based on premium level. In practice, customers do not move from an insurer to a cheaper one, and we observed inertia of the insurance demand, preventing an insurer from the Bertrand paradox and null profit. Indeed, customer behavior is much more complicated. In addition to customer loyalty, explain by insurers' reputation as well as search and switching costs (Chapter 2: Mouminoux et al. (2018), Schram and Sonnemans (2011)), Feldblum (2001) points out that there are barriers for a new insurer to enter the non-life insurance market, such as solvency constraint, underwriting standard,...

The pricing of insurance contracts is a classical research topic. In practice, insurance companies use various approaches including general principles of premium calculation, based on the expected claim expenses or to its moment (average, standard deviation,...), credibility theory and generalized linear models (GLM), see for instance Kaas et al. (2008).
However, in a highly competitive market which is dominated by a relatively small numbers of firms, insurance pricing does not reduce to compute technical premium: each insurer attempts to predict customers' reactions and takes advantages against the others. The actuarial premium is thus altered by the marketing and management departments for several reasons, such as the customers' loyalty and the market conditions.

Hence, game theory concepts seem to be an appropriate way to evaluate the strategic choices of insurers in the presence of competition. The use of game theory in actuarial science is not new. The first attempts go back to Borch (1962, 1974), Bühlmann (1980, 1984), and Lemaire (1984, 1991), who applied cooperative games in order to model risk transfer between insurers and reinsurers (see also others researches and review as Aase (1993), Brockett and Xiaohua (1997), Tsanakas and Christofides (2006), Boonen (2016)). Regarding to non-cooperative games, two types of models have been considered in non-life insurance markets: a) the Cournot oligopoly where insurers' strategies are based on the choice of volume of business (see Powers and Shubik (1998), Powers et al. (1998)) and b) the Bertrand oligopoly where insurers set premiums (see Polborn (1998), Rees et al. (1999) and Dutang et al. (2013)).

In this paper, we attempt to model competition in non-life insurance markets with non-cooperative game in order to extend the insurer-vs-market reasoning of Taylor (1986, 1987), see also Kliger and Levikson (1998), Emms et al. (2007) and Moreno-Codina and Gomez-Alvado (2008) for extensions. Extending the model of Dutang et al. (2013), we focus our analysis on consequences of customers' price sensitivity, in others words customers inertia, as well as consequences of solvency constraints, on price equilibrium, market share and their consequences on profits. Dutang et al. (2013) modelling seems to be the most appropriated to our analysis since they included a lapse model, an aggregate loss model as well as a solvency constraint functions for insurers. Integration of stochastic lapse behavior through a lapse probability function of the premiums offered by all insurers is a best way to avoid any Bertrand paradox as well as integer consumers' inertia in a game theoretic approach. In addition, because of strong regulation of insurance market, it is essential to consider in the insurer decision-making strategies a solvency constraint which will affect premium level as well as long run market patterns. Indeed, while this paper does not aim to analyze market pricing cycle but focus analysis on one-period market equilibrium according to customer behavior, many researches have demonstrate the presence of market cycle in different lines of insurance (Rantala (1988), Malinovskii (2010); Emms (2012), Boonen et al. (2018)). Hence, we also consider a repeated one-period game in order to emphasize convergence properties of the market using a Markov chains approach.

The rest of the paper is organized as follows. Section 3.2 remains the one-period game based on Dutang et al. (2013). Section 3.3 present general properties of the lapse and loss models as well as short and long run convergence measures of insurers' market share. A subsection is also dedicated to the analysis of a strongly regulated market under our customer behavior modelling. We also illustrate in this section some results using numerical simulations of the game in a simple market. In Section 3.4, we analytically analyze premium equilibrium of a simple duopolistic market by focusing results on customers' price sensitivity and impacts of comparative advantage of one insurers. We also show when solvency and prices constraints are actives and their consequences on market equilibrium. In Section 3.5, we display some ruins theory results in order to understand the long run market pattern regarding to the number of insurers present on the market. In Section 3.6, we provide numerical illustration of the repeated one-period game by commenting some pattern under stochastic simulations. We also provide a parameters sensitivity analysis. Finally, a conclusion and perspectives are given in Section 3.7.

3.2 Description of the repeated game

Consider J insurance companies competing on price in a market of N policyholders with one-year policies, where N is considered constant across periods. In the sequel, a subscript $j \in \{1, \dots, J\}$ will always denote a player index (i.e. an insurer) whereas a subscript $i \in \{1, \dots, N\}$ denotes an insured index. Vectors will be **bolded**. The "game" for each insurer $j \in \{1, ..., J\}$ consists in define premium $x_{j,t}$ at the beginning of each year t, in order to maximize their profits by selling identical policies to the insured market of size N. Let $\boldsymbol{x}_t = (x_{1,t}, \dots, x_{J,t}) \in [\underline{x}, \overline{x}]^J \subset \mathbb{R}^J$ be the insurers price vector, with $x_{j,t}$ representing premium of insurer j for year t. We thus denote by $\boldsymbol{x}_{-j,t} = (x_{1,t}, \dots, x_{j-1,t}, x_{j+1,t}, \dots, x_{J,t})$ the vector $\boldsymbol{x}$ without the jth component. In this section, we first describe the sequence of the repeated game and then explain each component in details .

3.2.1 Game overview

Game theorie of noncooperative games deals with two main equilibirum type: the Nash equilibrium and the Stackelberg equilibrium. The Nash equilibrium assumes that player actions are taken simultaneously while for the Stackelberg equilibrium actions take place sequentially, see e.g. Fudenberg and Tirole (1991),Osborne and Rubinstein (2006). In our setting, we consider the Nash equilibrium as the most appropriate concept. Indeed, in majority of insurance market, in particular in retail-mass market, it is difficult to admit that one insurer is always taking its decision at first. Pricing strategies are generally defined at the beginning of the year, after an update of past year observations such losses level, market shares at the end of the year, financial market features (i.e. inflation, time-value,...) and shareholders' expectations for the future year. We thus give below the definition of a Nash equilibrium and use this approach for the rest of our results.

Definition. *For a game with J players, with payoff functions O_j and constraint function g_j, a Nash equilibrium is a vector $\boldsymbol{x}^\star = (x_1^\star, \dots, x_J^\star)$, where x_j is also constrained by price lower and upper bounds $[\underline{x}, \overline{x}]$, such that for all $j = 1, \dots, J$, $x_j^\star$ solves the subproblem*

$$\max_{x_{j,t} \in [\underline{x}, \overline{x}]} O_j(x_j, \boldsymbol{x}^\star_{-j}) \ s.t. \ g_j(x_j) \geq 0.$$

where x_j and $\boldsymbol{x}_{-j}$ denote action of player j and the other players' action, respectively.

A Nash equilibrium is interpreted as a point at which each players, given the strategies of others players, have no incentive to change their strategies. Having reached Nash equilibrium a player will be worse off by changing its strategy. Note that the constraint function depends only on player action x_j and not on other players' strategies $\boldsymbol{x}_{-j}$. Otherwise, we would deal with a generalized Nash equilibrium.

The elements of the objective function $O_{j,t}$ corresponding to the expected profit of insurer j for time t and the definition of the constraint function $g_{j,t}$ corresponding to solvency constraints required by regulators are detailed in the next subsections.

We define the repeated game as the iteration of expression one-period game over T years. However, since regulator constraints insurers to be solvent, we pulled out them of the market when they have either a tiny market share (< 0.1%) or a negative capital. In the same way, we remove players from the game when the capital is below the minimum capital requirement (MCR), whereas we keep them if capital is between MCR and solvency capital requirement (SCR). As a reminder, MCR can be defined as a percentage of the SCR computed in the solvency constraint. In general in non-life insurance retail market MCR is between 25% and 45% of the SCR.

Repeated game. *For period $t = 1, \dots, T$, repeat*

1. *The insurers among J_t maximize their objective function subject to the solvency constraint*

$$\sup_{x_{j,t} \in [\underline{x}, \overline{x}]} O_{j,t}(x_{j,t}, \boldsymbol{x}_{-j,t}) \text{ s.t. } g_{j,t}(x_{j,t}) \geq 0.$$

2. *Once the premium equilibrium vector $\boldsymbol{x}_t^\star$ is determined, customers randomly lapse or renew. We get a realization $n_{j,t}^\star$ of the random portfolio size $N_{j,t}(\boldsymbol{x}_t^\star)$.*
3. *Aggregate claim amounts $S_{j,t}$ are randomly drawn according to the chosen loss model and the portfolio size $n_{j,t}^\star$.*
4. *The underwriting result for Insurer j is computed by $UW_{j,t} = n_{j,t}^\star \times x_{j,t}^\star \times (1 - e_{j,t}) - S_{j,t}$, where $e_{j,t}$ correspond the rate of handling costs of Insurer j at time t.*
5. *We update the capital by the following equation $K_{j,t+1} = K_{j,t} + UW_{j,t}$.*
6. *We update the set of competitors J_{t+1} by removing bankrupted insurers or too small insurers. We defined small insurers, those who have a market share lower than 0.1%. We assume that tiny insurers will not be able to face future losses and handling costs. Indeed, they will not benefit from mutualisation concepts, essential for insurance market and thus will decide to run-off their business.*

In the current framework and for simplicity, we make the following implicit simplifying assumptions: (i) the pricing procedure is done (only) once a year (on January 1), (ii) all policies start at the beginning of the year, (iii) all premiums are collected on January 1, (iv) every claim is (fully) paid on December 31 and (v) there is no inflation and (vi)no stock/bond market to invest premium.

In practice, these assumptions do not hold: (i) pricing by actuarial and marketing departments can be done more frequently, e.g. every 6 months, (ii) policies start and are renewed all over the year, (iii) premium is collected all over the year, (iv) claims are settled every day and there are reserves for incurred-but-not-reported claims and (v) there is inflation on both claims and premiums, (vi)the time between the premium payment and a possible claim payment is used to invest in stock/bond markets. However, we need the above simplifications to have a sufficiently simple and tractable model.

In the next subsections, we present in detail the four components of the game: a lapse model, a loss model, an objective function and a solvency constraint function.

3.2.2 Components of the game for policyholders

In the following subsections, we state the precise assumptions made for the behavior of policyholders, both in terms of lapse/renewal probabilities and in terms of loss.

3.2.2.1 Assumptions for the lapse model

Being with insurer j in the previous period $t-1$, the insurer choice of insured i for the current period t follows an J-dimensional multinomial distribution $\mathcal{M}_J(1, \boldsymbol{p}_{j\rightarrow}(t))$ with probability vector $\boldsymbol{p}_{j\rightarrow}(t) = (p_{j\rightarrow 1}(t), \dots, p_{j\rightarrow J}(t))$ where components sum to 1.

It seems natural and it has been verified empirically (Duijmelinck et al. (2015))that the probability to choose an insurer is highly influenced by the choice of the previous period . In other words, the probability $p_{j\rightarrow k}(t)$ with $k \neq j$ is generally much lower than the probability to renew $p_{j\rightarrow j}(t)$. To our knowledge, only the extreme UK market shows lapse rates for non-life insurance above 50%. Those probabilities have to depend on the premiums x_j, x_k proposed by insurer j and k, respectively. Therefore, we assume that $p_{j\rightarrow k}(t) = p_{j\rightarrow k}(\boldsymbol{x}_t)$.

In the economics literature, $p_{j\rightarrow k}$ is considered in the framework of discrete choice models. In the random utility maximization setting, McFadden (1981) or Anderson et al. (1989) propose multinomial logit and probit probability choice models. In this paper, we choose a multinomial logit model, since the probit link function does not really enhance the choice model despite its additional complexity. Working with unordered choices, we arbitrarily set the insurer reference category for $p_{j\rightarrow k}$ to j, the current insurer. As in Dutang et al. (2013), we define the probability for a customer to go from insurer j to k given the price vector $\boldsymbol{x}$ by the following multinomial logit model

$$p_{j\rightarrow k}(\boldsymbol{x}) = \begin{cases} \frac{1}{1+\sum\limits_{l\neq j} e^{f_j(x_j,x_l)}} & \text{if } j = k, \\ \frac{e^{f_j(x_j,x_k)}}{1+\sum\limits_{l\neq j} e^{f_j(x_j,x_l)}} & \text{if } j \neq k, \end{cases} \tag{3.1}$$

where the sum is taken over the set $\{1, \dots, J\}$ and f_j is a price sensitivity function. In the following, we consider two types of price functions

$$\bar{f}_j(x, y) = \mu_j + \alpha_j \frac{x}{y} \text{ and } \tilde{f}_j(x, y) = \tilde{\mu}_j + \tilde{\alpha}_j(x - y). \tag{3.2}$$

The first function $\bar{f}_j$ assumes a price sensitivity with the ratio of the proposed premium x_j and competitor premium x_l, whereas $\tilde{f}_j$ works with the premium difference $x_j - x_l$. The two models are denoted by premium ratio (PR) and premium difference (PD) lapse models. Parameters μ_j, α_j represent a base lapse level and price sensitivity. We assume that insurance products display positive price-elasticity of demand $\alpha_j > 0$. Clearly $\sum_{k=1}^{J} p_{j\rightarrow k}(\boldsymbol{x}) = 1$. The previous expression of $p_{j\rightarrow k}$ can be rewritten as

$$p_{j\rightarrow k}(\boldsymbol{x}) = p_{j\rightarrow j}(\boldsymbol{x}) \left(\delta_{jk} + (1-\delta_{jk}) e^{f_j(x_j,x_k)} \right),$$

with δ_{ij} denoting the Kronecker product. This representation helps in deriving analytic properties of the lapse model.

We use the following notation for year t. For customers, $C_{i,t}$ is the choice of customer i at time t take value in $\{1, \dots, J\}$, and is randomly chosen according to probabilities $p_{j\rightarrow l}(t)$ knowing that $C_{i,t-1} = j$. For instance, $(C_{i,t})_t = (1, 1, 1, 2, 2, 3, \dots)$ indicates that customer i chooses Insurer 1 for period 1, 2, 3 and Insurer 2 for period 4,5; etc...

The following assumptions are made

- A1: The customer choice of insurer at time t depends only on the previous choice at time $t-1$.
- A2: Customers are independent, i.e. $(C_{i,t})_i$ are independent variables.
- A3: Customer behaviors are identical over time, i.e. $p_{j\rightarrow k}(\boldsymbol{x}_t)$ depends on t through premium $\boldsymbol{x}_t$.
- A4: No customer can enter or exit the market, the total market size N is constant.

Hence, being with current insurer j during the $t-1$ period, the insurer choice $C_{i,t}$ of insured i for the year t follows an J-dimensional multinomial distribution $\mathcal{M}_J(1, \boldsymbol{p}_{j\rightarrow}(\boldsymbol{x}_t))$ with probability vector $\boldsymbol{p}_{j\rightarrow}(\boldsymbol{x}_t)$. The probability mass function is given by $P(C_{i,t} = k \mid C_{i,t-1} = j) = p_{j\rightarrow k}(\boldsymbol{x}_t)$.

In fact, the random choice $(C_{i,t})_t$ of policyholder i is governed by a (discrete-time) Markov chain with transition matrix

$$P_{\rightarrow}(\boldsymbol{x}_t) \triangleq \begin{pmatrix} p_{1\rightarrow 1}(\boldsymbol{x}_t) & \dots & p_{1\rightarrow J}(\boldsymbol{x}_t) \\ & \ddots & \\ p_{J\rightarrow 1}(\boldsymbol{x}_t) & \dots & p_{J\rightarrow J}(\boldsymbol{x}_t) \end{pmatrix},$$

see Proposition 3.3.1 for details. In general, the Markov chain is time-inhomogeneous as $P_{\rightarrow}$ depends on the price vector $\boldsymbol{x}_t$ which evolves over time. Note that using Markov chains to study customer behavior was also done in Marker (1998), where a special duopolistic situation is considered: one insurer versus the market.

From an insurer point of view, we are interested in the number of policyholders at time t. Let $n_{j,t-1}$ be the portfolio size at the previous period $t-1$. We define $\overline{\boldsymbol{C}}_{j,t} = (\overline{C}_{j,1,t}, \dots, \overline{C}_{j,J,t})$ as the random assignment of customers of Insurer j at time t, where $\overline{C}_{j,k,t}$ denotes the (random) number of policyholders moving from Insurer j to Insurer k.

Therefore, policyholders of Insurer j will choose insurers according to a multinomial distribution $\overline{\boldsymbol{C}}_{j,t} \sim \mathcal{M}_I(n_{j,t-1}, \boldsymbol{p}_{j\rightarrow}(\boldsymbol{x}_t))$ given the portfolio size $n_{j,t-1}$. Let $\boldsymbol{N}_t = \big(N_{1,t}, \dots, N_{J,t}\big)$ be the vector of (random) portfolio sizes at time t. They are obtained by summing the choices of each Insurers' customers

$$\boldsymbol{N}_t = \sum_{j=1}^{J} \overline{\boldsymbol{C}}_{j,t} = \begin{pmatrix} \overline{C}_{1,1,t} \\ \vdots \\ \overline{C}_{1,J,t} \end{pmatrix} + \dots + \begin{pmatrix} \overline{C}_{J,1,t} \\ \vdots \\ \overline{C}_{J,J,t} \end{pmatrix} = \begin{pmatrix} N_{1,t} \\ \vdots \\ N_{J,t} \end{pmatrix}.$$

In other words, the portfolio sizes are obtained a sum of J independent multinomial vectors $\boldsymbol{N}_t = \overline{\boldsymbol{C}}_{1,t} + \dots + \overline{\boldsymbol{C}}_{J,t}$. We denote by $\boldsymbol{n}_t = \big(n_{1,t}, \dots, n_{J,t}\big)$ the realizations of the random vector.

For Insurer j, his portfolio size $N_{j,t}$ is a sum of independent variables: $N_{j,t} = \sum_{k=1}^{J} \overline{C}_{k,j,t}$, where $\overline{C}_{j,j,t}$ corresponds to renewed policies and the rest of the sum corresponds to new business coming from other insurers. It is important to note that the insurers' portfolio sizes are not independent, since the total market size remains constant (A4).

For instance, consider a three-insurer market with $\boldsymbol{n}_0 = (100, 50, 60)$. $\overline{\boldsymbol{C}}_{1,1} = (70, 20, 10)$ denotes that 70 customers choose to renew, 20 go to Insurer 2, 10 to Insurer 3. If in addition $\overline{\boldsymbol{C}}_{2,1} = (18, 30, 12)$ and $\overline{\boldsymbol{C}}_{3,1} = (2, 8, 50)$, then $\boldsymbol{n}_1 = (90, 58, 74)$ is the realized portfolio size at time 1, i.e. Insurer 1 gets 90 customers, Insurer 2 gets 58 and Insurer 3 gets 74.

Finally, the process $(\boldsymbol{N}_t)_t$ is a J-dimensional discrete-time Markov process, see Proposition 3.3.4 for details. The process $(\boldsymbol{N}_t)_t$ takes values in the set of portfolio sizes

$$\mathcal{S}_N = \left\{ \boldsymbol{n} \in \mathbb{N}^J, \sum_{j=1}^{J} n_j = N \right\},$$

which has $\binom{N+J-1}{N}$ elements, see e.g. Breuer and Baum (2005).

The lapse/renewal process of policyholders at each time period considered can be seen as a closed Markovian network of discrete-time queues with batch services (see e.g. (Boucherie and van Dijk, 2011, Chap. 6)), for which the service time corresponds to the number of years a policyholder stays with the same insurer. In that context, $p_{j\to k}$ are called routing probabilities, an insurer is a server and a policyholder is a customer.

3.2.2.2 Assumptions for the loss model

Now we turn our attention to the loss model faced by policyholders. Let $Y_{i,t}$ be the aggregate loss of policy i during the coverage period t. The following assumptions are made

- A5: There is no adverse selection, i.e. $Y_{i,t}$ are independent and identically distributed (i.i.d.) random variables, $\forall i = 1, \dots, N$.
- A6: Catastrophic events are excluded and Y_i follows a frequency – average severity loss model.
- A7: The insurance business is short-tailed, i.e. the loss Y_i is paid in total on December 31 of each year.

Assumption A5 allows us to simplify the simulation process because the i.i.d. assumption implies that individual losses $Y_{i,t}$ do not need to be simulated. The aggregate claim amount per insurer $S_{j,t}$ could be simulated directly. Assumption A6 means that we assume a simple frequency – average severity loss model

$$Y_{i,t} = \sum_{l=1}^{M_{i,t}} Z_{i,l,t},$$

where the claim number $M_{i,t}$ is independent of the claim severity $Z_{i,l,t}$. Therefore, the aggregate claim amount for insurer j is

$$S_{j,t}(\boldsymbol{x}_t) = \sum_{i=1}^{N_{j,t}(\boldsymbol{x}_t)} Y_{i,t},$$

where $N_{j,t}(\boldsymbol{x}_t)$ is the portfolio size of insurer j given the price vector $\boldsymbol{x}_t$. Assumption A5 means that both claim numbers $M_{i,t}$ and claim amounts $Z_{i,l,t}$ are i.i.d. random variables. As already mentioned, we focus on short-tail business and so Assumption A7 is in line with our objective of modelling personal business lines.

For the numerical illustrations, we assume two different distributions for both claim frequencies and claim severities. Claim numbers follow either a Poisson or a negative binomial distribution, whereas claim amounts follow either a lognormal or a gamma distributions. Hence, we consider four different cases for the loss model. Taking into account the time t, we obtain the following cases

- PG: a Poisson-gamma with $M_{j,t} \sim \mathcal{P}(\lambda)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{G}(\mu_1, \sigma_1)$,
- NBG: a negative binomial-gamma with $M_{j,t} \sim \mathcal{NB}(r, p)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{G}(\mu_1, \sigma_1)$.
- PLN: a Poisson-lognormal with $M_{j,t} \sim \mathcal{P}(\lambda)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{LN}(\mu_2, \sigma_2^2)$,
- NBLN: a negative binomial-lognormal with $M_{j,t} \sim \mathcal{NB}(r, p)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{LN}(\mu_2, \sigma_2^2)$.

Let $\widetilde{M}_{j,t} = \sum_{i=1}^{N_{j,t}(\boldsymbol{x}_t)} M_{i,t}$ be the total number of claims for Insurer j in period t. We assume that $(\widetilde{M}_{1,t}, \dots, \widetilde{M}_{I,t})$ is a vector with independent margins.

We could have considered correlated loss models by introducing dependence either among claim numbers $(\widetilde{M}_{j,t})_j$ at portfolio level or claim frequency $(M_{1,t}, \dots, M_{N,t})$ or on the claim severity $(Z_{j,i,1}, \dots, Z_{j,i,m_i})$ at individual level. This procedure is not considered here.

3.2.3 Components of the game for insurers

3.2.3.1 Objective function

In Section 3.2.2.1, we assumed that the price elasticity of demand for the insurance product is positive. Thus, if the entire market underwrites below costs, any action of a particular insurer to get back to profitability will result in a reduction of his business volume. This has two consequences for the choice of the objective function: (i) it should involve a decreasing demand function of price $x_{j,t}$ given the competitors' price vector $\boldsymbol{x}_{-j,t}$ and (ii) it should depend on an assessment of the insurer break-even premium $\pi_{j,t}$ per unit of exposure.

Insurers have a history of past premium levels $x^\star_{j,t}$, gross written premium $\text{GWP}_{j,t}$, portfolio size $n_{j,t}$ and capital $K_{j,t}$ at the beginning of year t. Let d be the past number of years considered for which economic variables (e.g. market premium) will be computed.

The parameter $\pi_{j,t}$ corresponds to the estimated mean of overall costs including handling costs and claims and depends on the assessment of the loss expectation by insurer j. We thus define $\pi_{j,t}$ as

$$\pi_{j,t} = \omega_j \bar{a}_{j,t-1} + (1-\omega_j)\overline{m}_{t-1}, \tag{3.3}$$

where, at the beginning of each time period, the average market premium is determined as

$$\overline{m}_{t-1} = \frac{1}{d}\sum_{u=1}^{d} m_{t-u}, \text{ with } m_{t-u} = \frac{\sum_{j=1}^{J} \text{GWP}_{j,t-u} \times x^\star_{j,t-u}}{\text{GWP}_{.,t-u}},$$

which is the mean of last d market premiums and $\text{GWP}_{.,t-u} = \sum_{j=1}^{I} \text{GWP}_{j,t-u}$. With current portfolio size $n_{j,t-1}$ and initial capital $K_{j,t-1}$, each insurer computes its actuarially based premium as

$$\bar{a}_{j,t-1} = \frac{1}{1-e_{j,t}}\frac{1}{d}\sum_{u=1}^{d} \underbrace{\frac{S_{j,t-u}}{n_{j,t-u}}}_{\text{avg ind loss}},$$

where $S_{j,t}$ denotes the observed aggregate loss of insurer j during year t and $e_{j,t}$ denotes the expense rate as a percentage of gross written premium. Note that the claim amount is not adjusted against large claims (i.e. $y_{i,t}$ are not capped).

$\overline{a}_{j,t}$ represents the actuarial premium based on the past loss experience of Insurer j, $\overline{m}_t$ is the market premium, available for instance, via rating agencies or through insurer associations and $\omega_j \in [0,1]$ is the credibility factor of insurer j. ω_j reflects the confidence of insurer j in its own loss experience: the closer to 1, the more confident insurer j is. Note that π_j takes into account expenses implicitly via the actuarial and the market premiums.

As in Dutang et al. (2013) ,we choose the demand function as

$$D_{j,t}(\boldsymbol{x}_t) = \frac{n_{j,t-1}}{N}\left(1-\beta_j\left(\frac{x_{j,t}}{m_j(\boldsymbol{x}_t)}-1\right)\right), \tag{3.4}$$

where $\beta_j > 0$ is the elasticity parameter and $m_j(\boldsymbol{x})$ is a market premium proxy. Indeed, we assume that the insurance product is a normal product where price elasticity of consumers is negative. In this form, $D_j(\boldsymbol{x}_t)$ approximates the expected market share $E(N_{j,t}(\boldsymbol{x}_t))/N$ presented in Section 3.2.2.1.

As the elasticity parameter β_j is positive, a premium increase (of insurer j) will result in a decrease of the demand for insurance. The market proxy used in Equation (3.4) is the mean price of the other competitors

$$m_j(\boldsymbol{x}) = \frac{1}{J-1}\sum_{k\neq j} x_k. \tag{3.5}$$

The market proxy aims to assess other insurers' premiums without specifically targeting one competitor. It can be interpreted as the premium of an ideal medium competitor. Consequently, Insurer j typically does not target the cheapest, the most expensive or the leader insurers. From a mathematical point of view, we will lose the continuity of the demand function if we choose the cheapest premium $\min_{k\neq j} x_k$. Furthermore, the term $\frac{x_j}{m_j(\boldsymbol{x})} - 1$ in the demand function is closed related to the average of the relative premium differences since

$$\frac{1}{J-1}\sum_{k\neq j}(\frac{x_k}{x_j} - 1) = \frac{1}{x_j}\frac{1}{J-1}\sum_{k\neq j} x_k - \frac{J-1}{J-1} = \frac{m_j(\boldsymbol{x})}{x_j} - 1.$$

Consider now an alternative market proxy value depending on the previous period market share of insurers. Hence, firms do not attribute the same weight to each competitor. We thus consider that firms put more importance on the biggest competitors' price. In the following, we analyze the following market proxy being the weighted mean of other competitors' prices

$$m_j(\boldsymbol{x}, \boldsymbol{n}) = \frac{1}{N - n_j}\sum_{k\neq j} n_k x_k. \tag{3.6}$$

Now we state the objective function defined as

$$O_{j,t}(\boldsymbol{x}_t) = \frac{n_{j,t}}{N}\left(1 - \beta_j\left(\frac{x_{j,t}}{m_j(\boldsymbol{x}_t, \boldsymbol{n}_t)} - 1\right)\right)(x_{j,t} - \pi_{j,t}), \tag{3.7}$$

where $\pi_{j,t}$ is the break-even premium j in (3.3) and $m_j(\boldsymbol{x}_t, \boldsymbol{n}_t)$ is either (3.5) or (3.6).

The objective function $O_{j,t}$ defined as the product of a demand function and an expected profit per policy represents a company-wide expected profit. Thus, maximising the objective function $O_{j,t}$ leads to a trade-off between increasing premium to favour higher projected profit margins and decreasing premium to defend the current market share. Note that $O_{j,t}$ has the nice property to be infinitely differentiable.

3.2.3.2 Constraint function

In our game context, we want to avoid the simplistic Solvency I framework and follow the Solvency II framework. However, we still want to keep the tractability for the SCR computation rule. We recall that the aggregate claim amount is assumed to be a frequency - average severity model and catastrophic events are excluded. A simplification is to approximate a q-quantile $Q(n, q)$ of aggregate claim amount of n i.i.d. risks by a bilinear function of n and $\sqrt{n}$

$$Q(n, q) = E(Y)n + k_q\sigma(Y)\sqrt{n}, \tag{3.8}$$

where the coefficient k_q has to be determined and Y is the generic individual claim severity variable. The first term corresponds to the mean of the aggregate claim amount, while the second term is related to the standard deviation.

Numerical experiments show that the normal approximation is less conservative for high quantiles (i.e. $k_q^N < k_q^P$) when the claim number follows a negative binomial distribution, and the reverse for the Poisson distribution. Based on this study, we choose to approximate quantiles at 85% and 99.5% levels with coefficients $k_{85} = 1$ and $k_{995} = 3$ corresponding respectively to the computation of the MCR and the SCR. Thus, using the approximation (3.8), the minimum and solvency capital requirement (MCR and SCR) is deduced as

$$\mathrm{SCR}_q \approx k_q\sigma(Y)\sqrt{n}.$$

Therefore, we decide to choose the adapted solvency constraint function

$$g_{j,t}^1(x_{j,t}) = \frac{K_{j,t-1} + n_{j,t-1}(x_{j,t} - \pi_{j,t})}{k_{995}\sigma(Y)\sqrt{n_{j,t-1}}} - 1, \tag{3.9}$$

where k_{995} is the solvency coefficient. Indeed, insurers are constrained to have capital above the SCR. When insurers get capital between the MCR and SCR, regulator completely monitor them while when capital drops down the MCR, regulators constraint them to run-off their business. The numerator corresponds to the sum of current capital K_j and expected profit on the in-force portfolio (without taking into account new business). It is easy to see that the constraint $g_j^1(x) \geq 0$ is equivalent to $K_j + n_j(x_{j,t} - \pi_j) \geq k_{995}\sigma(Y)\sqrt{n_j}$, but g_j^1 is normalized with respect to capital, providing a better numerical stability.

3.3 Properties of policyholder behaviors and consequences on insurer portfolio sizes and losses

3.3.1 Properties of the lapse model

In this first subsection, we study the theoretical properties of the lapse model at both policyholder level and insurer level. We derive three propositions for the process $(C_{i,t})_t$ of Policyholder i as well as two propositions for the process $(N_{j,t})_t$ of Insurer j. All proofs are postponed to Appendix 3.8.1.

3.3.1.1 Properties of the lapse model at policy level

For a single policyholder i, the following results show that the insurer choices $(C_{i,t})_t$ of that customer follow a time-inhomogeneous Markov chain.

Proposition 3.3.1. *The choice $(C_{i,t})_t$ of customer i at time t is a time-inhomogeneous Markov chain. Therefore, $P_{\rightarrow}^{(t)} = P_{\rightarrow}(\boldsymbol{x}_1) \times \cdots \times P_{\rightarrow}(\boldsymbol{x}_t)$ is the transition matrix of $C_{i,t}$ knowing $C_{i,0}$. The Markov chain $(C_{i,t})_t$ has an invariant measure.*

This result can be extended to all policyholders simultaneously:

Proposition 3.3.2. *The choice vector $(C_{1,t}, \dots, C_{N,t})_t$ of all customers at time t is a time-inhomogeneous Markov chain with transition matrix $P_{\rightarrow}(\boldsymbol{x}_t)\otimes^n$ (the n-times Kronecker product of the matrix $P_{\rightarrow}(\boldsymbol{x}_t)$).*

From this, the distribution of policyholders for each insurers can be defined.

Proposition 3.3.3. *For all $t \in \mathbb{N}^\star$, $\overline{\boldsymbol{C}}_{j,t} \sim \mathcal{M}_I(n_{j,0}, \tilde{\boldsymbol{p}}_j)$ given $N_{j,0} = n_{j,0}$ where $\tilde{\boldsymbol{p}}_j$ is the jth row of the matrix $P_{\rightarrow}^{(t)}$. In particular, $\overline{\boldsymbol{C}}_{j,t} \sim \mathcal{M}_I(n_{j,t-1}, \boldsymbol{p}_{j\rightarrow}(\boldsymbol{x}_t))$ given $N_{j,t-1} = n_{j,t-1}$.*

3.3.1.2 Properties of the lapse model at portfolio size level

Following Prop. 3.3.3 and Prop. 3.3.1, we show that insurer portfolio sizes $(\boldsymbol{N}_t)_t$ is a (multidimensional) Markov chain in the set of all possible portfolio sizes $\mathcal{S}_N$. Then we also characterize the conditional distribution $\boldsymbol{N}_t \mid \boldsymbol{N}_{t-1} = \boldsymbol{n}$ which in the limit is a multinomial distribution, see Prop. 3.3.4. In Prop. 3.3.5, we derive the conditional distribution of a particular portfolio size $N_{j,t} = m_j | \boldsymbol{N}_{t-1} = \boldsymbol{n}$ of Insurer j. It is difficult to derive other general properties of the distribution of a sum of multinomial or binomial variables with different probabilities $p_{l\rightarrow j}$, except when the size parameters n_j are reasonably large, in which case the normal approximation is appropriate.

Proposition 3.3.4. *The insurer portfolio size vector $(\boldsymbol{N}_t)_t$ is a time-inhomogeneous Markov chain with state space $\mathcal{S}_N$. Let μ be the invariant measure of $(C_{i,t})_t$ (policy level). $\boldsymbol{N}_t \mid \boldsymbol{N} = \boldsymbol{n}$ tends to a multinomial distribution $\mathcal{M}_J(N, \mu)$; and the invariant measure of $(\boldsymbol{N}_t)_t$ (insurer level) is the vector with all probabilities of that multinomial distribution $\mathcal{M}_J(N, \mu)$.*

The probability generating function of $\boldsymbol{N}_t \mid \boldsymbol{N}_{t-1} = \boldsymbol{n}$ is tractable and is given by

$$G^P_{\boldsymbol{N}_t|\boldsymbol{N}_{t-1}=\boldsymbol{n}}(\boldsymbol{z}) = \left(P_{\rightarrow}(\boldsymbol{x}_t) \times \boldsymbol{z}\right)^{\boldsymbol{n}},$$

where $\boldsymbol{z} \in \mathbb{R}^J$ and the exponentiation $\boldsymbol{n}$ is a component-wise operation on the vector $P_{\to}(\boldsymbol{x}_t) \times \boldsymbol{z}$. Note that $\boldsymbol{N}_t \mid \boldsymbol{N}_{t-1} = \boldsymbol{n}$ does not follow a multinomial distribution unless the rows of $P_{\to}(\boldsymbol{x}_t)$ become identical. This is easy to show at least asymptotically. Similarly, the probability generating function of $\boldsymbol{N}_t \mid \boldsymbol{N}_0 = \boldsymbol{n}$ is $G^P_{\boldsymbol{N}_t|\boldsymbol{N}_0=\boldsymbol{n}}(\boldsymbol{z}) = \left(P^{(t)}_{\to} \times \boldsymbol{z}\right)^{\boldsymbol{n}}$.
Finally, the probability mass function is

$$
\begin{aligned}
&P(\boldsymbol{N}_t = \boldsymbol{m}|\boldsymbol{N}_{t-1} = \boldsymbol{n}) \\
=& \sum_{\substack{0\le c_{11},\dots,c_{1J}\le N,\\ s.t.\ \sum_l c_{1l}=n_1}} \cdots \sum_{\substack{0\le c_{J1},\dots,c_{JJ}\le N,\\ s.t.\ \sum_l c_{Jl}=n_J}} \prod_{\substack{j=1,\\ \sum_k c_{kj}=m_j}}^{J} \frac{n_j!}{c_{j1}!\dots c_{jJ}!} \left(p_{j\to 1}(\boldsymbol{x}_t)\right)^{c_{j1}} \dots \left(p_{j\to J}(\boldsymbol{x}_t)\right)^{c_{jJ}},
\end{aligned}
$$

where $N = \sum_i n_i$.

Proposition 3.3.5. *The distribution of $N_{j,t} = m_j|\boldsymbol{N}_{t-1} = \boldsymbol{n}$ has probability mass function*

$$
P(N_{j,t} = m_j|\boldsymbol{N}_{t-1} = \boldsymbol{n}) = \sum_{\substack{0\le c_1,\dots,c_J\le n\\ s.t.\ \sum_l c_l=m_j}} \prod_{l\in J} \binom{n_l}{c_j} \left(p_{l\to j}(\boldsymbol{x}_t)\right)^{c_j} \left(1 - p_{l\to j}(\boldsymbol{x}_t)\right)^{n_l - c_j}, \tag{3.10}
$$

and probability generating function

$$
G^P_{N_{j,t}|\boldsymbol{N}_{t-1}=\boldsymbol{n}}(z) = (1 - p_{1\to j}(\boldsymbol{x}_t) + p_{1\to j}(\boldsymbol{x}_t)z)^{n_1} \times \dots \times (1 - p_{J\to j}(\boldsymbol{x}_t) + p_{J\to j}(\boldsymbol{x}_t)z)^{n_J}. \tag{3.11}
$$

In particular, $E(N_{j,t}|\boldsymbol{N}_{t-1} = \boldsymbol{n}) = n_j \times p_{j\to j}(\boldsymbol{x}_t) + \sum_{l\neq j} n_l \times p_{l\to j}(\boldsymbol{x}_t)$.

3.3.1.3 Additional properties for a constant regulated price vector

Now we turn our attention to the special case of a strongly regulated market. In a regulated market, premiums are either set by regulators to a particular level or subject to an upper and lower bound. Insurance price regulation has important consequences on competition, capital and thus on insurers' solvability (Klein et al. (2002)). Using convergence measure, we investigate long-run market share distribution among insurers benefiting from different consumers demand patterns. In our setting, we focus on the case where regulators set the premium at a single level (for a given line of business). Since premiums are the same for all insurers, premium level defined by the regulators does not have consequences on our lapse model, i.e. $\forall t > 0, \boldsymbol{x}_t = (x, \dots, x)$, and that the regulated price evolves from year to another, i.e. $\boldsymbol{x}_1 = (x, \dots, x)$ $\boldsymbol{x}_2 = (y, \dots, y)$ $\boldsymbol{x}_3 = (z, \dots, z)$,.... Indeed the price sensitivity functions remains the same: both $\overline{f}_j(x_j, x_l) = \mu_j + \alpha_j$ and $\widetilde{f}_j(x_j, x_l) = \tilde{\mu}_j$ from (3.2).

In this subsection, we assume that the price vector is a constant vector $\boldsymbol{x}_t = (x, \dots, x)$ and time dependence on t is thus omitted. Thus, $p_{j\to k}$ simplifies to

$$
p_{j\to k}(\boldsymbol{x}) = \begin{cases} \frac{1}{1+\sum_{l\neq j} e^{f_j}} & \text{if } j = k, \\ \frac{e^{f_j}}{1+\sum_{l\neq j} e^{f_j}} & \text{if } j \neq k, \end{cases} = \begin{cases} \frac{1}{1+(J-1)e^{f_j}} & \text{if } j = k, \\ \frac{e^{f_j}}{1+(J-1)e^{f_j}} & \text{if } j \neq k, \end{cases} = \begin{cases} p_{j\to j} & \text{if } j = k, \\ p_{j\neq} & \text{if } j \neq k, \end{cases} \tag{3.12}
$$

Note that this expression is only function of j (and not of k). Since $\sum_k p_{j\to k} = 1$, we get $p_{j\neq} = (1 - p_{j\to j})/(J - 1)$.

The two following results are directly derived from Propositions 3.3.4 and 3.3.5 in the general case.

Proposition 3.3.6. *$\boldsymbol{N}_t|\boldsymbol{N}_{t-1} = \boldsymbol{n}$ has the following transition probabilities*

$$
P(\boldsymbol{N}_t = \boldsymbol{m}|\boldsymbol{N}_{t-1} = \boldsymbol{n}) = \sum_{\substack{0\le c_{11},\dots,c_{1J}\le N,\\ s.t.\ \sum_l c_{1l}=n_1}} \cdots \sum_{\substack{0\le c_{J1},\dots,c_{JJ}\le N,\\ s.t.\ \sum_l c_{Jl}=n_J}} \prod_{\substack{j=1,\\ \sum_k c_{kj}=m_j}}^{J} \frac{n_j!}{c_{j1}!\dots c_{jJ}!} p_{=}^{c_{jj}} p_{j\neq}^{n_j - c_{jj}}, \tag{3.13}
$$

where $p_= = p_{j\to j}$, $p_{j\neq} = \frac{1-p_=}{J-1}$.

Proposition 3.3.7. *$N_{j,t}|\boldsymbol{N}_{t-1} = \boldsymbol{n}$ is a sum of two binomially distributed random variables $\mathcal{B}(n_j, p_=)$ and $\mathcal{B}(n-n_j, \frac{1-p_=}{J-1})$. In particular, given $\boldsymbol{N}_{t-1} = \boldsymbol{n}$, the probability generating function is*

$$P(N_{j,t} = m_j | N_{j,t} = n_j) = \sum_{k=0^+}^{n_j^-} \binom{n_j}{k} p_=^k (1-p_=)^{n_j-k} \binom{n-n_j}{m_j-k} p_{j\neq}^{m_j-k} (1-p_{j\neq})^{n-n_j-m_j+k}, \quad (3.14)$$

where $p_= = p_{j\to j}$, $p_{j\neq} = \frac{1-p_=}{J-1}$, $0^+ = \max(0, (m_j - (n-n_j))$ *and* $n_j^- = \min(n_j, m_j)$.

The two following propositions at policy level are derived from the corresponding propositions in the general case.

Proposition 3.3.8. *The choice $(C_{i,t})_t$ of customer i at time t is a time-homogeneous Markov chain when $\boldsymbol{x}_t = \boldsymbol{x}$. In particular, $P_\to^{(t)} = (P_\to(\boldsymbol{x}))^t$. There exists a unique invariant measure μ for $(C_{i,t})_t$ given by*

$$\mu = \left(\frac{c_1^\Pi}{c_1^\Pi + \cdots + c_J^\Pi}, \ldots, \frac{c_J^\Pi}{c_1^\Pi + \cdots + c_J^\Pi} \right) \text{ with } c_i^\Pi = \prod_{j=1, j\neq i}^{J} p_{j\neq}. \quad (3.15)$$

If in addition, the choice probabilities $p_{j\to k}$ are identical for all Insurer j, then $\mu = (1/J, \ldots, 1/J)$.

Proposition 3.3.9. *The choice vector $(C_{1,t}, \ldots, C_{N,t})_t$ of all customers at time t is a time-homogeneous Markov chain when $\boldsymbol{x}_t = \boldsymbol{x}$. The invariant measure is obtained by applying N times the Kronecker product $\mu\otimes^N$ of μ defined in Equation (3.15). If, in addition, the choice probability $p_{j\to k}$ are identical across insurers, then $\mu\otimes^N = (1/J^N, \ldots, 1/J^N)$.*

Therefore, we can deduce two propositions at portfolio level from the two previous propositions.

Proposition 3.3.10. *For all $t \in \mathbb{N}^\star$, $\overline{\boldsymbol{C}}_{j,t} \sim \mathcal{M}_J(n_{j,0}, \tilde{\boldsymbol{p}}_j)$ given $N_{j,0} = n_{j,0}$ where $\tilde{\boldsymbol{p}}_j$ is the jth row of the matrix $(P_\to)^t$. In particular, $\overline{\boldsymbol{C}}_{j,t} \sim \mathcal{M}_J(n_{j,t-1}, \boldsymbol{p}_{j\to}(\boldsymbol{x}))$ given $N_{j,t-1} = n_{j,t-1}$.*

Proposition 3.3.11. *The portfolio size vector $(\boldsymbol{N}_t)_t$ at time t is a time-homogeneous Markov chain with state space $\mathcal{S}_N$. The probability generating function of $\boldsymbol{N}_t \mid \boldsymbol{N}_{t-1} = \boldsymbol{n}$ is $G^P_{\boldsymbol{N}_t|\boldsymbol{N}_{t-1}=\boldsymbol{n}}(\boldsymbol{z}) = (P_\to \times \boldsymbol{z})^{\boldsymbol{n}}$, where $\boldsymbol{z} \in \mathbb{R}^J$ and the exponentiation $^{\boldsymbol{n}}$ is a component-wise operation.*

Let μ be the invariant measure at individual level in (3.15). $\boldsymbol{N}_t \mid \boldsymbol{N}_0 = \boldsymbol{n}$ converges in distribution to a multinomial distribution with parameters $\mathcal{M}_J(N, \mu)$. Therefore, the invariant measure of $(\boldsymbol{N}_t)_t$ (insurer level) is the vector with all probabilities of that multinomial distribution $\mathcal{M}_J(N, \mu)$.

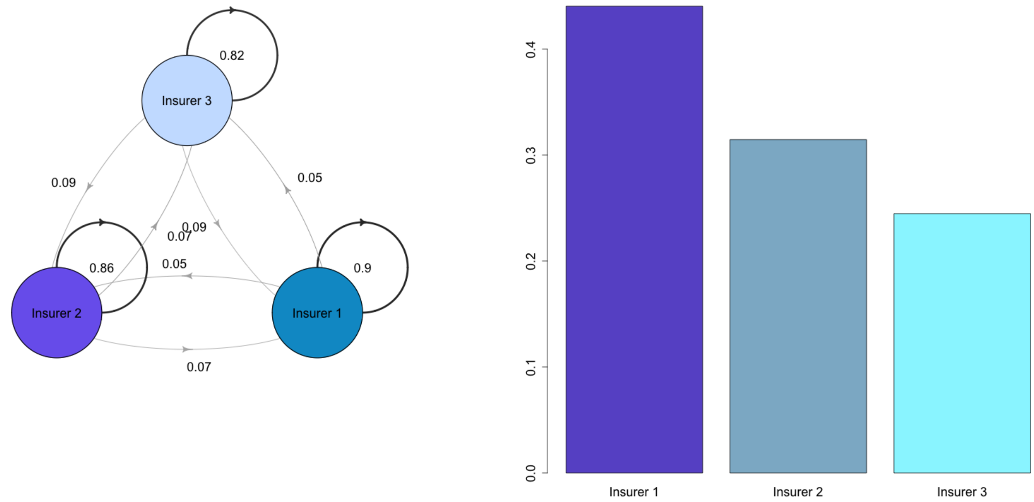
In other words for a constant price vector $\boldsymbol{x}$ and a large t, the distribution of $\boldsymbol{N}_t$ is independent of $\boldsymbol{N}_0$ and for $\boldsymbol{n} \in \mathcal{S}_N$, the probability mass function is

$$P(\boldsymbol{N}_t = \boldsymbol{n}) = \frac{N!}{n_1! \ldots n_J!} \mu_1^{n_1} \ldots \mu_J^{n_J} \text{ with } \mu \text{ defined in (3.15).} \quad (3.16)$$

Furthermore, $E(N_{i,t}) = \mu_i N$ and $Var(N_{i,t}) = \mu_i(1-\mu_i)N$. The last equation is in line with the so-called product form solution of a closed Markovian network of queues with batch services. In fact, Theorem 2 of Henderson et al. (1990) show a similar form to (3.16), where the invariance condition $\mu^T = \mu^T P_\to$ is called the balance equation.

In the special case of identical insurers leading to $\mu = (1/J, \ldots, 1/J)$, we further obtain

$$P(\boldsymbol{N}_t = \boldsymbol{n}) = \frac{N!}{n_1! \ldots n_J!} \left(\frac{1}{I}\right)^N, \; E(N_{i,t}) = \frac{N}{J}, \; Var(N_{i,t}) = \frac{N(J-1)}{J^2}.$$



(a) Transition matrix for a policyholder (b) Invariant measure

Figure 3.1 – The Markov chain of a single policyholder $(C_{i,t})_t$

3.3.1.4 Illustration purposes of a simple market

For illustration, let us consider a small three-insurer market with 6 policyholders. For a single client, the Markov chain $(C_{i,t})_t$ is plotted in Figure 3.1a.
The lapse/renewal function (3.1) is computed using a price sensitivity function $\tilde{f}_j(x, y)$ in (3.2).

The lapse probabilities have been chosen such that the central lapse rates are 10%, 14% and 18% respectively. If all insurers propose the same premium (1.25 when 1 is the pure premium), and lapse rates increase by 5 points when premium increase by 5%. These lapse parameters correspond to those used in Dutang et al. (2013). In this setting, we assume that Insurer 1 has a better reputation (solvency, brand value) than Insurer 2 and Insurer 3 so that Insurer 1 retains more customers than competitors (when premium are identical). As plotted in Figure 3.1a, policyholders of Insurer 1 have a renewal probability of 90% and lapse towards Insurer 2 or Insurer 3 with a probability of $p_{1\neq} = 5\%$.

From Equation (3.15), the invariant measure for a single policyholder rewrites as

$$\mu = \left(\frac{p_{2\neq}p_{3\neq}}{p_{2\neq}p_{3\neq}+p_{1\neq}p_{3\neq}+p_{1\neq}p_{2\neq}}, \frac{p_{1\neq}p_{3\neq}}{p_{2\neq}p_{3\neq}+p_{1\neq}p_{3\neq}+p_{1\neq}p_{2\neq}}, \frac{p_{1\neq}p_{2\neq}}{p_{2\neq}p_{3\neq}+p_{1\neq}p_{3\neq}+p_{1\neq}p_{2\neq}} \right).$$

Considering the parametrization $p_{1\neq} = 0.05$, $p_{2\neq} = 0.07$, $p_{3\neq} = 0.09$ leads to $\mu = (0.441, 0.314, 0.245)$. Figure 3.1b, depicts a barplot with the invariant measure of a (single) policyholder. For instance, in the long run, the probability of a policyholder to be with Insurer 1 is 44.1%. This convergence is relatively quick. Below, we compute the transition probability (at policy level) after 10, 20 and 30 periods. We observe the geometric convergence towards μ.

$$P_{\rightarrow}^{(10)} = \begin{pmatrix} 0.520 & 0.263 & 0.218 \\ 0.368 & 0.390 & 0.242 \\ 0.392 & 0.312 & 0.296 \end{pmatrix}, \ P_{\rightarrow}^{(20)} = \begin{pmatrix} 0.452 & 0.307 & 0.241 \\ 0.429 & 0.324 & 0.246 \\ 0.434 & 0.317 & 0.249 \end{pmatrix}, \ P_{\rightarrow}^{(30)} = \begin{pmatrix} 0.442 & 0.314 & 0.244 \\ 0.439 & 0.316 & 0.245 \\ 0.440 & 0.315 & 0.245 \end{pmatrix}.$$

The set of portfolio sizes is

$$\mathcal{S}_N = \left\{ (n_1, n_2, n_3) \in \mathbb{N}^3, n_1 + n_2 + n_3 = 6 \right\},$$

which has $\binom{6+3-1}{6} = 28$ elements. In other words, all possible portfolio sizes are integer-valued 3-component vectors that sum up to 6. All possible portfolio sizes are given in Table 3.1. The Markov chain of insurers portfolio sizes $(\boldsymbol{N}_t)_t$ is plotted in Figure 3.2.

As already mentionned, all states of the Markov chain of insurers portfolio sizes $(\boldsymbol{N}_t)_t$ are communicating and recurrent. Therefore, any probability to go from $\boldsymbol{n} = (n_1, n_2, n_3)$ to $\boldsymbol{m} = (m_1, m_2, m_3)$ is strictly positive for $\boldsymbol{n}, \boldsymbol{m} \in \mathcal{S}_N$. For readability, we do not plot all transition probabilities of the 28×28 transition matrix of $(\boldsymbol{N}_t)_t$ in Figure 3.2. The full transition matrix is given in Appendix 3.8.1.5. In Figure 3.2, we set a threshold of 5% and plot only arrows with the probability value above that threshold. In Appendix 3.8.1.5, we plot the transition matrix with a threshold of 1%, see Figure 3.12. To emphasize the highest probabilities, the thickness and the darkness of an arrow are increasing with its probability value, see the legend of Figure 3.2.

Irrespective of the state, we observe that the highest transition probabilities are the probabilities to remain in the same state, that is policyholders' moves are neutral for Insurers. However, for the state $(0, 0, 6)$, the probability to remain in that state is only 39% whereas for the state $(6, 0, 0)$, it is 59%. This is a direct consequence of $p_{i\neq}$ since Insurer 1 is the preferred insurer.

6,0,0	5,0,1	3,1,2	0,3,3
5,1,0	4,1,1	2,2,2	2,0,4
4,2,0	3,2,1	1,3,2	1,1,4
3,3,0	2,3,1	0,4,2	0,2,4
2,4,0	1,4,1	3,0,3	1,0,5
1,5,0	0,5,1	2,1,3	0,1,5
0,6,0	4,0,2	1,2,3	0,0,6

Table 3.1 – All possible portfolio sizes with 3 insurers and 6 policyholders

Using Equation (3.13), we compute the invariant measure of insurers portfolio sizes $(\boldsymbol{N}_t)_t$. The invariant measure is plotted in Figure 3.3. Colors for the state in Figure 3.2 are chosen such that the darkest (from light blue to slate blue) colors correspond to the most probable states according to the invariant measure of $(\boldsymbol{N}_t)_t$. According to the invariant measure, the top 5 most probable states are $(4, 1, 1)$ $(2, 3, 1)$ $(3, 1, 2)$ $(2, 2, 2)$ $(3, 2, 1)$. In these states, Insurer 1 is either an absolute leader (portfolio size greater than 2) or a co-leader (all insurer of size 2) except the fourth most probable state $(2, 3, 1)$ see Table 3.2.

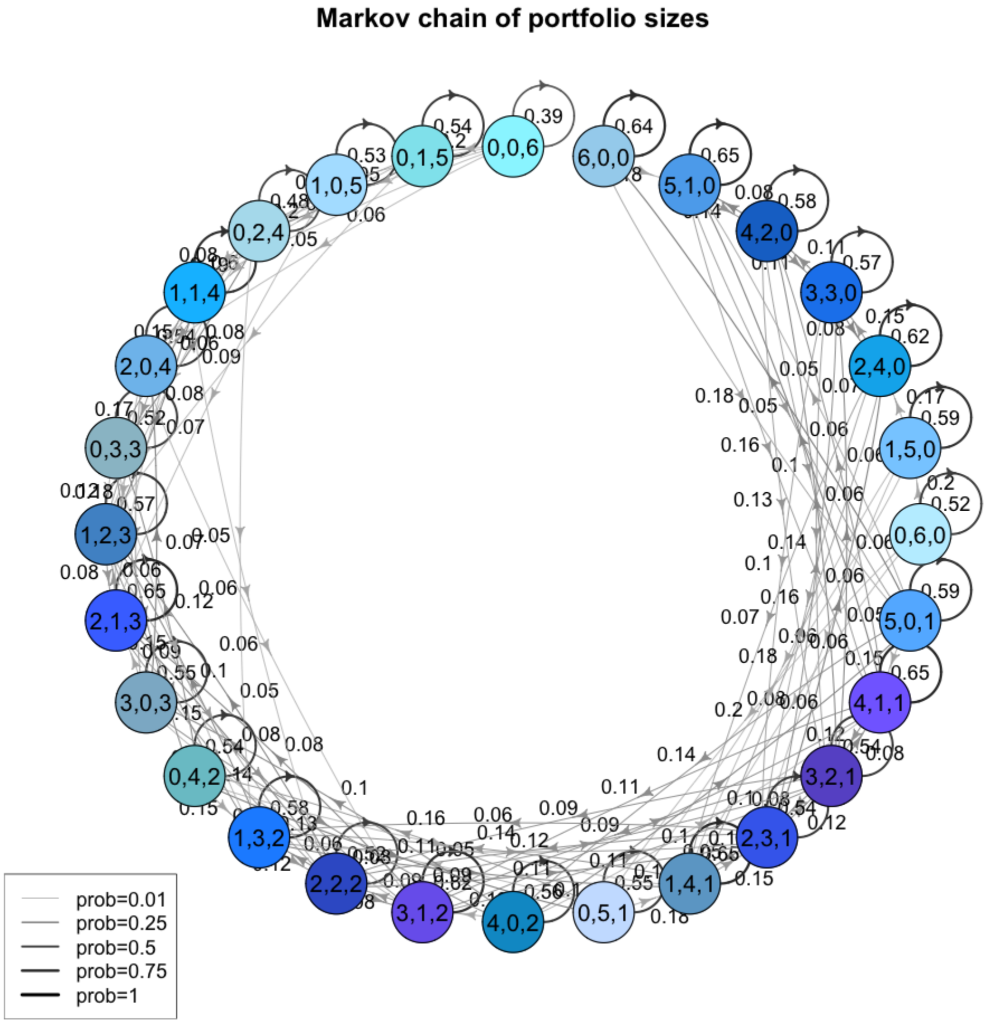


Figure 3.2 – The Markov chain of insurers portfolio sizes $(\boldsymbol{N}_t)_t$ (transition probabilities lower than 5% are not plotted and line thickness increases the probability value)

	Leadership			co-leadership		
	states	nb. states	cum. prob.	states	nb. states	cum. prob.
Ins. 1	(6,0,0);(5,1,0);(4,2,0);(5,0,1) (4,1,1);(3,2,1);(4,0,2);(3,1,2)	8	0.46	(3,0,3);(3,3,0) (2,2,2)	3	0.18
Ins. 2	(2,4,0);(1,5,0);(0,6,0);(2,3,1) (1,4,1);(0,5,1);(1,3,2);(0,4,2)	8	0.22	(0,3,3);(3,3,0) (2,2,2)	3	0.17
Ins. 3	(2,1,3);(1,2,3);(2,0,4);(1,1,4) (0,2,4);(1,0,5);(0,1,5);(0,0,6)	8	0.13	(0,3,3);(3,0,3) (2,2,2)	3	0.14

Table 3.2 – Long-term leadership and co-leadership probabilities by insurer

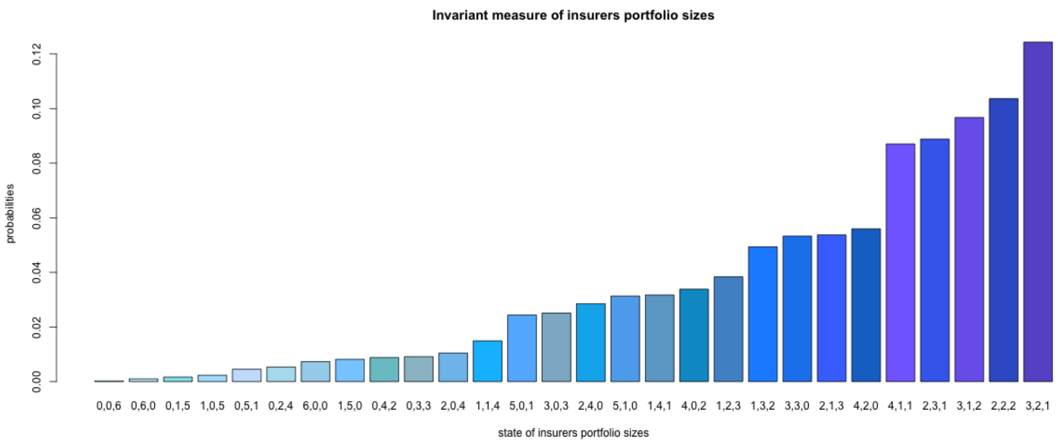


Figure 3.3 – The invariant measure of insurer portfolio sizes $(\boldsymbol{N}_t)_t$

3.3.1.5 Additional properties for a time-independent price vector

Finally, we consider the case of a deviation from a regulated price or a market-accepted level by one insurer, yet the other competitors remain at the same level. Therefore, we restrict our analysis to $\boldsymbol{x}_t = (x, \rho x, \dots, \rho x)$ with $\rho > 0$ a fixed parameter. Note that the case where $\boldsymbol{x} = (x_1, \dots, x_J)$ without assuming that some insurers propose the same price is very complex and should be solved numerically. We cannot use the simplification of transition probability $p_{j\to k}$ in this case, yet the transition matrix $P_{\to}$ will be a circulant matrix.

The dependence on time t is omitted. In that case using (3.2), we have two cases to consider either

$$\begin{cases} \overline{f}_1(x_1, x_l) = \mu_1 + \alpha_1 \frac{x_1}{x_l} = \mu_1 + \frac{\alpha_1}{\rho} \\ l \neq 1, \overline{f}_j(x_j, x_l) = \mu_j + \alpha_j \frac{x_j}{x_l} = \mu_j + \alpha_j \\ l = 1, \overline{f}_j(x_j, x_1) = \mu_j + \alpha_j \frac{x_j}{x_1} = \mu_j + \alpha_j \rho \end{cases} \text{ or } \begin{cases} \widetilde{f}_1(x_1, x_l) = \mu_1 + \alpha_1 (x_1 - x_l) = \mu_1 + \alpha_1 (1-\rho) x \\ l \neq 1, \widetilde{f}_j(x_j, x_l) = \mu_j + \alpha_j (x_j - x_l) = \mu_j \\ l = 1, \widetilde{f}_j(x_j, x_1) = \mu_j + \alpha_j (x_j - x_1) = \mu_j + \alpha_j (\rho - 1) x. \end{cases} \tag{3.17}$$

So there are three possible exponents denoted respectively $f_1, f_j, f_{j,\rho}$. Therefore, $p_{j\to k}$ is only a function of j and not of k and simplifies to

$$p_{1\to j}(\boldsymbol{x}) = \begin{cases} \frac{1}{1+\sum\limits_{l\neq j} e^{f_1}} & \text{if } j = 1, \\ \frac{e^{f_1}}{1+\sum\limits_{l\neq j} e^{f_1}} & \text{if } j \neq 1, \end{cases} = \begin{cases} \frac{1}{1+(I-1)e^{f_1}} & \text{if } j = 1, \\ \frac{e^{f_1}}{1+(I-1)e^{f_1}} & \text{if } j \neq 1, \end{cases} = \begin{cases} p_{1\to 1} & \text{if } j = 1, \\ p_{1\neq} & \text{if } j \neq 1, \end{cases} \tag{3.18}$$

$$p_{j\to l}(\boldsymbol{x}) = \begin{cases} \frac{1}{1+(I-2)e^{f_j}+e^{f_{j,\rho}}} & \text{if } j=l, \\ \frac{e^{f_j}}{1+(I-2)e^{f_j}+e^{f_{j,\rho}}} & \text{if } j\neq l\neq 1, \\ \frac{e^{f_{j,\rho}}}{1+(I-2)e^{f_j}+e^{f_{j,\rho}}} & \text{if } j\neq 1, l=1, \end{cases} = \begin{cases} p_{j\to j} & \text{if } j=k, \\ p_{j\neq} & \text{if } j\neq l\neq 1, \\ p_{j\to 1} & \text{if } j\neq 1, l=1, \end{cases} \tag{3.19}$$

Now we are able to derive the invariant measure in the following proposition for the case $\boldsymbol{x}_t = (x, \rho x, \dots, \rho x)$.

Proposition 3.3.12. *The choice $(C_{i,t})_t$ of customer i at time t is a time-homogeneous Markov chain when $\boldsymbol{x}_t = \boldsymbol{x}$. In particular, $P_{\to}^{(t)} = (P_{\to}(\boldsymbol{x}))^t$. There exists a unique invariant measure μ for $(C_{i,t})_t$ given by*

$$\mu_1 = \frac{d_{-1}^{\Pi} - \sum_{j=2}^{J} d_{-1,-j}^{\Pi} p_{j\neq}}{d_{-1}^{\Pi} + \sum_{j=2}^{J} d_{-1,-j}^{\Pi}(p_{1\neq} - p_{j\neq})}, \mu_j = \frac{d_{-1,-j}^{\Pi} p_{1\neq}}{d_{-1}^{\Pi} + \sum_{j=2}^{J} d_{-1,-j}^{\Pi}(p_{1\neq} - p_{j\neq})}, j = 2,\dots,J. \tag{3.20}$$

with $d_l = (J-1)p_{l\neq} + p_{l\to 1}$ and $d_{-1,-j}^{\Pi} = \prod_{l=2,l\neq j}^{J} d_l$, $d_{-1}^{\Pi} = \prod_{l=2}^{J} d_l$.

In the special case of identical insurers, the invariant measure becomes $\mu_1 = \frac{p_{2\to 1}}{p_{2\to 1}+(J-1)p_{1\neq}}$, $\mu_j = \frac{p_{1\neq}}{p_{2\to 1}+(J-1)p_{1\neq}}$ for $j = 2,\dots,I$.

Let us analyze the case $\rho > 1$, i.e. Insurer 1 is cheaper than others. From (3.17), we deduce that $f_j < f_{j,\rho}$. Therefore, we can order the transition probabilities

$$e^{f_j} < e^{f_{j,\rho}} \Rightarrow \forall l, j \neq 1, p_{j\neq} = p_{j\to l} < p_{j\to 1}.$$

In order to easily compare transition probabilities, we further assume that insurers are identical, with lapse parameters $\mu_j = \mu_1$ and $\alpha_j = \alpha_1$. So $f_1 < f_2 < f_{2,\rho}$ yields

$$\begin{cases} e^{f_{2,\rho}} > e^{f_1} \\ e^{f_1+f_{2,\rho}} > e^{f_1+f_2} \end{cases} \Rightarrow e^{f_{2,\rho}} + (I-2)e^{f_1+f_{2,\rho}} > e^{f_1} + (I-2)e^{f_1+f_2}$$

$$\Leftrightarrow e^{f_{2,\rho}}(1+(I-1)e^{f_1}) > e^{f_1}(1+(I-2)e^{f_2}+e^{f_{2,\rho}}) \Leftrightarrow \frac{e^{f_{2,\rho}}}{1+(I-2)e^{f_2}+e^{f_{2,\rho}}} > \frac{e^{f_1}}{1+(I-1)e^{f_1}}.$$

Since the invariant measure simplifies in that case to $\mu_1 = \frac{p_{2\to 1}}{p_{2\to 1}+(I-1)p_{1\neq}}$, $\mu_j = \frac{p_{1\neq}}{p_{2\to 1}+(I-1)p_{1\neq}}$, we have $\mu_1 > \mu_j$ for $j = 2,\dots,I$.

Proposition 3.3.13. *The portfolio sizes (vector) $(\boldsymbol{N}_t)_t$ at time t is a time-homogeneous Markov chain with state space $\mathcal{S}_N$. Let μ be the invariant measure at individual level in (3.20). $\boldsymbol{N}_t \mid \boldsymbol{N}_0 = \boldsymbol{n}$ converges in distribution to a multinomial distribution with parameters $\mathcal{M}_J(N,\mu)$. Therefore, the invariant measure of $(\boldsymbol{N}_t)_t$ (insurer level) is the vector with all probabilities of that multinomial distribution $\mathcal{M}_J(N,\mu)$.*

Using the same setting as in Section 3.3.1.4, we illustrate this result. Consider a premium vector $\boldsymbol{x} = (1.25, 1.25, 1.1)$, that Insurer 3 (the least preferred insurer in the previous setting) proposes the lowest premium, $\rho = 1.136364$. We can then apply Propostion 3.3.12 to derive the invariant measure. Note that it is not Insurer 1 who deviates from ρx but Insurer 3.

From Equation (3.20), taking into account the index change (i.e. $d_{-3,-2} = d_1 = 2p_{1\neq} + p_{1\to 3}$, $d_{-3,-1} = d_2 = 2p_{2\neq} + p_{2\to 3}$, $d_{-3} = d_1 d_2$) the invariant measure for a single policyholder rewrites as

$$\mu_3 = \frac{(2p_{1\neq}+p_{1\to 3})(2p_{2\neq}+p_{2\to 3}) - p_{1\neq}(2p_{2\neq}+p_{2\to 3}) - p_{2\neq}(2p_{1\neq}+p_{1\to 3})}{\sum_i \mu_i},$$

$$\mu_2 = \frac{p_{3\neq}(2p_{1\neq} + p_{1\to 3})}{\sum_i \mu_i}, \mu_1 = \frac{p_{3\neq}(2p_{2\neq} + p_{2\to 3})}{\sum_i \mu_i}.$$

Numerically, we get

$$P_{\to} = \begin{pmatrix} 0.82 & 0.05 & 0.14 \\ 0.06 & 0.78 & 0.15 \\ 0.05 & 0.05 & 0.91 \end{pmatrix}, \mu = \begin{pmatrix} 0.295 \\ 0.228 \\ 0.477 \end{pmatrix}.$$

Therefore, Insurer 3 is largely preferred by policyholders both in short-term and long-term horizons.

Using $P_{\to}$, we can also compute the transition matrix of insurers' portfolio size $(\boldsymbol{N}_t)_t$. And using μ, we can again compute the invariant measure of $(\boldsymbol{N}_t)_t$. In Figure 3.4, we plot the transition matrix with the same graphic procedure. At one-period level, Insurer 3 is the most preferred insurer: most states on the left-hand of Figure 3.4 have darkest colors. This contrasts with the situation in Figure 3.2.

The invariant measure is plotted in Figure 3.5 and the leadership probabilities are given in Table 3.3. In Figure 3.5, we observe that the most probable state is $(1, 1, 4)$ when Insurer 3 gets two thirds of the market. In the top 5 most probable states, Insurer 3 is always a leader (strictly). In Table 3.3, the leadership of Insurer 3 is even more striking with 76% of leadership compared to competitors.

So, Insurer 3 is largely the big winner of the price setting. Asking a premium 1.1 when competitors set 1.25 changes completely the previous situation of equalled premium of 1.25. Obviously, competitors will not remain inactive or blind of that situation and promptly react to an insurer stealing the market.

	Leadership			co-leadership		
	states	nb. states	cum. prob.	states	nb. states	cum. prob.
Ins. 1	(6,0,0);(5,1,0);(4,2,0);(5,0,1) (4,1,1);(3,2,1);(4,0,2);(3,1,2)	8	0.08	(3,0,3);(3,3,0) (2,2,2)	3	0.10
Ins. 2	(2,4,0);(1,5,0);(0,6,0);(2,3,1) (1,4,1);(0,5,1);(1,3,2);(0,4,2)	8	0.05	(0,3,3);(3,3,0) (2,2,2)	3	0.08
Ins. 3	(2,1,3);(1,2,3);(2,0,4);(1,1,4) (0,2,4);(1,0,5);(0,1,5);(0,0,6)	8	0.76	(0,3,3);(3,0,3) (2,2,2)	3	0.12

Table 3.3 – Long-term leadership and co-leadership probabilities by insurer

3.3.2 Properties of the loss model

In this subsection, we focus on the properties of the loss model.

Proposition 3.3.14. *The aggregate claim amount per insurer $S_{j,t}$ at period t is a compound distribution of the same kind as the individual loss amount $Y_{i,t}$.*

Hence, the insurer aggregate claim amount $S_{j,t}(\boldsymbol{x}_t)$ is a compound distribution $\sum_{l=1}^{\widetilde{M}_{j,t}(\boldsymbol{x}_t)} Z_l$ such that the total claim number $\widetilde{M}_{j,t}$: (i) a Poisson-lognormal with $\widetilde{M}_{j,t}(\boldsymbol{x}_t) \sim \mathcal{P}(N_{j,t}(\boldsymbol{x}_t)\lambda)$; (ii) a negative binomial-lognormal with $\widetilde{M}_{j,t}(\boldsymbol{x}_t) \sim \mathcal{NB}(N_{j,t}(\boldsymbol{x}_t)r, p)$.

Taking into account the time t, the insurer aggregate claim amount t $S_{j,t}$ for year is the compound $\sum_{l=1}^{\widetilde{M}_{j,t}} Z_{j,l,t}$ given that $N_{j,t} = n_{j,t}$. We assume that the claim number $M_{j,t}$ and claim severity $Z_{j,l,t}$ follow

- PG: a Poisson-gamma with $\widetilde{M}_{j,t} \sim \mathcal{P}(n_{j,t}\lambda)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{G}(\mu_1, \sigma_1)$,
- NBG: a negative binomial-gamma with $\widetilde{M}_{j,t} \sim \mathcal{NB}(n_{j,t}r, p)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{G}(\mu_1, \sigma_1)$.
- PLN: a Poisson-lognormal with $\widetilde{M}_{j,t} \sim \mathcal{P}(n_{j,t}\lambda)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{LN}(\mu_2, \sigma_2^2)$,
- NBLN: a negative binomial-lognormal with $\widetilde{M}_{j,t} \sim \mathcal{NB}(n_{j,t}r, p)$ and $(Z_{j,l,t})_l \overset{\text{i.i.d.}}{\sim} \mathcal{LN}(\mu_2, \sigma_2^2)$.

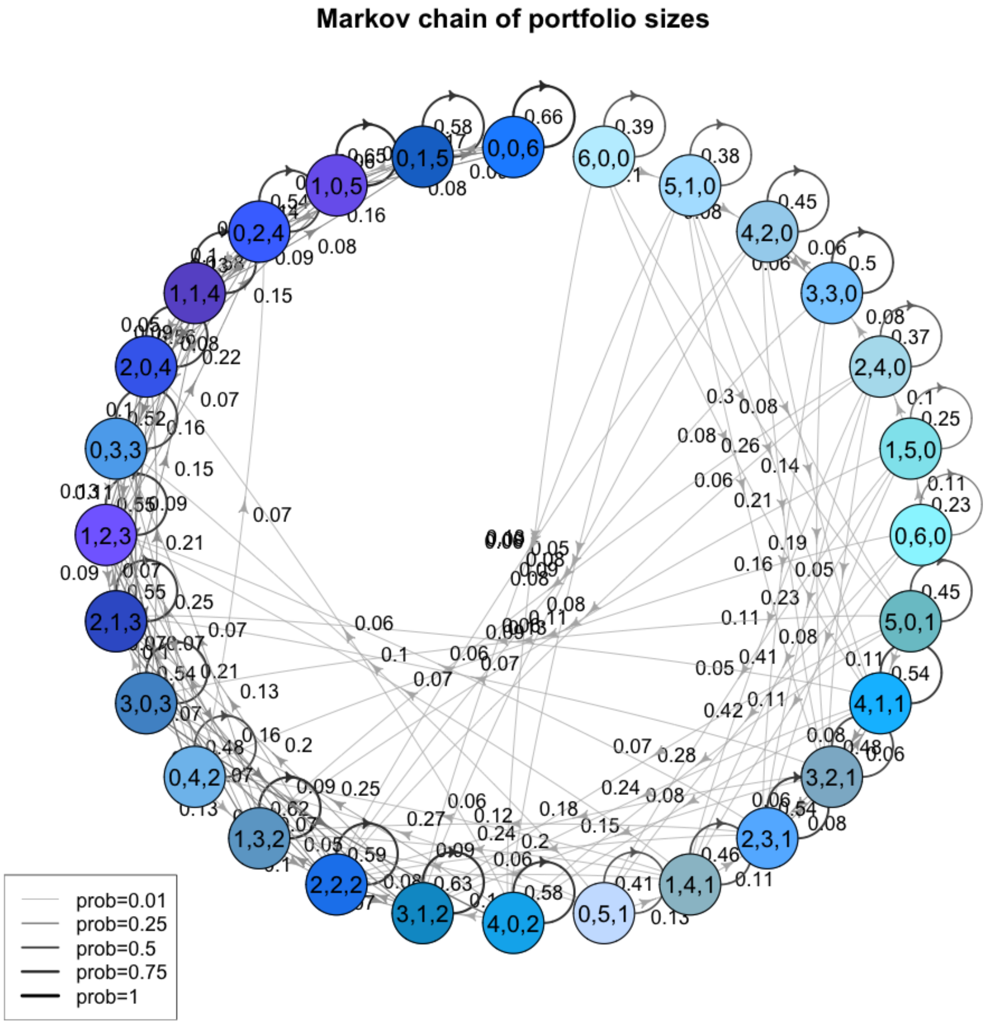


Figure 3.4 – The Markov chain of insurers portfolio sizes $(\boldsymbol{N}_t)_t$ (transition probabilities lower than 5% are not plotted and line thickness increases the probability value)

Therefore, if we assume $M_i \overset{\text{i.i.d.}}{\sim} \mathcal{P}(\lambda)$, then $\widetilde{M}_j$ follows a Poisson distribution $\mathcal{P}(n_j\lambda)$, while if we assume $M_i \overset{\text{i.i.d.}}{\sim} \mathcal{NB}(r,p)$, then $\widetilde{M}_j$ follows a negtive binomial distribution $\mathcal{NB}(rn_j,p)$. In addition to considering two frequency distributions (Poisson and negative binomial), we also consider two loss severity distributions: (i) gamma and (ii) lognormal.

The expectation and the variance of the aggregate claim amount can be easily derived

$$E(S_{j,t}) = E(\widetilde{M}_{j,t})E(Z),\ Var(S_{j,t}) = Var(\widetilde{M}_{j,t})E^2(Z) + E(\widetilde{M}_{j,t})Var(Z).$$

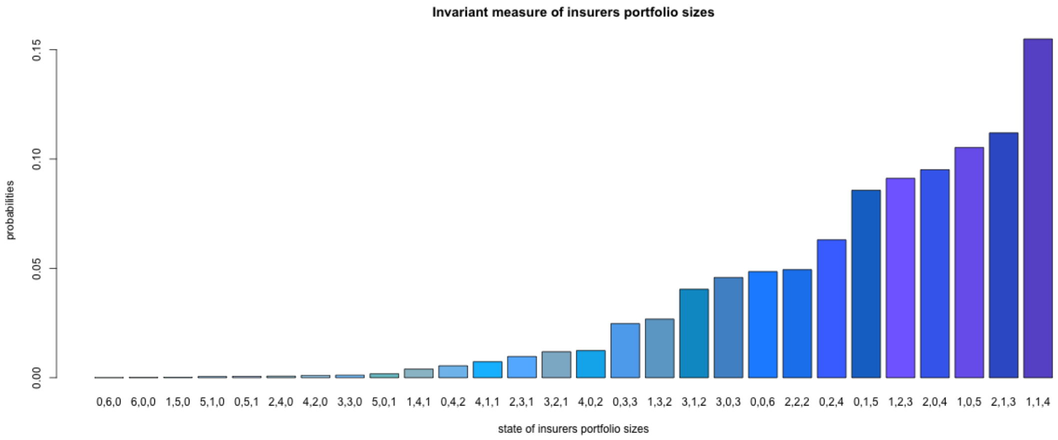


Figure 3.5 – The invariant measure of insurers' portfolio sizes $(\boldsymbol{N}_t)_t$

The survival function of the aggregate claim amount is given by

$$P(S_{j,t} > s) = \sum_{0\leq m\leq n} P(N_{j,t} = m) \sum_{0\leq k} P\left(\widetilde{M}_{j,t} = k | N_{j,t} = m\right) P\left(\sum_{l=1}^{k} Z_l > s\right). \tag{3.21}$$

The expression is explicit in the case of gamma variables Z_l, where the term $\sum_{l=1}^{k} Z_l$ still follows a gamma distribution $\mathcal{G}(k\mu_1, \sigma_1^2)$. When it exists, the moment generating function is given by

$$G^M_{S_{j,t}}(z) = G^P_{N_{j,t}}\left(G^P_{\widetilde{M}_{j,t}}\left(G^M_Z(z)\right)\right),$$

where $G^M(.)$ denotes the moment generating function with $G^M_Z(z) = (\mu_1/(\mu_1 - z))^{\sigma_1}$. If we consider lognormal losses, the moment generating function is not explicit. Furthermore, the distribution of the sum $\sum_l Z_l$ is not a lognormal distribution, but a large number of approximations exist, see Asmussen et al. (2012) and the references therein.

In the special case where $\boldsymbol{x} = (x, \dots, x)$ is a constant vector and f_j is independent of j, using (3.14) we have for all s

$$\begin{aligned} P(S_j > s) \quad = \quad & \sum_{0\leq m\leq n} \sum_{l=0^+}^{n_j^-} \binom{n_j}{l} (p_=)^l (1-p_=)^{n_j - l} \binom{n-n_j}{m-l} \left(\frac{1-p_=}{J-1}\right)^{m-l} (p_{j\neq})^{n-n_j-m+l} \\ & \sum_{0\leq k} P\left(\widetilde{M}_j = k | N_j = m\right) P\left(\sum_{l=1}^{k} Z_l > s\right), \end{aligned}$$

where $0^+ = 0 \vee (m_j - (n - n_j))$ and $n_j^- = n_j \wedge m_j$. As explained above, the random variable $\widetilde{M}_j = k \mid N_j = m$ follows a Poisson distribution $\mathcal{P}(m\lambda)$ or a negative binomial distribution $\mathcal{NB}(mr, p)$. Therefore, the probability can be computed explicitly when Z_l follows a gamma distribution or approximatively when Z_l follows a lognormal distribution.

When $\boldsymbol{x} = (x, \rho x, \dots, \rho x)$, we can also have an explicit expression of the distribution of $N_j(\boldsymbol{x})|\boldsymbol{N}_{t-1} = \boldsymbol{n}$ which will lead to a similar expression of the survival function of the aggregate loss.

Proposition 3.3.15. *If Insurer j is the cheapest insurer (i.e. for all $k \neq j$, $x_j \leq x_k$), then the loss average by policy of Insurer j is stochastically ordered as*

$$\forall k \neq j, \frac{1}{N_j(\boldsymbol{x})} \sum_{i=1}^{N_j(\boldsymbol{x})} Y_i \leq_{cx} \frac{1}{N_k(\boldsymbol{x})} \sum_{i=1}^{N_k(\boldsymbol{x})} Y_i.$$

This proposition remains us results from Wang et al. (2010), where they find in a dynamic model that larger firms experience less premium variation than smaller firms. Indeed, since premium equilibrium is highly correlated to loss history, we can reasonably expect that, in the long run, firm proposing lowest premium benefits from a largest market share. The loss average is therefore less volatile, allowing insurer to be less constrained by solvency regulation and potential loss shocks and leading, in a dynamic pattern, to a more stable premium.

3.4 Properties of insurer underwriting strategies

In this section, we derive theoretical properties of the insurance game with respect to insurers. Firstly, we focus on the one-period game in a general setting, then we study the special case of two insurers. As for the previous section, all proofs are postponed to Appendix 3.8.2.

3.4.1 Properties of the premium equilibrium in the one-period game (the general case)

The first proposition shows that the premium equilibrium of the one-period game admits a unique premium equilibrium.

Proposition 3.4.1. *Consider the J-player insurance game with objective function* (3.7) *and solvency constraint* (3.9) *where the market proxy is either the arithmetic mean* (3.5) *or the weighted mean* (3.6) *of the competitors' price. The game admits a unique (Nash) premium equilibrium when $\beta_j > 0$ for $j = 1, \dots, J$.*

Furthermore, in a very special case of no active constraint, a simple linear system can be solved to get the premium equilibrium, see Appendix 3.8.2.1 for details. In this case, the premium equilibrium does not depend (directly) on capital levels and the loss model. It may depend on portfolio sizes only when the market proxy is the weighted mean (3.6) of the competitors' price, not for the arithmetic mean (3.5).

Proposition 3.4.2. *Let $\boldsymbol{x}^\star$ be the premium equilibrium of the J-player insurance game with objective function and solvency constraint function defined in Equations* (3.7) *and* (3.9)*, respectively, where the market proxy is either the arithmetic mean* (3.5) *or the weighted mean* (3.6) *of the competitors' price. When no constraint functions are active, the premium equilibrium $\boldsymbol{x}^\star$ solves a linear system of equations $M\boldsymbol{x}^\star = \boldsymbol{v}$.*

It is important to note that the linear system deduced in the previous proposition is **not equivalent** to the original Nash equilibrium since it relies on the strong assumptions of non-active constraint functions. Nevertheless, it helps to understand the effect of some parameters on the premium equilibrium in that particular case.

Getting a linear system for the premium equilibrium, we are looking for a **necessary and sufficient** condition. The linear system for the premium equilibrium has a solution when the determinant of M is non null. Using $M = M_1 M_2$ from Appendix 3.8.2.1, we have $\det(M) = \det(M_1)\det(M_2)$. M_2 being diagonal yields a positive determinant $\det(M_2) = \prod_i w_i > 0$ for the weights considered. So $\det(M_0) = 0$ is equivalent to $\det(M_1) = 0$. As shown in Appendix 3.8.2.4, we have

$$\det(M_1) = 0 \Leftrightarrow 2\beta_1 w_{-1}^{\Sigma} \tilde{\beta}_1 = w_1 \sum_{j=2}^{J} (1 + \beta_j)\tilde{\beta}_j.$$

Many β_j solve this equation. But there is a unique solution when $w_j = w$ is constant and β_j's are all identical. Indeed, $\beta_j = 1$ is a solution for $\det(M_1) = 0$ both when the market proxy is the arithmetic mean (3.5) or the weighted mean (3.6).

Again, when the determinant of M cancels, it does not mean that premium equilibrium does not exist, but that the premium equilibrium does not solve that linear system.

A **sufficient** condition for the linear system to have a solution is given now. A sufficient condition for the linear system to have a (unique) solution is M to be diagonally dominant. That is $\forall j = 1, \dots, J$,

$$|2b_j| > \sum_{k \neq j} |-a_j w_k| \Leftrightarrow 2b_j > a_j \sum_{k \neq j} w_k \Leftrightarrow 2b_j > a_j w^{\Sigma}_{-j}$$

In the case of objective function (3.7), we choose $a_j = 1 + \beta_j$, $b_j = \beta_j w^{\Sigma}_{-j}$. So the sufficient condition is

$$2\beta_j w^{\Sigma}_{-j} > (1 + \beta_j) w^{\Sigma}_{-j} \Leftrightarrow \beta_j > 1.$$

In that case, one can check that the determinant $\det(M_1)$ is strictly positive. This fact was also seen in Dutang et al. (2013).

Proposition 3.4.3. *Let $\boldsymbol{x}^\star$ be the premium equilibrium of the J-player insurance game. For each Insurer j, the premium equilibrium $x_j^\star$ depends on the parameters as given in Table 3.4, where each column analyzes the different situations of active constraints. As the break-even premium π_j is increasing in $\bar{a}_{j,t-1}$ and $\overline{m}_{t-1}$, the effect of $\bar{a}_{j,t-1}$ and $\overline{m}_{t-1}$ on the premium equilibrium $x_j^\star$ is identical to the effect of π_j.*

	$x_j^\star = \underline{x}, \overline{x}$	$x_j^\star$ solv. constr.	no act. constraint	
z	$z \mapsto x_j^\star(z)$	$z \mapsto x_j^\star(z)$	$z \mapsto x_j^\star(z)$ if (3.5)	$z \mapsto x_j^\star(z)$ if (3.6)
π_j	——	$\nearrow$	$\nearrow$	$\nearrow$
β_j	——	——	$\searrow$	$\searrow$
n_j	——	unknown	——	$\nearrow$ if $\pi_j > 2$, $\searrow$ oth.
K_j	——	$\searrow$	——	——
$\sigma(Y)$	——	$\nearrow$	——	——

Table 3.4 – Sensitivity analysis of premium $x_j^\star$

3.4.2 Properties of the premium equilibrium in the one-period game : $J = 2$

The objective of this section is to better understand price equilibrium, in a simple case where $I = 2$. We first look at parameters conditions leading to constraints activation. As sum up in Table 3.5, we determine bounds of costs $_i$ according to price constraint level as well as consumers sensitivity parameters β_j. We thus determine the value of Nash Equilibrium (table6) , which is under active constraints, for at least one competitor.

In a second part, we investigate the case where constraints are not active (i.e. price equilibrium of each firm depends on their consumer sensitivity and competitor consumer sensitivity). We thus analyze effect of competitor price elasticity on price equilibrium. Interestingly, explicit formula allow us to show that marginal effect is not monotone and highly depend on comparative competitive advantage in term of consumers' inertia.

Considering a two-insurer market $\boldsymbol{x} = (x_1, x_2)$, $\boldsymbol{n} = (n_1, n_2)$ leads to the same market proxy. Indeed using Equations (3.5) and (3.6), we obtain

$$m_j(\boldsymbol{x}, \boldsymbol{n}) = \frac{1}{n_1 + n_2 - n_j} n_{-j} x_{-j} = x_{-j} = m_j(\boldsymbol{x}),$$

where the index $-j$ denotes the other player when considering player j. The objective function (3.7) simplifies to

$$O_j(x_j, x_{-j}) = \frac{n_j}{n_1 + n_2} \left(1 - \beta_j \left(\frac{x_j}{x_{-j}} - 1\right)\right)(x_j - \pi_j).$$

The computation of the premium equilibrium involves the use of Lagrange multipliers to take into account constraints in the KKT system. Table 3.21 of Appendix 3.8.2.5 sums up the 9 different cases for the KKT system (3.26): $(x_1^\star, x_2^\star)$. As shown in Appendix 3.8.2.5, we get the following Proposition 3.4.4 for $\beta_1, \beta_2 > 1$. We refer to Appendix 3.8.2.5 for other cases.

Proposition 3.4.4. *The premium equilibrium of the two-player insurance game when solvency constraints are not active is given in Table 3.5 with premium value given in Table 3.6.*

		π_1				
		$. < l_1(\beta_1)$	$l_1(\beta_1) < . < l_2(\beta_1)$	$l_2(\beta_1) < . < l_3(\beta_1)$	$l_3(\beta_1) < . < l_4(\beta_1)$	$l_4(\beta_1) < .$
	$. < l_1(\beta_2)$	$(\underline{x}, \underline{x})$	$(\underline{x}, \underline{x})$	$(\tilde{x}_a, \underline{x})$	$(\tilde{x}_a, \underline{x})$	$(\underline{x}, \overline{x})$
	$l_1(\beta_2) < . < l_2(\beta_2)$	$(\underline{x}, \underline{x})$	$(\underline{x}, \underline{x})$	$(\tilde{x}_a, \underline{x})$	$(\tilde{x}_a, \underline{x})$ or $(\overline{x}, \tilde{x}_c)$	$(\overline{x}, \tilde{x}_c)$
	$l_2(\beta_2) < . < l_3(\beta_2)$	$(\underline{x}, \tilde{x}_b)$	$(\underline{x}, \tilde{x}_b)$	(x_1^{NC}, x_2^{NC})	$(\overline{x}, \tilde{x}_c)$	$(\overline{x}, \tilde{x}_c)$
π_2	$l_3(\beta_2) < . < l_4(\beta_2)$	$(\underline{x}, \tilde{x}_b)$	$(\underline{x}, \tilde{x}_b)$ or $(\tilde{x}_d, \overline{x})$	$(\tilde{x}_d, \overline{x})$	$(\overline{x}, \overline{x})$	$(\overline{x}, \overline{x})$
	$l_4(\beta_2) < .$	$(\underline{x}, \overline{x})$	$(\tilde{x}_d, \overline{x})$	$(\tilde{x}_d, \overline{x})$	$(\overline{x}, \overline{x})$	$(\overline{x}, \overline{x})$

Table 3.5 – Summary of all cases for a unique equilibrium $(x_1^\star, x_2^\star)$ (only valid for $\beta_j > 1$), with $l_1(\beta) = 2\underline{x} - \frac{1+\beta}{\beta}\overline{x}$, $l_2(\beta) = \frac{\beta-1}{\beta}\underline{x}$, $l_3(\beta) = \frac{\beta-1}{\beta}\overline{x}$, $l_4(\beta) = 2\overline{x} - \frac{1+\beta}{\beta}\underline{x}$.

Value of $x_j^\star$	no constraint	$x_{-j}^\star = \underline{x}$	$x_{-j}^\star = \overline{x}$
Player 1	$x_1^{NC} = \frac{2\beta_1\beta_2\pi_1 + (1+\beta_1)\beta_2\pi_2}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)}$	$\tilde{x}_a = \frac{\beta_1\pi_1 + (1+\beta_1)\underline{x}}{2\beta_1}$	$\tilde{x}_d = \frac{\beta_1\pi_1 + (1+\beta_1)\overline{x}}{2\beta_1}$
Player 2	$x_2^{NC} = \frac{2\beta_1\beta_2\pi_2 + (1+\beta_2)\beta_1\pi_1}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)}$	$\tilde{x}_b = \frac{\beta_2\pi_2 + (1+\beta_2)\underline{x}}{2\beta_2}$	$\tilde{x}_c = \frac{\beta_2\pi_2 + (1+\beta_2)\overline{x}}{2\beta_2}$

Table 3.6 – Unconstrained premium equilibrium value

We see in Table 3.5 that Nash Equilibrium of our game highly depends on price bounds $(\underline{x}, \overline{x})$ but also on consumers costs sensitivity β_j, $j = \{1, 2\}$.First and as expected an increase of $\underline{x}$ or a decrease of $\overline{x}$ lead to constraint NE (Nash Equilibrium).

More interestingly, competitive advantage resulting from low costs π_j is limited by regulatory constraint. Indeed, since x_j^{NC} is an increasing function of π_j, low π_j could lead firm j to have first best price x_j^{NC} below $\underline{x}$, in particular when their consumers are sensitive (i.e. β_j large enough).

In response, competitor $-j$ of insurer j, which is unconstrained by bounds will not propose the unconstrained prices x_{-j}^{NC} since it could benefit from this limitation by proposing highest price. Inversely, an important costs π_j lead firm j to be limited by upper bounds in particular when consumers are not sensitive.

From Table 3.6, we find that when firm j is contrained, the best response of firm $-j$ does not depend on j consumers' sentivitity. We thus focus our analysis on the special case where there are no active constraints. In that case, $\boldsymbol{x}^\star = (x_1^{NC}, x_2^{NC})$ by Table 3.5 or as shown directly from the linear system in Appendix 3.8.2.6. From Table 3.4, we already know that $\pi_j \mapsto x_j^\star$ is an increasing function. From the explicit value, it is easy to show that $\pi_{-j} \mapsto x_j^\star$ is an increasing affine function when $\det(M) > 0$ or a decreasing affine function otherwise. From Table 3.4, we already know that an increase of consumers' price sensitivity with respect to firm j decrease the price equilibrium of firm j.

However, an increase of consumers' price sensitivity with respect to firm $-j$ could have positive or negative impact depending on β_2 based level. Indeed from Appendix 3.8.2.6, we have

$$\frac{\partial x_1^\star}{\partial \beta_2} = \frac{\beta_2^2\pi_2(3\beta_1 - 1) - 2\beta_2\pi_2(1+\beta_1) - 2\beta_1\pi_1(1+\beta_1) - \pi_2(1+\beta_1)}{(3\beta_1\beta_2 - 1 - \beta_1 - \beta_2)^2}.$$

The numerator admits two real roots $\tilde{\beta_2^1}$, $\tilde{\beta_2^2}$ such that $\tilde{\beta_2^1} < 0$ and $\tilde{\beta_2^2} > (1+\beta_1)/(3\beta_1 - 1)$. Thus, there always exists a threshold of $\tilde{\beta_2}^2$ for which the derivative of firm j price equilibrium changes sign with respect to the price sensitivity of competitor $-j$. This means that when $(1+\beta_1)/(3\beta_1 - 1) < \beta_2 < \tilde{\beta_2}^2$, an increase of price sensitivity of consumers with respect to competitor $-j$ lead to a decrease of price equilibrium for both insurers. Intuitively, the decrease of price competitor $-j$ over pass the price elasticity advantage of j. However, when $\beta_2 > \tilde{\beta_2}^2$, an increase of β_2 leads to a decrease of $x_2^\star$ and an increase of $x_1^\star$. Hence, when β_2 is high enough, a marginal increase of this parameter leads to an important comparative advantage for firm 1.

The simple case consider here (i.e $J = 2$), allows us to analytically demonstrate the importance of consumers' price sensitivity, in particular when one firm benefit from highest inertia. It conducts either to determine importance of regulatory constraints on market equilibrium but also on premium level, particularly when firms get important difference in their consumers sensitivity.

First, firms displaying important (resp. weak) consumers' price sensitivity will be more constraint by lower (resp. upper) bounds. In addition, when they are not constrained, consumers' price sensitivity has also an important impact on market equilibrium. While we consider here a Bertrand competition (competition in price), taking into account stochastic demand of consumers by including sensitivity parameter β allow us to avoid the "Bertrand Paradox". This is particularly evident when firms have important difference in consumers' price sensitivity.

3.5 Theoretical properties of the repeated game

The two following propositions give us some insights of the complexity of the repeated game. The first proposition ensures that the game will end in a long term horizon, whereas the second proposition shows that the cheapest insurer at one period has an advantage on its underwriting result by policy from a stochastic order point of view.

Proposition 3.5.1. *For the repeated $J-$player insurance game defined in the previous subsection, the probability that there are at least two non-bankrupt insurers at time t decreases geometrically as t increases.*

Proposition 3.5.2. *For the repeated $J-$player insurance game defined in the previous subsection, if for all $k \neq j$ among non-bankrupt insurers, $x_j \leq x_k$ and $x_j(1-e_j) \leq x_k(1-e_k)$, then the underwriting result by policy is ordered $UW_j \leq_{icx} UW_k$ where UW_j is the random variable*

$$UW_j = x_j(1-e_j) - \frac{1}{N_j(x)} \sum_{i=1}^{N_j(x)} Y_i.$$

3.6 Empirical evidences

All numerical applications are carried out with the R software, R Core Team (2018) : we refer to Appendix 3.8.4.1 for computation details. This section is divided into two parts: one part for a two-insurer market and the other for a three-insurer market. In each subsection, we begin by presenting the base parameters for which simulates the repeated game; and then we follow by analyzing single random paths and over a large number of runs. Finally we investigate the effect of parameters on the repeated game.

3.6.1 A duopoly

3.6.1.1 Description of the setting

We consider a duopoly where two insurers fight for a market of $N = 500$ policyholders. In the central setting, Insurer 1 has 50% more policies than Insurer 2. Thus Insurer 1 will be called the leader, whereas Insurer 2 the challenger. Objective and constraint functions are given in Equations (3.7) and (3.9) with parameters given in Table 3.9. Policyholders which are the source of randomness (both for lapse and loss) have parameters given in Tables 3.7 and 3.8 respectively loss and lapse. Lapse rates and price elasticity are similar to those of Dutang et al. (2013). Price are normalized to 1 as it is the pure premium without any expense.

	Loss frequency		Loss severity		
	dist.	λ	dist.	μ_1	σ_1
Policyholder	Poisson	0.1	lognormale	1.931616	0.8613578
(Expec. & Var.)	$(E(M) = Var(M) = 0.1)$		$(E(Z) = 10, Var(Z) = 110)$		

Table 3.7 – Parameters of the loss model PLN

	Lapse MLN PD model				
	f_j	μ_j	α_j	$1 - p_{j\to j}((1,1))$	$1 - p_{j\to j}((1,1.05))$
Policy of Insurer 1	$\tilde{f}_j$	-2.1973	7.4020	10%	15%
Policy of Insurer 2	$\tilde{f}_j$	-1.8153	5.8445	14%	19%

Table 3.8 – Parameters of the lapse model

	Obj. and constr. param.						
	β_j	ω_j	e_j	$\pi_{j,0}$	$n_{j,0}$	$K_{j,0}$	solv. ratio
Insurer 1	3	1	30%	1.28494	300	317.49	133%.
Insurer 2	4	1	30%	1.28612	200	259.23	133%
	$\underline{x}$	$\overline{x}$	N	$E(Y)$	$Var(Y)$	k_{95}	d
Market	1	3	500	1	4.5826	3	3

Table 3.9 – Parameters of insurers

3.6.1.2 Analysis of some random paths

Let us consider a single random path (that can be obtained by setting the seed). In Figure 3.6, we observe that the challenger's premium is lower for the first four periods than the leader's premium.

This leads to a change of leadership both in terms of portfolio sizes and gross written premium. However in period 4, Insurer 2 experiences a large loss depicted by a loss ratio around 120% damaging the solvency ratio. In period 5, Insurer 2 is forced to readjust the premium to a high level: this is a situation favorable to Insurer 1 which gets back some market share. After period 6, Insurer 1 retrieves the leadership and holds it until period 10.

In this simple setting, we notice that the competition is driven by many effects that are temporarily prevalent or temporarily insignificant. The first effect playing a major role is the solvency constraint, see e.g. periods 4 and 5. The second effect is the portfolio size which mitigates the loss volatility.

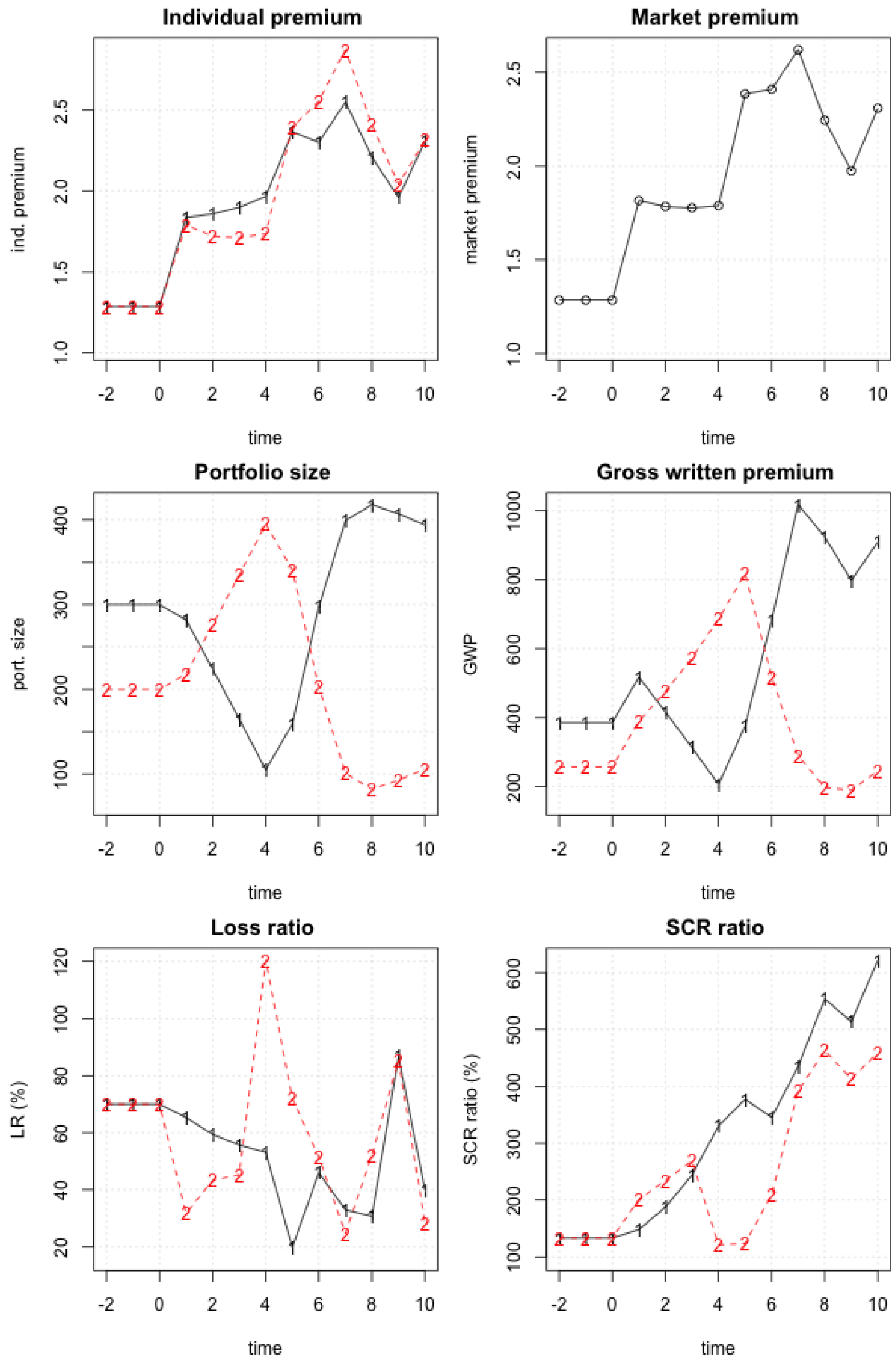


Figure 3.6 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2

3.6.1.3 Analysis of an aggregate number of random paths

Now we analyze 100 runs of the repeated game. For each plot in Figure 3.7, we compute the first quartile, the median and the third quartile over 100 runs at each period. From Figure 3.7, we can first note that most of the time premium of Insurer 1 are always higher than Insurer 2's premium, since all quartiles of Insurer 1 are higher than those of Insurer 2. This leads to a huge difference in terms of portfolio sizes, for which at period 10 the first quartile of Insurer 2 is even higher than the third quartile of Insurer 1. This situation is a little tempered when looking gross written premium.

Secondly, the level of premium equilibria are higher than initial premium. Typically the medians of premium equilibria are around 2 when initial premiums are 1.25. This leads to a drop in terms of loss ratio and a gain for solvency ratios.

Thirdly, Insurer 2 having a higher number of policies, in particular for the last five periods, reduces the interquartile range of claim frequencies by policies. The third quartiles of Insurer 2 claim numbers tends towards the median, while the opposite effect is observed for Insurer 1.

3.6.1.4 Sensitivity analysis with respect to parameters

In this subsection, we make a sensitivity analysis with respect parameters in the following order: the loss model, the lapse model, the portfolio size, the total market size and the objective parameters. Most figures and tables are put in Appendix 3.8.7.1.

Let us start with the loss distribution. Recalling that the base model is a Poisson - lognormal compound distribution. We consider two other models : a negative binomial - lognormal compound distribution labelled NBLN and a Poisson - gamma compound distribution labelled PG. The new set of parameters of the frequency distribution or the severity distribution are given in Table 3.10. These two concurrent loss models analyze either an increase of the frequency variance (Poisson vs. negative binomial) or a decrease of the severity tail (lognormal vs. gamma). In the first case, the resulting variance of the aggregate claim distribution is increased, while in the second case the resulting variance is reduced.

	Loss frequency for NBLN			Loss severity for PG		
	dist.	r	p	dist.	μ_2	σ_2
Policyholder	neg. binom.	0.014286	0.125	gamma	1.51515	0.151515
(Expec. & Var.)	$(E(M) = 0.1, Var(M) = 0.81)$			$(E(Z) = 10, Var(Z) = 66)$		

Table 3.10 – New parameters of loss models NBLN and PG

On a single run, the NBLN loss model produces a similar pattern for the volume indicators (portfolio size, gross written premium) but the premium equilibrium and the loss ratio are more erratic for both insurers, see Figure 3.14 vs. Figure 3.6. The range of the premium and the loss ratio are higher than for the base model. For the PG loss model, portfolio sizes and gross written premium of insurers have also similar pattern but the premium equilibrium and the loss ratio are far less erratic for both insurers, see Figure 3.15 vs. Figure 3.6. Therefore the SCR ratio is higher and more stable for the PG model.

Over 100 runs, the NBLN loss model has a major impact of the quartiles of premium equilibrium. Indeed the higher volatility of the frequency distribution increases the claim number by policy (the interquartile range triples from 4 to 12 percentage points) implying a more volatile break-even premium $\pi_{j,t}$ which leads to more erratic premium equilibrium, see Figure 3.23 vs. Figure 3.7. This effect is logical from the sensitivity Table 3.4. By a domino effect, the portfolio size and hence the gross written premium are more unstable for both insurers (the interquartile range nearly doubles).

The opposite effect is observed for the PG loss model. For that case, the lower volatility of the severity distribution slightly decreases the volatility of the loss ratio. (the interquartile range triples

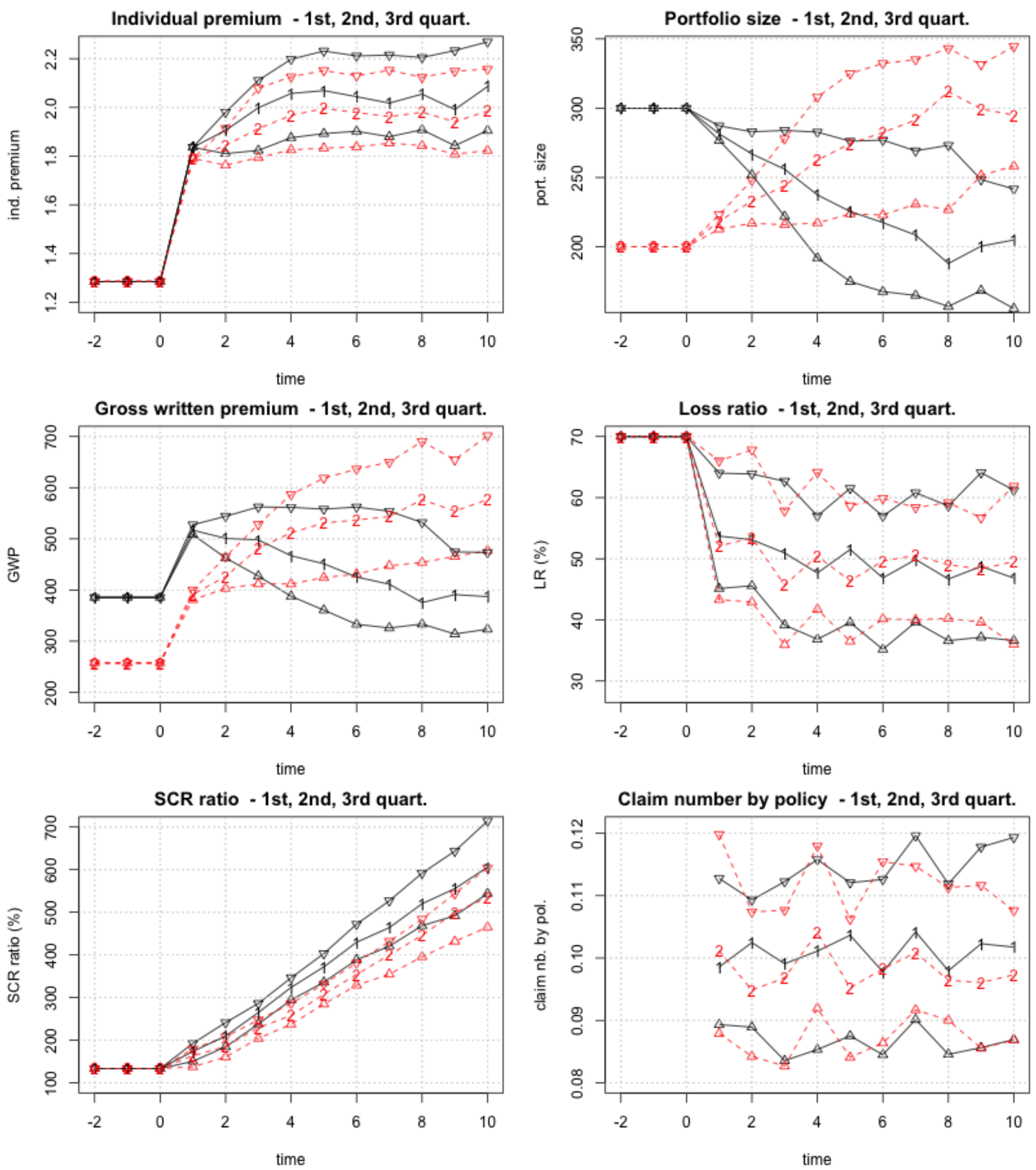


Figure 3.7 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles

from 4 to 12 percentage points). However, the PG loss model has a relative small impact on other quantities, see Figure 3.24 vs. Figure 3.7.

Let us continue with the lapse model. Recalling that the base model is a multinomial logit model based on the differences of premium values called MLN PD. We test two lapse models : a multinomial logit model based on the differences of ratio premium values called MLN PR and a multinomial logit model based on the differences of premium values with a new set of parameters leading to different central lapse rates called MLN PD 2.

	Lapse MLN PR model				
	f_j	μ_j	α_j	$1 - p_{j\to j}((1,1))$	$1 - p_{j\to j}((1,1.05))$
Policy of Insurer 1	$\underline{f}_j$	-11.4497	9.2525	10%	15%
Policy of Insurer 2	$\bar{f}_j$	-9.1209	7.3056	14%	19%
	Lapse MLN PD 2 model				
	f_j	μ_j	α_j	$1 - p_{j\to j}((1,1))$	$1 - p_{j\to j}((1,1.05))$
Policy of Insurer 1	$\tilde{f}_j$	-2.1973	7.4020	10%	15%
Policy of Insurer 2	$\tilde{f}_j$	-2.1973	7.4020	10%	15%

Table 3.11 – New parameters of lapse models MLN PR and MLN PD 2

On a single run, the MLN PR lapse model produces a similar pattern for the portfolio size but the premium equilibrium and the loss ratio seems more cyclical for both insurers, see Figure 3.16 vs. Figure 3.6. Furthermore, the premium values are more closed to the initial premium leading to a more stable gross written premium. Customers are less elastic to price changes for the MLN PR lapse model. For the MLN PD 2 lapse model exhibits very different patterns of premium and portfolio size values. In this particular path, Insurer 2 takes and keeps the leadership on the market, see Figure 3.17 vs. Figure 3.6.

Over 100 runs, we note that the quartiles of the premium equilibrium for the MLN PR are closer than the base situation (MLN PD). Thus the quartiles of the portfolio sizes tends to be identical between the two insurers, Figure 3.25 vs. Figure 3.7. After 6 periods, the gross written premium, the loss ratio and the claim numbers have very similar quartiles between both insurers.

For the MLN PD 2 lapse model, the leadership of Insurer 2 after 5 periods is more overwhelming than in the base situation: the first quartile of Insurer 2's portfolio size is higher than the third quartile of Insurer 3's portfolio size after 5 periods. The MLN PD 2 lapse model seems to accelerate the leadership of Insurer 2 which yields to a more stable claim numbers by policy and loss ratio, see Figure 3.26 vs. Figure 3.7.

Let us pursue with the initial parameters. Recalling that the base model assumes initial portfolio sizes $\boldsymbol{n}_0 = (300, 200)$ and initial solvency ratios of 133%. We study first another value of initial portfolio sizes $\boldsymbol{n}_0 = (250, 250)$ and then another values of solvency ratios 100%.

On a single run, we observe that insurers have both a five-period leadership, however the domination of Insurer 1 is striking on this scenario. For instance, both the portfolio size and the gross written premium of Insurer 1 are dominant which leads to better solvency ratio, see Figure 3.18 vs. Figure 3.6.

Over 100 runs, Insurer 2 takes most of the time the leadership in terms of gross written premium or portfolio sizes. But the tendency towards of a full leadership of Insurer 2 is observed as in the base setting, see Figure 3.27 vs. Figure 3.7.

When considering new solvency ratios (of 100%), the effect is negligible compared to the base scenario for a single path, see Figure 3.19 vs. Figure 3.6. Over 100 runs, the pattern for all indicators are identical between the two situations, see Figure 3.28 vs. Figure 3.7. The novelty appears by the fact that insurers have been bankrupted in some random path: 4% for Insurer 2 of ruin and 1% for Insurer 1 in the first two periods, no ruin after.

Let us continue with the total market size. Originally the total market size is $N = 500$. We test the effect of portfolio sizes by doubling it. On a single run, the range of the premium equilibrium as well as the premium differences between the two insurers are much lower than in the base situation, see Figure 3.20 vs. Figure 3.6. This implies that the market shares make more periods to change so that Insurer 1 almost remains the leader.

Over 100 runs, we note that the interquartile range decreases for many indicators, in particular loss ratios, claim number by policies. The direct consequence is an increase of the solvency ratios for all quartiles, see Figure 3.29 vs. Figure 3.7.

Let us finally consider the lapse sensitivity parameters β_j and the credibility factor ω_j. In the base situation, $\beta_1 = 3$ and $\beta_2 = 4$. We test the opposite situation. This has a major influence on the premium equilibrium value both on a single run or over 100 runs. For instance, on Figure 3.21, we observe that Insurer 1 is more competitive and remains the leader for 8 out of 10 periods. But the drawback is that Insurer 1's solvency ratio is worse for this alternative situation than in the base scenario, see Figure 3.21 vs. Figure 3.6.

Over 100 runs, Insurer 1 remains most of the time the leader, see the quartiles of the gross written premium on Figure 3.30. Consequently both the loss ratio and the claim number by policy are more stable: the interquartile range falls for Insurer 1 and widens for Insurer 2.

Finally we study another set of values for credibility factors ω_j. In the base scenario, they are set to zero, while in the alternative scenario we assume that $\omega_j = 1/2$. That is the break even premium π_j is the mean between past market premium and past actuarial premium.

On a single run, we observe that the break even premium increases a lot leading to large premium equilbrium. The premium upper bound $\overline{x}$ is reached after 6 periods, see Figure 3.22. As soon as both insurers ask the same premium, only the lapse model plays a role : we see a convergence towards an equilibrium depicted in Proposition 3.3.11. The loss model has no longer any effect: the loss ratios shrinks and the solvency ratios explodes.

Over 100 runs, a similar effect is observed on Figure 3.31. The first quartiles of premium equilibrium reach almost the upper bound so that the portfolio sizes quartiles towards the invariant measure. However, we see that Insurer 2 takes the leadership very soon. Again the solvency ratios and the loss ratios appear abnormal. This is due to the fact that the total market size is constant and cannot exit the market as it would happen in such situation.

3.6.2 A three-insurer market

3.6.2.1 Description of the setting

We consider a game where three insurers fight for a market of $N = 500$ policyholders. In the central setting, Insurer 1 is the leader with more than one half of the market, whereas Insurer 2 is the challenger with 30% of policyholders and Insurer 3 the outsider with the last 10% of policyholders. Objective and constraint functions are still given in Equations (3.7) and (3.9) with parameters given in Table 3.13. In this three-player setting, we also study both market proxies $m(\boldsymbol{x}, \boldsymbol{n})$ used in the objective function given in Equations (3.5), (3.6). Policyholders face the same of the loss model as the previous subsection, see Table 3.7, but follow a new multinomial logit model with parameters given in Table 3.12.

3.6.2.2 Analysis of some random paths

In Figure 3.8 for a simple market proxy (see (3.5)), we observe that Insurer 1 remains the leader for the first six periods, until competitors have a better loss experience with very low loss ratios around

	Lapse MLN PD model				
	f_j	μ_j	α_j	$1 - p_{j\to j}((1,1))$	$1 - p_{j\to j}((1,1.05))$
Policy of Insurer 1	$\bar{f}_j$	-2.890372	7.401976	10%	15%
Policy of Insurer 2	$\tilde{f}_j$	-2.508437	5.844477	14%	19%
Policy of Insurer 3	$\tilde{f}_j$	-2.209495	4.928581	18%	23%

Table 3.12 – Parameters of the lapse model

	Obj. and constr. param.						
	β_j	ω_j	e_j	$\pi_{j,0}$	$n_{j,0}$	$K_{j,0}$	solv. ratio
Insurer 1	3	1	30%	1.28494	300	317.49	133%.
Insurer 2	3	1	30%	1.28612	150	224.49	133%
Insurer 3	4	1	40%	1.48396	50	97.21	100%
	$\underline{x}$	$\overline{x}$	N	$E(Y)$	$Var(Y)$	k_{95}	d
Market	1	3	500	1	4.5826	3	3

Table 3.13 – Parameters of insurers

20% compared to 60% in Periods 5 and 6. Hence inevitably, Insurer 1 sets a very high premium in Period 1 due to a high break-even premium and Insurer 3 capture half of the market. Globally on this run, Insurer 2 keeps a very stable market share. When we compare to the weighted market proxy (see (3.6)), the pattern mainly different. For the whole period, Insurer 1 remains the leader both in terms of portfolio size and gross written premium by setting premium slightly lower than competitors, see Figure 3.9. The weighted market proxy seems to induce higher values of premium equilibrium: we almost reach the upper bound $\overline{x}$ in Figure 3.9.

3.6.2.3 Analysis of an aggregate number of random paths

Now we turn our attention on the analysis of 100 runs of the repeated game. Again, for each plot in Figures 3.10 and 3.11, we compute the first quartile, the median and the third quartile over 100 runs at each period. From Figure 3.10, we observe that premium equilibrium are very similar with almost the same quartiles for every insurers.

However quartiles of portfolio sizes, gross written premium and solvency ratios are ordered: for instance third quartiles of Insurer 1 is greater than third quartiles of Insurer 2 which is in turn greater than third quartiles of Insurer 3. Hence Insurer 1 seem to stay leader in most scenarios. In addition, we notice that that the interquartile range of loss ratio and claim numbers per policy is lower than those of Insurers 2 and 3.

In Table 3.14, we observe that the leadership of Insurer 1 is greater than those of competitors, yet there is a decreasing trend over time. The longer the repeated game is player, the lower the probability that Insurer 1 is the leader is. In Table 3.14, the ruin probability is also computed: having a lower initial solvency ratio, Insurer 3 is more exposed to the insolvency risk (13% against 1% for Insurer 2 and 0% for Insurer 1).

Now we analyze 100 runs of the repeated game when the market proxy is the weighted mean (3.6). In Figure 3.11, the pattern observed are very similar to those of Figure 3.10. The quartiles of premium equilibrium are similar, yet for some periods an order seems to appear: Insurer 1 has the highest first quartiles for the first five period but the lowest for the last four periods.

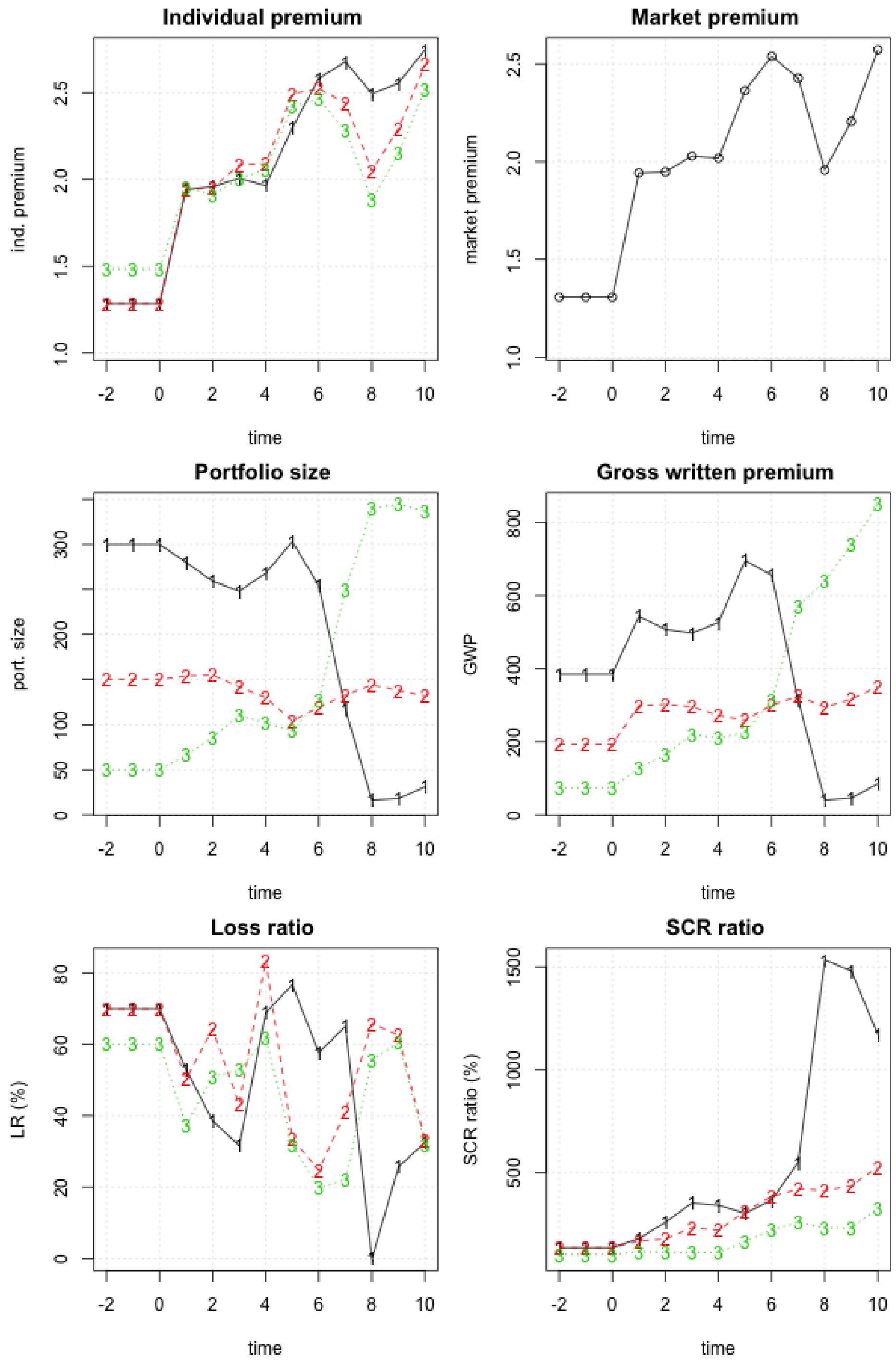


Figure 3.8 – Some indicators of the repeated game for a single run, black solid line for Insurer 1, red dashed line for Insurer 2, green dotted line for Insurer 3

In terms of volume indicators (portfolio sizes and gross written premium), Insurer 1 has the highest values for all quartiles. In particular for portfolio sizes, Insurer 3's third quartiles are most of time

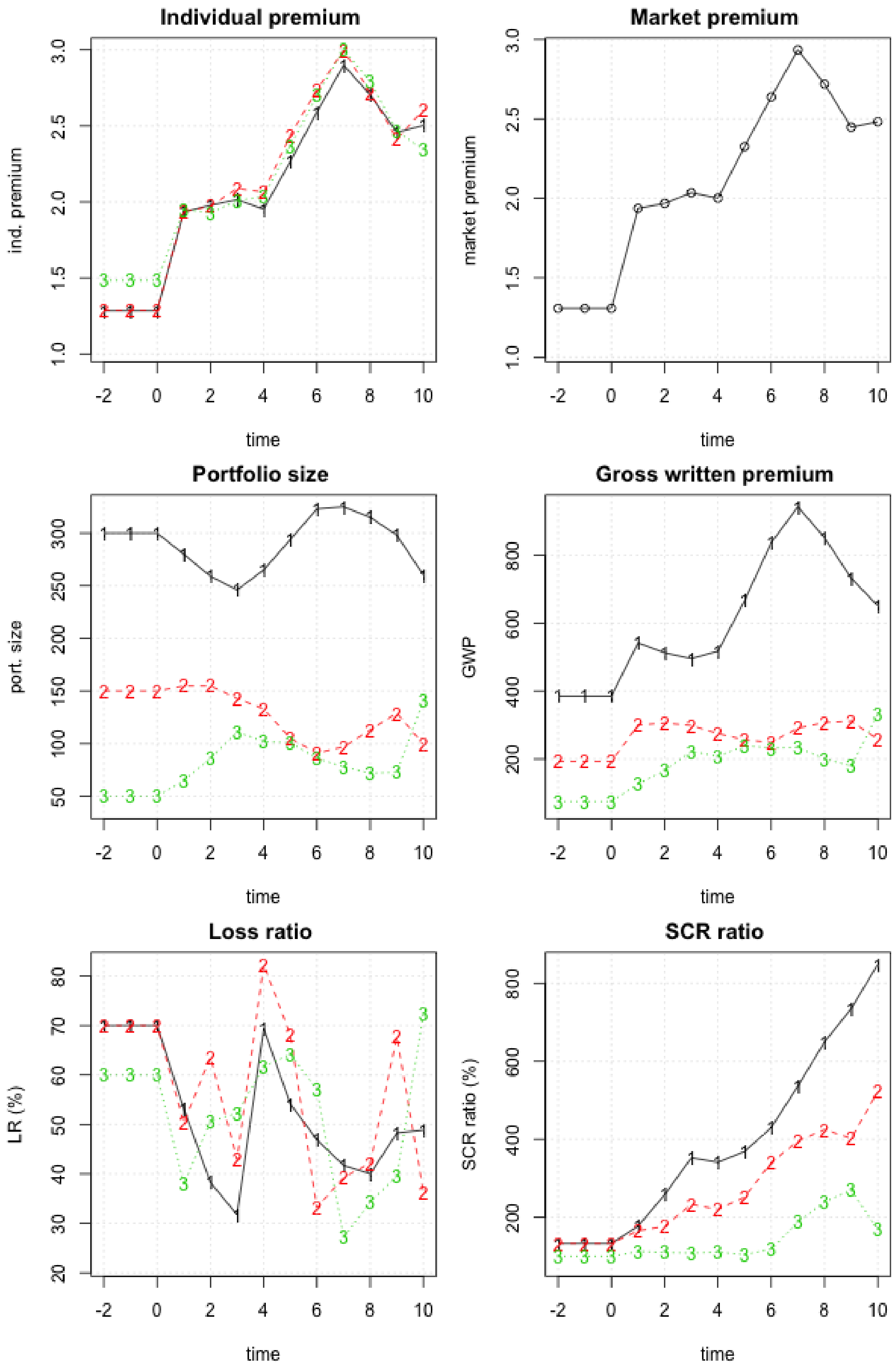


Figure 3.9 – Some indicators of the repeated game for a single run, black solid line for Insurer 1, red dashed line for Insurer 2, green dotted line for Insurer 3

lower than Insurer 1's first quartiles. The domination of Insurer 1 makes that the interquartile range of Insurer 1's loss ratio and claim numbers by policy is alway lower than those of competitors.

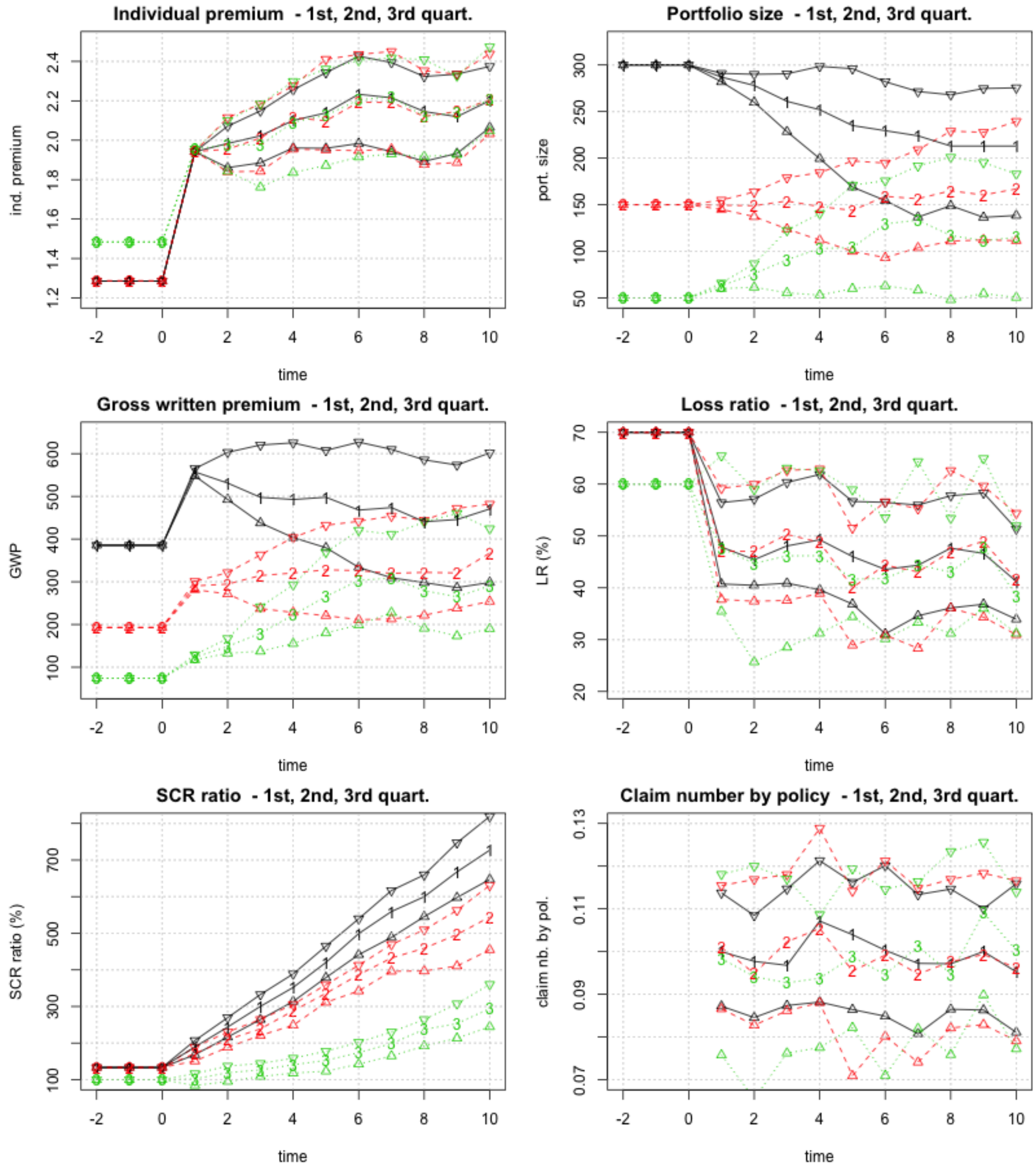


Figure 3.10 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1, red dashed line for Insurer 2, green dotted line for Insurer 3; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles

Looking at Table 3.15, we observe similarly to Table 3.14 that Insurer 1 is the leader and is never ruined. At the opposite, Insurer 3 gets bankrupted more often as time evolves (from 2% to 14%) and has few chances to become leader (around 10%). Finally Insurer 2 remains the best competitor of Insurer 1 with a significant of probability of leadership.

	Ruin probabilities			Leadership probabilities		
Period	Insurer 1	Insurer 2	Insurer 3	Insurer 1	Insurer 2	Insurer 3
1	0.00	0.01	0.05	1.00	0.00	0.00
2	0.00	0.01	0.09	0.94	0.00	0.00
3	0.00	0.01	0.10	0.86	0.02	0.03
4	0.00	0.01	0.12	0.73	0.12	0.06
5	0.00	0.01	0.12	0.59	0.16	0.14
6	0.00	0.01	0.13	0.52	0.18	0.16
7	0.00	0.01	0.13	0.45	0.23	0.18
8	0.00	0.01	0.13	0.42	0.25	0.18
9	0.00	0.01	0.13	0.38	0.25	0.21
10	0.00	0.01	0.13	0.44	0.24	0.15

Table 3.14 – Empirical probabilities of ruin and leadership over 100 runs, 3-player game with simple market proxy

	Ruin probabilities			Leadership probabilities		
Period	Insurer 1	Insurer 2	Insurer 3	Insurer 1	Insurer 2	Insurer 3
1	0.00	0.01	0.02	1.00	0.00	0.00
2	0.00	0.01	0.04	0.98	0.00	0.00
3	0.00	0.01	0.05	0.88	0.04	0.03
4	0.00	0.01	0.04	0.65	0.15	0.09
5	0.00	0.01	0.05	0.62	0.17	0.10
6	0.00	0.01	0.07	0.60	0.18	0.11
7	0.00	0.01	0.09	0.57	0.21	0.10
8	0.00	0.01	0.10	0.59	0.20	0.09
9	0.00	0.01	0.11	0.55	0.20	0.13
10	0.00	0.01	0.14	0.56	0.23	0.08

Table 3.15 – Empirical probabilities of ruin and leadership over 100 runs, 3-player game with weighted market proxy

3.6.2.4 Sensitivity analysis with respect to parameters

Every figures and tables of the sensitivity analysis are put in Appendix 3.8.7.2. Let us start the sensitivity analysis with respect to the loss model. For the 3-insurer game, we only test the NBLN loss model where the claim frequency follows a negative binomial compared to a Poisson distribution, see Tables 3.10 and 3.7 respectively.

Comparing Figures 3.10 and 3.32 for a simple market proxy, we note that the interquartile ranges of all indicators largely increase. In particular, portfolio sizes and claim numbers by policy are more erratic for all insurers, even for Insurer 1. In this more randomized setting, the leadership is harder to get. Typically quartiles of Insurer 1's gross written premium are more closed to those of competitors, see Table 3.23 for leadership probabilities. This increase of loss randomness translates to a lower solvency ratios. For instance, the medians of Insurer 2's solvency ratios is increasing from 133% to 300% for NBLN loss model while for PLN loss model it rockets to 500%. In many situations, Insurer 3 goes to bankrupt. In Table 3.23, we compute the ruin probabilities and observe very large probabilities of ruin for Insurer 3.

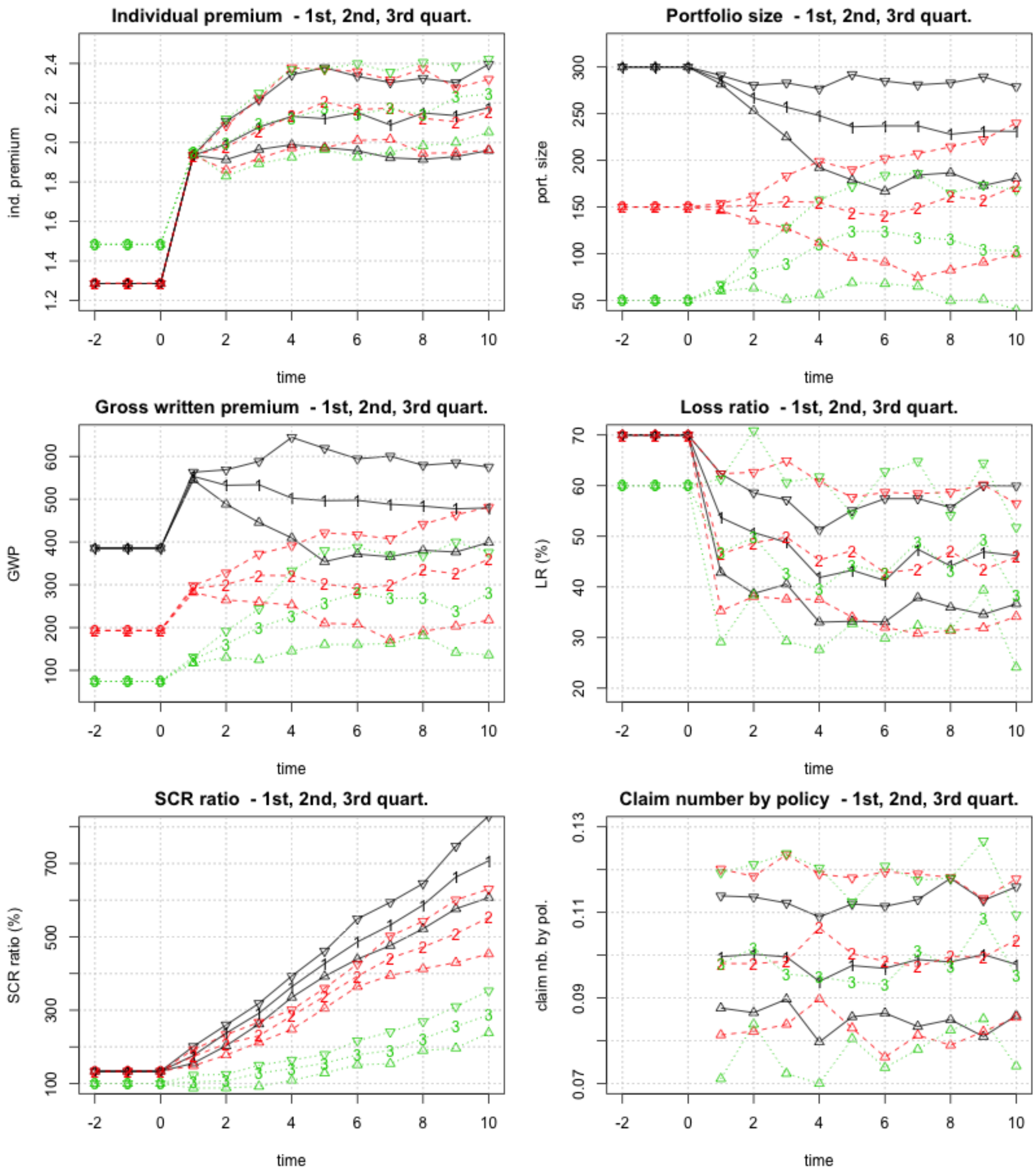


Figure 3.11 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1, red dashed line for Insurer 2, green dotted line for Insurer 3; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles

Studying the weighted market proxy, we observe similar patterns between Figures 3.11 and 3.33. For examples, quartiles of the premium equilibrium are similar across insurers, yet Insurer 3 seem below most of the time. Premium equilibrium have also higher values. The increase in loss randomness also deteriorates the solvency ratios, where first quartiles are much lower on Figure 3.33 than on Figure 3.11. In Table 3.24, the empirical ruin probabilities are higher even for Insurers 1 and 2.

Regarding the lapse model, we compare the MLN PD lapse model given in Table 3.12 with the MLN PR lapse model given in Table 3.16. Reference lapse rates are identical, only the sensitivity function f_j changes from premium differences to premium ratios.

The lapse model seems to produce more favorable situations for Insurer 1, see Figure 3.34. Yet the quartiles of premium equilibrium for all insurers are very close, the quartiles of portfolio sizes are almost separated. Indeed first quartiles of Insurer 1's portfolio sizes are above third quartiles of Insurer 2's portfolio sizes. Therefore, Insurer 1 is almost always the leader, see quartiles of gross written premium in Figure 3.34 and Table 3.25. As a consequence the interquartile ranges fall for loss ratio and claim numbers by policy for Insurer 1.

	Lapse MLN PR model				
	f_j	μ_j	α_j	$1 - p_{j\to j}((1,1))$	$1 - p_{j\to j}((1,1.05))$
Policy of Insurer 1	$\bar{f}_j$	-12.14284	9.25247	10%	15%
Policy of Insurer 2	$\bar{f}_j$	-9.814033	7.305596	14%	19%
Policy of Insurer 3	$\bar{f}_j$	-8.370220	6.160726	18%	23%

Table 3.16 – New parameters of lapse models MLN PR and MLN PD 2

We do not comment the sensitivity with respect to initial portfolio sizes or the total market size as the effects are similar to the situation decreasing the volatily of the loss model. Finally we focus on the objective parameters β_j and ω_j. We consider a test with $\omega_j = 1/2$ and a test with $\beta_3 = 3$ rather than $\beta_3 = 4$.

Changing the credibility factors have the effect of smoothing quartiles of all indicators. As the break-even premium is more closed among insurers, the value of the premium equilibrium are very stable, see Figures 3.36 and 3.37. This leads to smooth quartiles of portfolio sizes and gross written premium. In this situation, Insurer 1 remains the leader in most situations. The values of premium equilibrium are also much higher and touch the upper limit $\overline{x}$. As soon as all insurers set the upper limit $\overline{x}$, then we get the convergence towards equilibrium described in Proposition 3.3.11. This effect was previously observed for the two-insurer game.

Comparing Figure 3.38 against Figure 3.10, we see that premium values are higher for Insurer 3 (quartiles of Insurer 3's premium are always higher than those of competitors). Hence Insurer 3 obtains a worse situation with fewer policies and the competition between Insurers 1 and 2 is fiercer. Similar patterns are also observed when studying Figure 3.39 against Figure 3.11 where a weighted market proxy is considered.

3.7 Conclusion and perspectives

We address in this paper an analysis of consumers' price-sensitivity impacts as well as solvency constraint effects on market premiums equilibrium in a competitive non-life insurance market. We extent the repeated non-cooperative game of Dutang et al. (2013), where consumers behaviors and loss are both stochastic. Thus, we consider that each insurer maximizes its own objective function (i.e. the profit under solvency constraint) while taking into account that its strategy will affect others insurers' strategies.

We show that price-sensitivity of consumers has a major effect on prices equilibrium. An insurer has a higher probability to be a leader in the long-run if it benefits from a comparative advantage on consumers inertia. In addition, we demonstrate that a largest portfolio size implies a lowest volatility of losses. Therefore, it reduces the level solvency constraint. Numerical simulations allow us to support its results by evincing how losses affect premiums equilibrium under a regulated market. Insurers suffering from losses shock on the previous period could be obliged to increase its market premium to the detriment of portfolio size. Therefore, insurer benefiting from important inertia of its consumers is less impacted by a losses shock and is be able to keep a relatively important size, which guarantee long-run profit.

Indeed, while important losses shocks are generally the main sources of insurers' ruin, it could also result from the fact that insurers are not able to reach a sufficient level of business to guarantee mutualization principle. Therefore, further researches about price-sensitivity of consumers in a competitive market could bring a new dimension to the ruin theory. For instance it could explain why, even under solvency constraint, insurer bankruptcy could not be avoid.

Of course, we could have considered different modellings to approach our research question. For instance, we could improve the objective function by considering correlation of consumers' lapse behaviors across time (Barsotti et al. (2016)).It could also be interesting to examine the game under a dividend rule such as a solvency coverage ratio upper bounded, with for instance

$$\frac{K_{j,t+1}}{k_{995}\sigma(Y)\sqrt{n_{j,t}}} \leq 300\%.$$

In reality, insurers have to guarantee a minimum level of dividend in order to keep investments from the financial market. In addition, they are not allow to transfer are all benefits into their capital. Indeed, by doing so, they would avoid to pay income taxes.

Another interesting extension of our model could also consists on take into account information asymmetry. For now, we consider that all players observe passed claims of others and adjust their strategies with respect to these information. We can imagine to add information asymmetry where each player j has beliefs about others' claims such $\hat{s}_{-j} = s_j$ and $\hat{e}_{-j} = e_j$.

Finally, we assume that consumers are independent and identically distributed. However, we know that insurance market is characterized by an important asymmetry of information between insureds and insurers, leading to adverse selection effect. Considering the extending model of Albrecher and Daily-Amir (2017), it could be interesting to analyze how price-sensitivity impacts on premiums equilibrium are different when considering different type of consumers.

3.8 Appendix

3.8.1 Proofs of Section 3.3

3.8.1.1 Properties of lapse model at customer level

Proof of Prop. 3.3.1. By A3, $(C_{i,t})_t$ is a Markov chain with transition matrix $P_{\rightarrow}(\boldsymbol{x}_t)$ defined as

$$P_{\rightarrow}(\boldsymbol{x}_t) = \begin{pmatrix} p_{1\rightarrow 1}(\boldsymbol{x}_t) & \dots & p_{1\rightarrow J}(\boldsymbol{x}_t) \\ & \dots & \\ p_{J\rightarrow 1}(\boldsymbol{x}_t) & \dots & p_{J\rightarrow J}(\boldsymbol{x}_t) \end{pmatrix}.$$

In fact, $P_{\rightarrow}$ is a matrix function. By Proposition 3.8.2, the transition matrix has no null terms. It is immediate that the transition from $C_{i,0}$ to $C_{i,t}$ is the multiplication of the t matrices $P_{\rightarrow}(\boldsymbol{x}_1)$,..., $P_{\rightarrow}(\boldsymbol{x}_t)$. On the finite state space $\{1,\dots,J\}$, the Markov chain is both irreducible and aperiodic using Proposition 3.8.2 in Appendix 3.8.3.2. □

Proof of Prop. 3.3.2. By (Norris, 1997, p. 41), the process $(C_{1,t}, C_{2,t})_t$ is still a Markov chain on the space $E^2 = \{1,\dots,J\}^2$ with transition matrix $P_{\rightarrow}(\boldsymbol{x}_t) \otimes P_{\rightarrow}(\boldsymbol{x}_t)$. Iterating $N-1$ more times leads to the result. □

Proof of Prop. 3.3.3. Let $\mathcal{N}_j$ be the set of customers of Insurer j at time 0. That is $\forall i \in \mathcal{N}_j, C_{i,0} = j$. As $(C_{i,t})_t$ is a Markov chain, the transition from Insurer j to Insurer k is governed by the jth row $\tilde{p}_j$ of the matrix

$$P^{(t)}_{\rightarrow} = P_{\rightarrow}(\boldsymbol{x}_1) \times \dots \times P_{\rightarrow}(\boldsymbol{x}_t),\ \tilde{p}_j = \left(P^{(t)}_{\rightarrow,j,1}, \dots, P^{(t)}_{\rightarrow,j,J}\right).$$

Thus $\forall i \in \mathcal{N}_j, C_{i,t} \mid C_{i,0} = j \sim \mathcal{M}_J(1, \tilde{\boldsymbol{p}}_j)$. By A1 and A2, those policyholders of Insurer j will choose insurers according to a multinomial distribution $\overline{\boldsymbol{C}}_{j,t} \sim \mathcal{M}_J(n_{j,t-1}, \tilde{\boldsymbol{p}}_j)$ given $N_{j,0} = n_{j,0}$. From period $t-1$ to period t, the transition matrix simplifies to $P_{\rightarrow}(\boldsymbol{x}_t)$ and $\tilde{\boldsymbol{p}}_j = \boldsymbol{p}_{j\rightarrow}(\boldsymbol{x}_t)$. □

3.8.1.2 Properties of lapse model at portfolio level

Proof of Prop. 3.3.4. The portfolio sizes vector is the sum of choice vectors $\boldsymbol{N}_t = \overline{\boldsymbol{C}}_{1,t} + \dots + \overline{\boldsymbol{C}}_{J,t}$. By A2, Proposition 3.3.3 and given $\boldsymbol{N}_{t-1} = \boldsymbol{n}$, $(\overline{\boldsymbol{C}}_{j,t})_j$ are independent multinomial vectors with parameters $\mathcal{M}_J(n_j, \boldsymbol{p}_{j\rightarrow}(\boldsymbol{x}_t))$ for $j = 1,\dots,J$. Therefore, $\boldsymbol{N}_t$ (obtained by summing over j) has a known distribution given $\boldsymbol{N}_{t-1} = \boldsymbol{n}$. Since $(\boldsymbol{N}_t)_t$ is a discrete-time process taking values in $\mathcal{N}$, $(\boldsymbol{N}_t)_t$ is a Markov chain. By recurrence, the number of elements of $\mathcal{S}_{ms}$ is $\text{Card}(\mathcal{S}_{ms}) = \binom{N+J-1}{n_{ms}}$. The transition matrix of size $\text{Card}(\mathcal{S}_{ms}) \times \text{Card}(\mathcal{S}_{ms})$ has a complex expression

$$\overline{P}_t = \left(P(\boldsymbol{N}_t = \boldsymbol{m} | \boldsymbol{N}_{t-1} = \boldsymbol{n})\right)_{\boldsymbol{n},\boldsymbol{m}}$$

where $\boldsymbol{n}, \boldsymbol{m} \in \mathcal{S}_{ms}$ and

$$\begin{aligned} & P(\boldsymbol{N}_t = \boldsymbol{m} | \boldsymbol{N}_{t-1} = \boldsymbol{n}) \\ = \ & P(N_{1,t} = m_1, \dots, N_{J,t} = m_J | N_{1,t-1} = n_1, \dots, N_{J,t-1} = n_J). \\ = \ & \sum_{\substack{0 \le c_{11},\dots,c_{1J} \le N, \\ \text{s.t. } \sum_l c_{1l} = n_1}} \dots \sum_{\substack{0 \le c_{J1},\dots,c_{JJ} \le N, \\ \text{s.t. } \sum_l c_{Jl} = n_J}} \prod_{j=1}^{J} P(\overline{\boldsymbol{C}}_{j,t} = \boldsymbol{c}_j | N_{j,t-1} = n_j) \mathbb{1}_{\left(\sum_k c_{kj} = m_j\right)} \\ = \ & \sum_{\substack{0 \le c_{11},\dots,c_{1J} \le N, \\ \text{s.t. } \sum_l c_{1l} = n_1}} \dots \sum_{\substack{0 \le c_{J1},\dots,c_{JJ} \le N, \\ \text{s.t. } \sum_l c_{Jl} = n_J}} \prod_{\substack{j=1, \\ \sum_k c_{kj} = m_j}}^{J} \frac{n_j!}{c_{j1}! \dots c_{jJ}!} \left(p_{j\rightarrow 1}(\boldsymbol{x}_t)\right)^{c_{j1}} \dots \left(p_{j\rightarrow J}(\boldsymbol{x}_t)\right)^{c_{jJ}}. \end{aligned}$$

The probability $P(\overline{\boldsymbol{C}}_{j,t} = \boldsymbol{c}_j | N_{j,t-1} = n_j)$ depends on the price vector $\boldsymbol{x}_t$, and therefore is time dependent. By A2, the probability generating function of $\boldsymbol{N}_t | \boldsymbol{N}_{t-1} = \boldsymbol{n}$ is in constrast simpler

$$\begin{aligned} G^P_{\boldsymbol{N}_t|\boldsymbol{N}_{t-1}=\boldsymbol{n}}(\boldsymbol{z}) &= G^P_{\overline{\boldsymbol{C}}_{1,t}}(\boldsymbol{z}) \times \cdots \times G^P_{\overline{\boldsymbol{C}}_{J,t}}(\boldsymbol{z}) \\ &= \left(\boldsymbol{z}^T p_{1\to}(\boldsymbol{x}_t)\right)^{n_1} \times \cdots \times \left(\boldsymbol{z}^T p_{J\to}(\boldsymbol{x}_t)\right)^{n_J} = \left(P_{\to}(\boldsymbol{x}_t) \times \boldsymbol{z}\right)^{\boldsymbol{n}}, \end{aligned}$$

where $z \in \mathbb{R}^J$, the exponentiation $^{\boldsymbol{n}}$ is a component-wise operation and $G^P(.)$ denotes the probability generating function. Using Proposition 3.3.3, we have $\overline{\boldsymbol{C}}_{j,t} \mid \boldsymbol{N}_{j,0} = \boldsymbol{n}$ follows a multinomial distribution with parameters $\mathcal{M}_J(n_j, \tilde{\boldsymbol{p}}_j)$. By similar arguments, $G^P_{\boldsymbol{N}_t|\boldsymbol{N}_0=\boldsymbol{n}}(\boldsymbol{z}) = (P^{(t)}_{\to} \times \boldsymbol{z})^{\boldsymbol{n}}$. If μ is the invariant measure of $(C_{i,t})_t$, then

$$P^{(t)}_{\to} \underset{t\to+\infty}{\longrightarrow} \begin{pmatrix} \mu \\ \dots \\ \mu \end{pmatrix} \Rightarrow P^{(t)}_{\to} \times \boldsymbol{z} = \begin{pmatrix} \mu^T \boldsymbol{z} \\ \dots \\ \mu^T \boldsymbol{z} \end{pmatrix}$$

$$\Rightarrow G^P_{\boldsymbol{N}_t|\boldsymbol{N}_{t-1}=\boldsymbol{n}}(\boldsymbol{z}) = (\mu^T \boldsymbol{z})^{n_1} \times \cdots \times (\mu^T \boldsymbol{z})^{n_J} = (\mu^T \boldsymbol{z})^{\sum_i n_i}.$$

In other words, the probability generating function of $\boldsymbol{N}_t \mid \boldsymbol{N}_0 = \boldsymbol{n}$ is the p.g.f. of a multinomial distribution. Since we obtain a limiting distribution for the Markov chain $(\boldsymbol{N}_t)_t$ is also its invariant measure, see e.g. (Norris, 1997, p. 33). □

Proof of Prop. 3.3.5. As a sum of independent binomially distributed variables, the mass probability function of the portfolio size $N_{j,t}$ is given by

$$P(N_{j,t} = m_j | \boldsymbol{N}_{t-1} = \boldsymbol{n}) = \sum_{\substack{0 \le c_1,\dots,c_J \le n \\ \text{s.t. } \sum_l c_l = m_j}} \prod_{l \in J} \binom{n_l}{c_j} (p_{l\to j}(\boldsymbol{x}_t))^{c_j} \left(1 - p_{l\to j}(\boldsymbol{x}_t)\right)^{n_l - c_j}.$$

By A4, the probability generating function of the sum constituting $N_{j,t}$ is the product of generating function of each binomially distributed random variables

$$G^P_{N_{j,t}|\boldsymbol{N}_{t-1}=\boldsymbol{n}}(z) = (1 - p_{1\to j}(\boldsymbol{x}_t) + p_{1\to j}(\boldsymbol{x}_t) z)^{n_1} \times \cdots \times (1 - p_{J\to j}(\boldsymbol{x}_t) + p_{J\to j}(\boldsymbol{x}_t) z)^{n_J}.$$

Differentiating with respect to z, we get

$$G^P_{N_{j,t}|\boldsymbol{N}_{t-1}=\boldsymbol{n}}{}'(z) = \sum_{k=1}^{J} n_k p_{k\to j}(\boldsymbol{x}_t)(1 - p_{k\to j}(\boldsymbol{x}_t) + p_{k\to j}(\boldsymbol{x}_t) z)^{n_k - 1} \prod_{l \neq k} (1 - p_{l\to j}(\boldsymbol{x}_t) + p_{l\to j}(\boldsymbol{x}_t) z)^{n_l}.$$

Taking $z = 1$ leads to the result. □

3.8.1.3 Properties of a constant regulated price vector

Proof of Prop. 3.3.8. By construction, $(C_{i,t})_t$ is a Markov chain with transition matrix $P_{\to}(\boldsymbol{x}_t)$. When $\boldsymbol{x}_t = \boldsymbol{x}$, $P_{\to}(\boldsymbol{x}_t)$ is fixed across time that is

$$P_{\to}(\boldsymbol{x}_t) = \begin{pmatrix} p_{1\to 1}(\boldsymbol{x}) & \dots & p_{1\to J}(\boldsymbol{x}) \\ & \dots & \\ p_{J\to 1}(\boldsymbol{x}) & \dots & p_{J\to J}(\boldsymbol{x}) \end{pmatrix} = P_{\to}.$$

When $\boldsymbol{x} = (x, \dots, x)$ using (3.12), we have

$$p_{j\to k}(\boldsymbol{x}) = \begin{cases} p_{j\to j} & \text{if } j = k, \\ p_{j\neq} & \text{if } j \neq k, \end{cases}$$

Since $\sum_k p_{j\to k} = 1$, we deduce $p_{j\neq} = (1-p_{j\to j})/(J-1)$. Therefore,

$$P_{\to} = \begin{pmatrix} p_{1\to 1} & \frac{1-p_{1\to 1}}{J-1} & \dots \\ & p_{j\to j} & \\ \dots & \frac{1-p_{J\to J}}{J-1} & p_{J\to J} \end{pmatrix}.$$

Since the number of state is finite (J) and the Markov chain is irreducible by Proposition 3.8.2, there exists a unique invariant measure μ, see e.g. Norris (1997).

Let us consider the general matrix $M \in \mathbb{R}^{J\times J}$ with general term $M_{i,j} = a_i(1-\delta_{ij}) + (1-(J-1)a_i)\delta_{ij}$. Note that the rows of M equal 1. In other words, M has only two different terms by rows

$$M = \begin{pmatrix} 1-(J-1)a_1 & a_1 & \dots & & \\ & \ddots & & & \\ \dots & a_i & 1-(J-1)a_i & & a_i \\ & & \ddots & & \\ & \dots & a_J & & 1-(J-1)a_J \end{pmatrix}.$$

The reversibility conditions for a measure μ are

$$\begin{cases} \mu_1 M_{1,2} = \mu_2 M_{2,1} \\ \dots \\ \mu_1 M_{1,J} = \mu_J M_{J,1} \end{cases} \Leftrightarrow \begin{cases} \mu_1 a_1 = \mu_2 a_2 \\ \dots \\ \mu_1 a_1 = \mu_J a_J \end{cases} \Leftrightarrow \begin{cases} \mu_1 a_1/a_2 = \mu_2 \\ \dots \\ \mu_1 a_1/a_J = \mu_J \end{cases}$$

Let $a^{\Pi}_{-i} = \prod_{j=1, j\neq i}^{J} a_j$. Using $\mu_1 + \dots + \mu_J = 1$, we get by multiplying both sides by a^{Π}_{-i}

$$\mu_1 + \sum_{i>2} \mu_1 \frac{a_1}{a_i} = 1 \Leftrightarrow \mu_1 a^{\Pi}_{-1} + \mu_1 \sum_{i>2} a^{\Pi}_{-i} = a^{\Pi}_{-1} \Leftrightarrow \mu_1 = \frac{a^{\Pi}_{-1}}{\sum_{i=1}^{J} a^{\Pi}_{-i}}.$$

Thus, the following measure μ is in detailed balance with M $\mu = \left(\frac{a^{\Pi}_{-1}}{\sum_{i=1}^{J} a^{\Pi}_{-i}}, \dots, \frac{a^{\Pi}_{-J}}{\sum_{i=1}^{J} a^{\Pi}_{-i}} \right)$. μ is also an invariant measure for M. Setting $a_j = p_{j\neq}$, we obtain the following invariant measure for $P_{\to}$

$$\mu_i = \frac{\prod_{j\neq i} p_{j\neq}}{\sum_{l=1}^{J} \prod_{j\neq l} p_{j\neq}}.$$

In the special case where $p_{j\to j}$ are identical across insurers, $p_{j\neq} = p_{\neq}$ is constant. Hence for all $j = 1, \dots, J$

$$\prod_{j\neq i} p_{j\neq} = (p_{\neq})^{J-1} \Rightarrow \mu_i = \frac{(p_{\neq})^{J-1}}{\sum_{l=1}^{J} (p_{\neq})^{J-1}} = \frac{(p_{\neq})^{J-1}}{J \times (p_{\neq})^{J-1}} = \frac{1}{J}.$$

□

Proof of Prop. 3.3.9. As the transition matrix of $(C_{1,t}, \dots, C_{N,t})_t$ is $P_t \otimes^N = P \otimes^N$, we get a time homogeneous Markov chain. By (Norris, 1997, p. 41), if two independent Markov chains have the same invariant measure, then the Markov chain product is the Kronecker product of the invariant measure. So the process $(C_{1,t}, \dots, C_{2,t})_t$ has the invariant measure $\mu \otimes \mu = (1/J^2, \dots, 1/J^2)$. Iterating $N-1$ more times leads to the result. □

Proof of Prop. 3.3.10. Direct application of Proposition 3.3.3. □

Proof of Prop. 3.3.11. Direct application of Proposition 3.3.4. □

Proof of Prop. 3.3.6. Using Proposition 3.3.4 and the particular form of the transition matrix

$$P_{\rightarrow} = \begin{pmatrix} p_{1\rightarrow 1} & \frac{1-p_{1\rightarrow 1}}{J-1} & \dots \\ & p_{j\rightarrow j} & \\ \dots & \frac{1-p_{J\rightarrow J}}{J-1} & p_{J\rightarrow J} \end{pmatrix}.$$

we deduce that the probabilities appearing in Proposition 3.3.4 simplify to

$$\begin{aligned} \frac{n_j!}{c_{j1}!\dots c_{jJ}!}(p_{j\rightarrow j})^{c_{jj}}(p_{j\neq})^{c_{j1}}\dots(p_{j\neq})^{c_{jJ}} &= \frac{n_j!}{c_{j1}!\dots c_{jJ}!}(p_{j\rightarrow j})^{c_{jj}}\left(\frac{1-p_{j\rightarrow j}}{J-1}\right)^{\sum\limits_{l\neq j} c_{jl}} \\ &= \frac{n_j!}{c_{j1}!\dots c_{jJ}!}(p_{j\rightarrow j})^{c_{jj}}\left(\frac{1-p_{j\rightarrow j}}{J-1}\right)^{n_j-c_{jj}}. \end{aligned}$$

Therefore we get the desired by summing over appropriate indexes. □

Proof of Prop. 3.3.7. For a constant price vector, identical players ($p_{j\rightarrow j} = p_=$ and $p_{j\neq} = p_{\neq}$) and Proposition 3.3.5, the probability generating function is

$$\begin{aligned} G^P_{N_{j,t}|\boldsymbol{N}_{t-1}=\boldsymbol{n}}(z) &= (1-p_{1\rightarrow j}+p_{1\rightarrow j}z)^{n_1}\times\dots\times(1-p_{J\rightarrow j}+p_{J\rightarrow j}z)^{n_J} \\ &= (1-p_=+p_=z)^{n_j}\prod_{l\neq j}(1-p_{\neq}+p_{\neq}z)^{n_l} \\ &= (1-p_=+p_=z)^{n_j}(1-p_{\neq}+p_{\neq}z)^{N-n_j}. \end{aligned}$$

In other words, $N_{j,t}|\boldsymbol{N}_{t-1} = \boldsymbol{n}$ is a sum of two binomially distributed random variables $\mathcal{B}(n_j, p_=)$ and $\mathcal{B}(n - n_j, \frac{1-p_=}{J-1})$. □

3.8.1.4 Properties of a time-independent price vector

Proof of Prop. 3.3.12. Since $\sum_k p_{j\rightarrow k} = 1$, we get $p_{j\neq} = (1-p_{j\rightarrow j}-p_{j\rightarrow 1})/(J-2)$, and $p_{1\neq} = \frac{1-p_{1\rightarrow 1}}{J-1}$. We use the following notation

$$P_{\rightarrow} = \begin{pmatrix} p_{1\rightarrow 1} & p_{1\neq} & \dots & & \\ p_{2\rightarrow 1} & p_{2\rightarrow 2} & p_{2\neq} & \dots & \\ p_{3\rightarrow 1} & p_{3\neq} & p_{3\rightarrow 3} & p_{3\neq} & \dots \\ & & & \ddots & \\ p_{J\rightarrow 1} & p_{J\neq} & \dots & p_{J\neq} & p_{J\rightarrow J} \end{pmatrix} = \begin{pmatrix} a_1 & b_1 & \dots & & \\ b_2 & a_2 & c_2 & \dots & \\ b_3 & c_3 & a_3 & c_3 & \dots \\ & & & \ddots & \\ b_J & c_J & \dots & c_J & a_J \end{pmatrix}. \quad (3.22)$$

A first series of equation for the invariant measure is obtained from $\mu' = \mu' P_{\rightarrow}$. That is

$$\begin{cases} \mu_1 = a_1\mu_1 + b_2\mu_2 + b_3\mu_3 + b_4\mu_4 + \dots + b_J\mu_J \\ \mu_2 = b_1\mu_1 + a_2\mu_2 + c_3\mu_3 + c_4\mu_4 + \dots + c_J\mu_J \\ \mu_3 = b_1\mu_1 + c_2\mu_2 + a_3\mu_3 + c_4\mu_4 + \dots + c_J\mu_J \\ \vdots \\ \mu_J = b_1\mu_1 + c_2\mu_2 + \dots + c_{J-1}\mu_{J-1} + a_J\mu_J \end{cases}$$

Ignoring the first equation and subtracting the second equation to all others, we get

$$\begin{cases} \mu_2 = b_1\mu_1 + a_2\mu_2 + c_3\mu_3 + c_4\mu_4 + \dots + c_J\mu_J \\ \mu_3 - \mu_2 = (c_2 - a_2)\mu_2 + (a_3 - c_3)\mu_3 \\ \vdots \\ \mu_J - \mu_2 = (c_2 - a_2)\mu_2 + (a_J - c_J)\mu_J \end{cases}$$

$$\Leftrightarrow \begin{cases} -b_1\mu_1 = -\mu_2 + a_2\mu_2 + c_3\mu_3 + c_4\mu_4 + \cdots + c_J\mu_J \\ \mu_3(1 - a_3 + c_3) = (c_2 - a_2 + 1)\mu_2 \\ \vdots \\ \mu_J(1 - a_J + c_J) = (c_2 - a_2 + 1)\mu_2 \end{cases}$$

The $J-2$ equations give

$$\mu_j = \mu_2 \frac{c_2 - a_2 + 1}{c_j - a_j + 1}, j > 2$$

Recalling that any row of M sums up to 1, $a_i + b_i + c_i(J-2) = 1$ for $i \neq 1$, we have

$$\forall j = 3, \dots, J, a_j = 1 - c_j(J-2) - b_j \Rightarrow c_j - a_j + 1 = 1 - (1 - c_j(J-2) - b_j) + c_j = (J-1)c_j + b_j = d_j$$

Note that in the special case $b_j = c_j$, $d_j = Jb_j$ leads back to $\mu_j = b_2\mu_2/b_j$. So in general, $\mu_j = \mu_2 \frac{d_2}{d_j}$ for $j = 3, \dots, J$. The first equation for μ_1 becomes

$$b_1\mu_1 = \mu_2 - a_2\mu_2 - \sum_{j=3}^{J} c_j\mu_2\frac{d_2}{d_j} \Leftrightarrow b_1\mu_1 = \mu_2(1 - a_2) - \mu_2\sum_{j=3}^{J} c_j\frac{d_2}{d_j}$$

$$\Leftrightarrow \mu_1 = \mu_2\frac{(b_2 + (J-2)c_2)}{b_1} - \mu_2\sum_{j=3}^{J} c_j\frac{d_2}{b_1 d_j}\mu_1 = \mu_2\frac{b_2}{b_1} + \mu_2\frac{d_2}{b_1}\sum_{j=3}^{J}(\frac{c_2}{d_2} - \frac{c_j}{d_j}).$$

The probability condition $\sum_i \mu_i = 1$ gives

$$\mu_2\frac{b_2}{b_1} + \mu_2\frac{d_2}{b_1}\sum_{j=3}^{J}(\frac{c_2}{d_2} - \frac{c_j}{d_j}) + \mu_2 + \mu_2\sum_{j>2}\frac{d_2}{d_j} = 1$$

$$\Leftrightarrow \mu_2 + \mu_2\frac{b_2}{b_1} + \mu_2\frac{d_2}{b_1}\sum_{j=3}^{J}(\frac{c_2}{d_2} - \frac{c_j}{d_j} + \frac{b_1}{d_j}) = 1 \Leftrightarrow \mu_2 = \left(1 + \frac{b_2}{b_1} + \frac{d_2}{b_1}\sum_{j=3}^{J}(\frac{c_2}{d_2} - \frac{c_j}{d_j} + \frac{b_1}{d_j})\right)^{-1}.$$

Reintroducing the product notation $d^{\Pi}_{-1,-j} = \prod_{l\neq 1,j} d_l$ yields the following reformulation

$$\begin{aligned} \mu_2 &= \frac{d^{\Pi}_{-1,-2}}{d^{\Pi}_{-1,-2} + d^{\Pi}_{-1,-2}\frac{b_2}{b_1} + d^{\Pi}_{-1}\frac{1}{b_1}\sum_{j=3}^{J}(\frac{c_2}{d_2} - \frac{c_j}{d_j} + \frac{b_1}{d_j})} \\ &= \frac{d^{\Pi}_{-1,-2}b_1}{d^{\Pi}_{-1,-2}(b_1 + b_2) + (J-2)c_2 d^{\Pi}_{-1,-2} + \sum_{j=3}^{J} d^{\Pi}_{-1,-j}(b_1 - c_j)} \\ &= \frac{d^{\Pi}_{-1,-2}b_1}{d^{\Pi}_{-1,-2}(b_1 + d_2 - c_2) + \sum_{j=3}^{J} d^{\Pi}_{-1,-j}(b_1 - c_j)} \\ &= \frac{d^{\Pi}_{-1,-2}b_1}{d^{\Pi}_{-1} + \sum\limits_{j=2}^{J} d^{\Pi}_{-1,-j}(b_1 - c_j)} \end{aligned}$$

Using $\mu_j = \frac{d_2}{d_j}\mu_2$, we obtain (also valid for $j = 2$)

$$\mu_j = \frac{d^{\Pi}_{-1,-j}b_1}{d^{\Pi}_{-1} + \sum\limits_{j=2}^{J} d^{\Pi}_{-1,-j}(b_1 - c_j)}, j > 2, \mu_1 = \frac{d^{\Pi}_{-1} + \sum\limits_{j=2}^{J} d^{\Pi}_{-1,-j}(-c_j)}{d^{\Pi}_{-1} + \sum\limits_{j=2}^{J} d^{\Pi}_{-1,-j}(b_1 - c_j)}.$$

Let us go back to the original transition matrix (3.22)

$$a_1 = p_{1\to 1}, b_1 = p_{1\neq} = \frac{1-p_{1\to 1}}{J-1}, \forall j>1, a_j = p_{j\to j}, b_j = p_{j\to 1}, c_j = p_{j\neq} = \frac{1-p_{j\to j}-p_{j\to 1}}{J-2}.$$

So $d_j = (J-1)c_j + b_j = (J-1)p_{j\neq} + p_{j\to 1}$

$$\mu_1 = \frac{d^{\Pi}_{-1} - \sum_{j=2}^{J} d^{\Pi}_{-1,-j} p_{j\neq}}{d^{\Pi}_{-1} + \sum_{j=2}^{J} d^{\Pi}_{-1,-j}(p_{1\neq} - p_{j\neq})}, \mu_j = \frac{d^{\Pi}_{-1,-j} p_{1\neq}}{d^{\Pi}_{-1} + \sum_{j=2}^{J} d^{\Pi}_{-1,-j}(p_{1\neq} - p_{j\neq})}, j = 2, \dots, J.$$

In the special case where of identical insurers, then $\forall j \neq 1, p_{j\neq} = p_{2\neq}$ and $p_{j\to 1} = p_{2\to 1}$. So $d_j = d_2$ $\Rightarrow d^{\Pi}_{-1,-j} = d_2^{J-2} \Rightarrow d^{\Pi}_{-1} = d_2^{J-1}$. Hence

$$\forall j \geq 2, \mu_j = \frac{d_2^{J-2} p_{1\neq}}{d_2^{J-1} + \sum_{j=2}^{J} d_2^{J-2}(p_{1\neq} - p_{2\neq})} = \frac{p_{1\neq}}{p_{2\to 1} + (J-1)p_{1\neq}}, \mu_1 = \frac{p_{2\to 1}}{p_{2\to 1} + (J-1)p_{1\neq}}.$$

□

Proof of Prop. 3.3.11. Direct application of Proposition 3.3.4. □

3.8.1.5 Transition matrix of insurers portfolio sizes

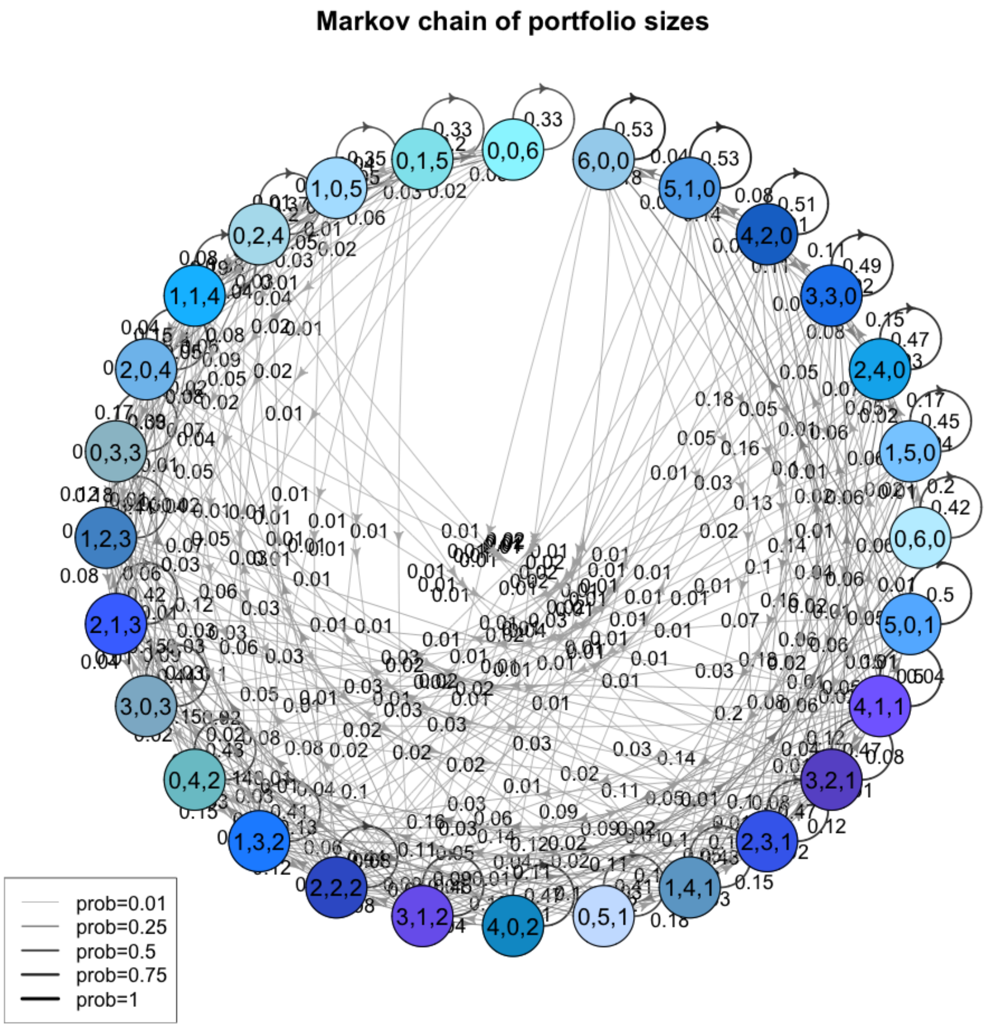


Figure 3.12 – The markov chain of insurers portfolio sizes $(\boldsymbol{N}_t)_t$ (transition probabilities lower than 1% are not plotted and line thickness increases the probability value)

	6,0,0	5,1,0	4,2,0	3,3,0	2,4,0	1,5,0	0,6,0	5,0,1	4,1,1	3,2,1
6,0,0	5.31e-01	1.77e-01	2.46e-02	1.82e-03	7.59e-05	1.69e-06	1.56e-08	1.77e-01	4.92e-02	5.47e-03
5,1,0	4.13e-02	5.19e-01	1.42e-01	1.57e-02	8.73e-04	2.42e-05	2.69e-07	5.28e-02	1.55e-01	3.28e-02
4,2,0	3.21e-03	7.97e-02	5.03e-01	1.09e-01	9.04e-03	3.34e-04	4.62e-06	7.14e-03	9.81e-02	1.28e-01
3,3,0	2.50e-04	9.26e-03	1.15e-01	4.83e-01	7.83e-02	4.31e-03	7.95e-05	7.92e-04	2.01e-02	1.35e-01
2,4,0	1.94e-05	9.58e-04	1.77e-02	1.46e-01	4.59e-01	4.97e-02	1.37e-03	8.00e-05	2.98e-03	3.75e-02
1,5,0	1.51e-06	9.30e-05	2.29e-03	2.82e-02	1.74e-01	4.33e-01	2.35e-02	7.65e-06	3.77e-04	7.00e-03
0,6,0	1.18e-07	8.67e-06	2.66e-04	4.36e-03	4.02e-02	1.98e-01	4.05e-01	7.06e-07	4.34e-05	1.07e-03
5,0,1	5.31e-02	6.79e-02	1.64e-02	1.73e-03	9.37e-05	2.56e-06	2.81e-08	4.99e-01	1.53e-01	1.85e-02
4,1,1	4.13e-03	5.58e-02	6.31e-02	1.23e-02	9.78e-04	3.54e-05	4.84e-07	4.27e-02	4.88e-01	1.18e-01
3,2,1	3.21e-04	8.27e-03	5.78e-02	5.80e-02	8.61e-03	4.59e-04	8.32e-06	3.63e-03	8.25e-02	4.73e-01
2,3,1	2.50e-05	9.49e-04	1.23e-02	5.91e-02	5.28e-02	5.33e-03	1.43e-04	3.06e-04	1.05e-02	1.19e-01
1,4,1	1.94e-06	9.76e-05	1.86e-03	1.63e-02	5.96e-02	4.76e-02	2.46e-03	2.56e-05	1.17e-03	2.00e-02
0,5,1	1.51e-07	9.44e-06	2.38e-04	3.03e-03	2.00e-02	5.96e-02	4.23e-02	2.13e-06	1.23e-04	2.80e-03
4,0,2	5.31e-03	1.18e-02	7.77e-03	1.38e-03	1.06e-04	3.75e-06	5.06e-08	9.80e-02	1.21e-01	2.49e-02
3,1,2	4.13e-04	5.97e-03	1.16e-02	6.89e-03	9.45e-04	4.88e-05	8.71e-07	8.01e-03	1.03e-01	1.12e-01
2,2,2	3.21e-05	8.58e-04	6.56e-03	1.12e-02	6.04e-03	5.72e-04	1.50e-05	6.54e-04	1.61e-02	1.07e-01
1,3,2	2.50e-06	9.73e-05	1.32e-03	7.07e-03	1.08e-02	5.21e-03	2.58e-04	5.32e-05	1.93e-03	2.40e-02
0,4,2	1.94e-07	9.95e-06	1.95e-04	1.80e-03	7.49e-03	1.03e-02	4.43e-03	4.32e-06	2.08e-04	3.79e-03
3,0,3	5.31e-04	1.68e-03	1.86e-03	8.12e-04	1.04e-04	5.19e-06	9.11e-08	1.46e-02	3.17e-02	1.98e-02
2,1,3	4.13e-05	6.36e-04	1.72e-03	1.75e-03	6.87e-04	6.13e-05	1.57e-06	1.18e-03	1.65e-02	3.10e-02
1,2,3	3.21e-06	8.88e-05	7.37e-04	1.74e-03	1.63e-03	5.71e-04	2.70e-05	9.45e-05	2.44e-03	1.81e-02
0,3,3	2.50e-07	9.97e-06	1.42e-04	8.31e-04	1.74e-03	1.50e-03	4.64e-04	7.58e-06	2.86e-04	3.78e-03
2,0,4	5.31e-05	2.18e-04	3.43e-04	2.49e-04	7.77e-05	6.56e-06	1.64e-07	1.94e-03	6.05e-03	6.50e-03
1,1,4	4.13e-06	6.75e-05	2.32e-04	3.34e-04	2.25e-04	6.23e-05	2.82e-06	1.55e-04	2.33e-03	6.17e-03
0,2,4	3.21e-07	9.19e-06	8.21e-05	2.43e-04	3.23e-04	2.02e-04	4.85e-05	1.24e-05	3.34e-04	2.70e-03
1,0,5	5.31e-06	2.69e-05	5.46e-05	5.61e-05	2.95e-05	6.79e-06	2.95e-07	2.42e-04	9.83e-04	1.51e-03
0,1,5	4.13e-07	7.14e-06	2.95e-05	5.49e-05	5.28e-05	2.58e-05	5.08e-06	1.92e-05	3.09e-04	1.04e-03
0,0,6	5.31e-07	3.19e-06	7.97e-06	1.06e-05	7.97e-06	3.19e-06	5.31e-07	2.91e-05	1.45e-04	2.91e-04

	2,3,1	1,4,1	0,5,1	4,0,2	3,1,2	2,2,2	1,3,2	0,4,2	3,0,3	2,1,3
6,0,0	3.04e-04	8.44e-06	9.38e-08	2.46e-02	5.47e-03	4.56e-04	1.69e-05	2.34e-07	1.82e-03	3.04e-04
5,1,0	2.69e-03	9.88e-05	1.37e-06	1.28e-02	1.84e-02	2.84e-03	1.53e-04	2.80e-06	1.35e-03	1.09e-03
4,2,0	1.96e-02	1.06e-03	1.92e-05	4.70e-03	2.00e-02	1.21e-02	1.17e-03	3.08e-05	8.36e-04	1.65e-03
3,3,0	9.84e-02	9.70e-03	2.58e-04	8.77e-04	1.25e-02	2.18e-02	6.54e-03	2.98e-04	3.82e-04	1.76e-03
2,4,0	1.64e-01	6.63e-02	3.18e-03	1.25e-04	3.20e-03	2.19e-02	1.87e-02	2.31e-03	9.10e-05	1.29e-03
1,5,0	5.80e-02	1.85e-01	3.31e-02	1.55e-05	5.79e-04	7.26e-03	3.15e-02	1.11e-02	1.60e-05	4.03e-04
0,6,0	1.31e-02	8.04e-02	1.98e-01	1.76e-06	8.67e-05	1.60e-03	1.31e-02	4.02e-02	2.35e-06	8.67e-05
5,0,1	1.11e-03	3.33e-05	3.97e-07	1.36e-01	3.18e-02	2.78e-03	1.08e-04	1.56e-06	1.50e-02	2.59e-03
4,1,1	1.07e-02	4.27e-04	6.38e-06	4.70e-02	1.15e-01	1.91e-02	1.10e-03	2.10e-05	9.15e-03	1.02e-02
3,2,1	8.49e-02	5.11e-03	1.02e-04	6.78e-03	8.69e-02	8.91e-02	9.52e-03	2.69e-04	3.99e-03	1.34e-02
2,3,1	4.53e-01	5.41e-02	1.63e-03	7.92e-04	1.90e-02	1.19e-01	6.07e-02	3.14e-03	7.94e-04	1.05e-02
1,4,1	1.51e-01	4.30e-01	2.57e-02	8.40e-05	2.98e-03	3.54e-02	1.43e-01	3.06e-02	1.19e-04	2.88e-03
0,5,1	3.19e-02	1.80e-01	4.03e-01	8.40e-06	3.96e-04	6.98e-03	5.46e-02	1.60e-01	1.53e-05	5.47e-04
4,0,2	2.08e-03	7.87e-05	1.13e-06	4.63e-01	1.23e-01	1.21e-02	5.17e-04	8.20e-06	9.98e-02	1.83e-02
3,1,2	1.75e-02	9.77e-04	1.86e-05	4.32e-02	4.53e-01	8.94e-02	5.81e-03	1.24e-04	4.14e-02	7.89e-02
2,2,2	1.02e-01	1.08e-02	3.05e-04	3.94e-03	8.34e-02	4.38e-01	5.72e-02	1.85e-03	6.36e-03	7.59e-02
1,3,2	1.09e-01	9.22e-02	5.01e-03	3.55e-04	1.14e-02	1.20e-01	4.18e-01	2.73e-02	7.81e-04	1.78e-02
0,4,2	3.17e-02	1.10e-01	8.22e-02	3.15e-05	1.36e-03	2.18e-02	1.53e-01	3.94e-01	8.66e-05	2.93e-03
3,0,3	2.81e-03	1.50e-04	2.76e-06	1.35e-01	1.60e-01	2.63e-02	1.54e-03	3.04e-05	4.24e-01	9.18e-02
2,1,3	1.74e-02	1.70e-03	4.61e-05	1.16e-02	1.42e-01	1.46e-01	1.63e-02	4.81e-04	4.28e-02	4.15e-01
1,2,3	3.00e-02	1.50e-02	7.68e-04	9.85e-04	2.32e-02	1.47e-01	1.33e-01	7.58e-03	4.18e-03	8.27e-02
0,3,3	1.95e-02	2.88e-02	1.28e-02	8.35e-05	2.91e-03	3.46e-02	1.50e-01	1.19e-01	3.97e-04	1.20e-02
2,0,4	2.63e-03	2.40e-04	6.31e-06	2.67e-02	5.65e-02	3.31e-02	3.36e-03	9.38e-05	1.64e-01	1.85e-01
1,1,4	6.07e-03	2.18e-03	1.06e-04	2.22e-03	3.01e-02	5.51e-02	2.87e-02	1.52e-03	1.47e-02	1.73e-01
0,2,4	6.23e-03	5.63e-03	1.78e-03	1.84e-04	4.61e-03	3.31e-02	5.32e-02	2.45e-02	1.30e-03	2.95e-02
1,0,5	1.05e-03	2.97e-04	1.37e-05	4.43e-03	1.35e-02	1.41e-02	5.20e-03	2.59e-04	4.04e-02	8.34e-02
0,1,5	1.47e-03	9.46e-04	2.32e-04	3.62e-04	5.32e-03	1.38e-02	1.31e-02	4.23e-03	3.47e-03	4.57e-02
0,0,6	2.91e-04	1.45e-04	2.91e-05	6.62e-04	2.65e-03	3.97e-03	2.65e-03	6.62e-04	8.04e-03	2.41e-02

	1,2,3	0,3,3	2,0,4	1,1,4	0,2,4	1,0,5	0,1,5	0,0,6
6,0,0	1.69e-05	3.13e-07	7.59e-05	8.44e-06	2.34e-07	1.69e-06	9.38e-08	1.56e-08
5,1,0	1.09e-04	2.91e-06	7.28e-05	3.22e-05	1.56e-06	1.99e-06	3.78e-07	2.19e-08
4,2,0	5.05e-04	2.31e-05	6.40e-05	6.18e-05	7.82e-06	2.27e-06	8.75e-07	3.06e-08
3,3,0	1.25e-03	1.43e-04	4.87e-05	9.35e-05	2.43e-05	2.44e-06	1.71e-06	4.29e-08
2,4,0	2.19e-03	5.57e-04	2.85e-05	1.18e-04	6.03e-05	2.40e-06	3.07e-06	6.00e-08
1,5,0	2.68e-03	1.69e-03	8.40e-06	1.14e-04	1.32e-04	1.93e-06	5.25e-06	8.40e-08
0,6,0	1.07e-03	4.36e-03	1.76e-06	4.34e-05	2.66e-04	7.06e-07	8.67e-06	1.18e-07
5,0,1	1.49e-04	2.84e-06	8.33e-04	9.49e-05	2.70e-06	2.31e-05	1.31e-06	2.56e-07
4,1,1	1.08e-03	3.01e-05	7.26e-04	4.00e-04	2.04e-05	2.62e-05	5.88e-06	3.59e-07
3,2,1	5.59e-03	2.78e-04	5.46e-04	7.41e-04	1.16e-04	2.82e-05	1.39e-05	5.02e-07
2,3,1	1.30e-02	2.01e-03	3.09e-04	1.02e-03	3.74e-04	2.75e-05	2.74e-05	7.03e-07
1,4,1	1.82e-02	8.30e-03	7.92e-05	1.03e-03	9.45e-04	2.18e-05	4.95e-05	9.84e-07
0,5,1	6.51e-03	2.58e-02	1.45e-05	3.49e-04	2.09e-03	7.04e-06	8.48e-05	1.38e-06
4,0,2	1.12e-03	2.25e-05	8.24e-03	9.78e-04	2.90e-05	3.04e-04	1.77e-05	4.20e-06
3,1,2	9.20e-03	2.75e-04	6.11e-03	4.65e-03	2.54e-04	3.25e-04	9.12e-05	5.88e-06
2,2,2	5.46e-02	3.06e-03	3.34e-03	7.93e-03	1.70e-03	3.15e-04	2.21e-04	8.24e-06
1,3,2	1.03e-01	2.79e-02	7.11e-04	8.68e-03	5.74e-03	2.45e-04	4.39e-04	1.15e-05
0,4,2	3.30e-02	1.23e-01	1.11e-04	2.57e-03	1.48e-02	6.81e-05	7.97e-04	1.61e-05
3,0,3	6.39e-03	1.45e-04	6.82e-02	8.74e-03	2.77e-04	3.74e-03	2.29e-04	6.89e-05
2,1,3	5.91e-02	2.05e-03	3.59e-02	4.80e-02	2.92e-03	3.60e-03	1.41e-03	9.65e-05
1,2,3	3.99e-01	2.83e-02	5.89e-03	6.54e-02	2.49e-02	2.75e-03	3.50e-03	1.35e-04
0,3,3	1.19e-01	3.79e-01	7.61e-04	1.64e-02	8.79e-02	6.30e-04	7.03e-03	1.89e-04
2,0,4	2.16e-02	6.66e-04	3.84e-01	5.97e-02	2.20e-03	4.12e-02	2.76e-03	1.13e-03
1,1,4	1.68e-01	1.01e-02	4.18e-02	3.74e-01	2.88e-02	3.08e-02	2.17e-02	1.58e-03
0,2,4	1.79e-01	1.51e-01	4.32e-03	8.06e-02	3.58e-01	5.40e-03	5.54e-02	2.22e-03
1,0,5	4.54e-02	2.48e-03	1.85e-01	1.98e-01	1.24e-02	3.44e-01	2.87e-02	1.85e-02
0,1,5	8.10e-02	3.88e-02	1.74e-02	1.95e-01	1.78e-01	4.02e-02	3.33e-01	2.60e-02
0,0,6	2.41e-02	8.04e-03	5.49e-02	1.10e-01	5.49e-02	2.00e-01	2.00e-01	3.04e-01

Table 3.17 – Values of the transition matrix

3.8.1.6 Properties of the loss model

Proof of Prop. 3.3.14. Using assumptions A5, A6, A7, we can show that the moment generating function of $S_{j,t}$ given that $N_{j,t} = n_j$ is

$$\forall u, G^M_{S_{j,t}}(u) = (G^M_{Y_{i,t}}(u))^{n_j}.$$

Since $Y_{i,t}$ is a compound distribution, we get

$$G^M_{Y_{i,t}}(u) = G^P_{M_{i,t}}(G^M_Z(u)).$$

As we use a classic frequency distribution (either Poisson or negative binomial), we obtain

$$G^P_{M_{i,t}}(u) = \exp(\lambda(u-1)) \text{ or } \left(\frac{p}{1-(1-p)u}\right)^r$$

leading to

$$G^M_{S_{j,t}}(u) = \exp(\lambda n_j (G^M_Z(u) - 1)) \text{ or } \left(\frac{p}{1-(1-p)G^M_Z(u)}\right)^{rn_j}.$$

That is the scale parameter of a Poisson distribution is λ and the scale parameter of a negative binomial is r, see also Appendix 3.8.5.3 for details. □

Proof of Prop. 3.3.15. When a price vector $\boldsymbol{x}$ is such that $x_j < x_k$ for all $k \neq j$, $p_{k\to j}(x) > p_{k\to l}(x)$ for $l \neq j$ given the initial portfolio sizes n_j's are constant, since the change probability $p_{k\to j}$ (for $k \neq j$) is a decreasing function (see Appendix 3.8.3.1). Using the stochastic orders ($\leq_{\text{st}}$, $\leq_{\text{cx}}$) and the majorization order ($\leq_{\text{m}}$) whose definitions and main properties are recalled in the Appendices 3.8.1.7 and 3.8.1.8 respectively, we can show a stochastic order of the portfolio size by applying the convolution property of the stochastic order J times: $N_k(\boldsymbol{x}) \leq_{\text{st}} N_j(\boldsymbol{x}), \forall k \neq j$. Let us consider the empirical loss average of an insurer with portfolio size n

$$\overline{A}(n) = \frac{1}{n}\sum_{i=1}^{n} Y_i,$$

where Y_i denotes the total claim amount per policy. Let $n < \tilde{n}$ be two policy numbers and $\boldsymbol{a}_n, \boldsymbol{b}_{\tilde{n}} \in \mathbb{R}^{\tilde{n}}$ be defined as

$$\boldsymbol{b}_{\tilde{n}} = \left(\frac{1}{\tilde{n}}, \dots, \frac{1}{\tilde{n}}\right) \text{ and } \boldsymbol{a}_n = \left(\underbrace{\frac{1}{n}, \dots, \frac{1}{n}}_{\text{size } n}, \underbrace{0, \dots, 0}_{\text{size } \tilde{n}-n}\right).$$

Since $\boldsymbol{b}_{\tilde{n}} \leq_{\text{m}} \boldsymbol{a}_n$ and $(Y_i)_i$'s are i.i.d. random variables, we have

$$\sum_i b_{\tilde{n},i} Y_i \leq_{\text{cx}} \sum_i a_{n,i} Y_i \Leftrightarrow \sum_{i=1}^{\tilde{n}} \frac{1}{\tilde{n}} Y_i \leq_{\text{cx}} \sum_{i=1}^{n} \frac{1}{n} Y_i \Leftrightarrow \overline{A}(\tilde{n}) \leq_{\text{cx}} \overline{A}(n).$$

Using Theorem 3.A.23 of Shaked and Shanthikumar (2007), except that for all ϕ convex, $E(\phi(\overline{A}(n)))$ is a decreasing function of n and $N_k(\boldsymbol{x}) \leq_{\text{st}} N_j(\boldsymbol{x})$, we can show $\overline{A}(N_j(\boldsymbol{x})) \leq_{\text{cx}} \overline{A}(N_k(\boldsymbol{x}))$. □

3.8.1.7 Notation and definition of classic stochastic orders

Using the notation of Shaked and Shanthikumar (2007), we denote by $\leq_{\text{st}}$ the stochastic order, which is characterised as $X \leq_{\text{st}} Y$ if $\forall x \in \mathbb{R}, P(X > x) \leq P(Y > x)$. They are various other *equivalent* characterizations, including expectectation order for all increasing function $E(\phi(X)) \leq E(\phi(Y))$, quantile order $F_X^{-1}(u) \leq F_Y^{-1}(u)$, distribution function order $F_X(x) \geq F_Y(x)$.

One important property of this stochastic order is the stability under convolutions, i.e. if $X \leq_{\text{st}} Y$ and $\tilde{X} \leq_{\text{st}} \tilde{Y}$, then $X + \tilde{X} \leq_{\text{st}} Y + \tilde{Y}$, see theorem 1.A.3 of Shaked and Shanthikumar (2007). By this mean, we can show that an ordering of binomial distributions. If $X \sim \mathcal{B}(n,p)$ and $Y \sim \mathcal{B}(n,q)$, such that $p \leq q$, then $X \leq_{\text{st}} Y$. Theorem 1.A.3 of Shaked and Shanthikumar (2007) also shows that the stochastic order is closed under mixtures.

The stochastic order is sometimes denoted by $\leq_1$ since $X \leq_1 Y$ requires that for all differentiable functions ϕ such that $\forall x, \phi^{(1)}(x) \geq 0$, we have $E(\phi(X)) \leq E(\phi(Y))$. With this reformulation in mind, we define the convex order denoted by $X \leq_2 Y$ or $X \leq_{\text{cx}} Y$ as $E(\phi(X)) \leq E(\phi(Y))$ for all convex functions ϕ. If we restrict to differentiable functions, it means $\forall x, \phi^{(2)}(x) \geq 0$. This explains the relation between notations $\leq_1$ and $\leq_2$.

Note that if expectations exist, then $X \leq_{\text{cx}} Y$ implies that $E(X) = E(Y)$, $Var(X) \leq Var(Y)$ and $E((X-a)_+) \leq E((Y-a)_+)$. By theorem 3.A.12 of Shaked and Shanthikumar (2007), the convex order is closed under mixtures and convolutions. We also have that $X \leq_{\text{cx}} Y$ is equivalent to $-X \leq_{\text{cx}} -Y$.

A third stochastic order is the increasing convex order: $X \leq_{\text{icx}} Y$ if for all increasing convex functions ϕ, $E(\phi(X)) \leq E(\phi(Y))$. For ϕ differentiable, it means that $\phi^{(1)}(x) \geq 0$, $\phi^{(2)}(x) \geq 0$.

3.8.1.8 Notation and definition of majorization orders

Using the book of Marshall and Olkin (1979), the majorization order $\leq_{\text{m}}$ is defined as $a \leq_{\text{m}} \tilde{a}$ if

$$\forall 1 \leq k \leq n-1, \sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} \tilde{a}_i \quad \text{and} \quad \sum_{i=1}^{n} a_i = \sum_{i=1}^{n} \tilde{a}_i.$$

A direct consequence of property B.2.c of Marshall and Olkin (1979) is that if $X_1, \dots, X_n$ are exchangeable and $a, \tilde{a} \in \mathbb{R}^n$, $a \leq_{\text{m}} \tilde{a}$ implies $\sum_i a_i X_i \leq_{\text{cx}} \sum_i \tilde{a}_i X_i$.

3.8.2 Proofs of Section 3.4

3.8.2.1 Existence and uniqueness properties

Proof of Prop. 3.4.1. Since the strategy set is $[\underline{x}, \overline{x}]^J$, it guarantees the market proxy $m_j = m_j(\boldsymbol{x})$ or $m_j = m_j(\boldsymbol{x}, \boldsymbol{n})$ to be positive. Therefore, given x_{-j}, the function $x_j \mapsto O_j(\boldsymbol{x})$ is a quadratic function with second-degree term $-\beta_j x_j^2/m_j$, hence concave. For all insurer, the constraint functions (3.9) are linear. By Theorem 1 of Rosen (1965), the existence of a premium equilibrium is guaranteed. The proof of uniqueness is exactly the same as Dutang et al. (2013). □

Proof of Prop. 3.4.2. Consider a generic objective function

$$O_j(\boldsymbol{x}) = \left(a_j - b_j \frac{x_j}{m_j(\boldsymbol{x})} \right) (x_j - c_j), \text{with } m_j(\boldsymbol{x}) = \sum_{i \neq j} w_i x_i,$$

with known weights $w_i > 0$ and positive constant $a_j, b_j, c_j > 0$. For simplicity, we remove the multiplicative term n_j/N. Note that the weights w_i do not sum up to 1. By assumption, no constraint functions are active, i.e. $\lambda^{j\star} = 0$. Thus if $\boldsymbol{x}^\star$ is a Nash equilibrium, $\boldsymbol{x}^\star$ must verify

$$\forall j, \nabla_{x_j} O_j(\boldsymbol{x}^\star) = 0 \Leftrightarrow -b_j \frac{1}{m_j(\boldsymbol{x})}(x_j - c_j) + a_j - b_j \frac{x_j}{m_j(\boldsymbol{x})} = 0 \Leftrightarrow -b_j(2x_j - c_j) + a_j m_j(\boldsymbol{x}) = 0$$

$$\Leftrightarrow 2b_j x_j - a_j \sum_{i \neq j} w_i x_i = b_j c_j \Leftrightarrow M\boldsymbol{x} = v,$$

with

$$M = \begin{pmatrix} 2b_1 & -a_1 w_1 & -a_1 w_2 & \\ -a_2 w_2 & 2b_2 & -a_2 w_3 & \dots \\ & & \ddots & \\ & \dots & -a_J w_{J-1} & 2b_J \end{pmatrix}, v = \begin{pmatrix} b_1 c_1 \\ \vdots \\ b_J c_J \end{pmatrix}.$$

The matrix M can be rewritten as $M = M_1 M_2$ with

$$M_1 = \begin{pmatrix} 2b_1/w_1 & -a_1 & \dots & & \\ -a_2 & 2b_2/w_2 & -a_2 & \dots & \\ & & \ddots & & \\ & \dots & -a_J & 2b_J/w_J \end{pmatrix}, M_2 = \begin{pmatrix} w_1 & 0 & \dots \\ & \ddots & \\ \dots & 0 & w_J \end{pmatrix}.$$

In the case of objective function (3.7), we choose $a_j = 1+\beta_j$, $b_j = \beta_j \sum_{i\neq j} w_i$ and $c_j = \pi_j$. When the market proxy is the arithmetic mean (3.5), we set $w_j = 1$ and $b_j = \beta_j(J-1)$ leading to

$$v = \begin{pmatrix} \pi_1\beta_1(J-1) \\ \vdots \\ \pi_J\beta_J(J-1) \end{pmatrix}, M_1 = \begin{pmatrix} 2\beta_1(J-1) & -1-\beta_1 & \dots \\ & \ddots & \\ \dots & -1-\beta_J & 2\beta_J(J-1) \end{pmatrix}, M_2 = \begin{pmatrix} 1 & 0 & \dots \\ & \ddots & \\ \dots & 0 & 1 \end{pmatrix}.$$

When the market proxy is the weighted mean (3.6), we set $w_j = n_j$ and $b_j = \beta_j(N-n_j)$ leading to

$$v = \begin{pmatrix} \pi_1\beta_1(N-n_1) \\ \vdots \\ \pi_J\beta_J(N-n_J) \end{pmatrix}, M_1 = \begin{pmatrix} 2\beta_1(N-n_1) & -1-\beta_1 & \dots \\ & \ddots & \\ \dots & -1-\beta_J & 2\beta_J(N-n_J) \end{pmatrix}, M_2 = \begin{pmatrix} n_1 & 0 & \dots \\ & \ddots & \\ \dots & 0 & n_J \end{pmatrix}.$$

□

3.8.2.2 Recursive formula for the determinant of a multidiagonal matrix

Proposition 3.8.1. *Consider the following multi-diagonal matrix for $n \geq 2$*

$$M_n = \begin{pmatrix} u_1 & v_1 & \dots & & \\ v_2 & u_2 & v_2 & \dots & \\ & & \ddots & & \\ & \dots & v_{n-1} & u_{n-1} & v_{n-1} \\ & & \dots & v_n & u_n \end{pmatrix}.$$

Let $w_1 = u_1$ and $w_j = v_j, \forall j = 2, \dots, n$. The determinant is given by

$$det(M_n) = (-1)^{n+1} \sum_{j=1}^{n} w_j \prod_{k\neq j} (v_k - u_k).$$

Proof. Let us find a recursive formula

$$
\begin{aligned}
\det(M_n) &= \begin{vmatrix} u_1 & v_1 & \dots & & v_1-u_1 \\ v_2 & u_2 & v_2 & & 0 \\ & & & \ddots & \vdots \\ & & & u_{n-1} & 0 \\ & & \dots & v_n & u_n - v_n \end{vmatrix} \\
&= (u_n-v_n)\det(M_{n-1}) + (-1)^{n+1}(v_1-u_1)\begin{vmatrix} v_2 & u_2 & v_2 & \\ & \ddots & & \\ & & & u_{n-1} \\ & & \dots & v_n \end{vmatrix} \\
&= (u_n-v_n)\det(M_{n-1}) + (-1)^{n+1}(v_1-u_1)\begin{vmatrix} v_2-u_2 & u_2 & v_2 & & & \\ 0 & v_3 & u_3 & v_3 & \dots & \\ 0 & v_4 & v_4 & u_4 & v_4 & \dots \\ \vdots & & & & \ddots & \\ \vdots & & & & & u_{n-1} \\ 0 & & \dots & & & v_n \end{vmatrix} \\
&= (u_n-v_n)\det(M_{n-1}) + (-1)^{n+1}(v_1-u_1)(v_2-u_2)\begin{vmatrix} v_3 & u_3 & v_3 & \dots & \\ v_4 & v_4 & u_4 & v_4 & \dots \\ & & & \ddots & \\ & & & & u_{n-1} \\ & \dots & & & v_n \end{vmatrix} \\
&= (u_n-v_n)\det(M_{n-1}) + (-1)^{n+1}(v_1-u_1)(v_2-u_2)\dots(v_{n-2}-u_{n-2})\begin{vmatrix} v_{n-1} & u_{n-1} \\ v_n & v_n \end{vmatrix} \\
&= (u_n-v_n)\det(M_{n-1}) + (-1)^{n+1}(v_1-u_1)(v_2-u_2)\dots(v_{n-1}-u_{n-1})v_n
\end{aligned}
$$

Let us guess that $\det(M_n) = (-1)^{n+1}\sum_{j=1}^n w_j \prod_{k\neq j}(v_k-u_k)$. We can check

$$
\begin{aligned}
\det(M_{n+1}) &= (u_{n+1}-v_{n+1})\det(M_n) + (-1)^{n+1}(v_1-u_1)(v_2-u_2)\dots(v_n-u_n)v_{n+1} \\
&= (u_{n+1}-v_{n+1})(-1)^{n+1}\sum_{j=1}^n w_j \prod_{k\neq j}(v_k-u_k) + (-1)^{n+1}(v_1-u_1)(v_2-u_2)\dots(v_n-u_n)v_{n+1} \\
&= (-1)^{n+2}\sum_{j=1}^{n+1} w_j \prod_{k\neq j}(v_k-u_k).
\end{aligned}
$$

Let us check the recurrence for $n=2$: $\det(M_2) = u_1u_2 - v_1v_2 = (u_2-v_2)u_1 + (-1)^{2+1}(v_1-u_1)v_2$. □

3.8.2.3 Sensitivity analysis

Proposition 2.2 of Dutang et al. (2013) state the sensitivity of premium equilibrium with respect to parameters when the objective function is (3.7) and the market proxy is the arithmetic mean (3.5). For sake of completeness, we unify the proof for both market proxies the arithmetic mean (3.5) or the weighted mean (3.6).

Proof of Prop. 3.4.3. The premium equilibrium $\boldsymbol{x}^\star$ solves the necessary Karush-Kuhn-Tucker conditions: for all Insurer j,

$$
\begin{aligned}
&\nabla_{x_j} O_j(\boldsymbol{x}^\star) + \sum_{1\leq l\leq 3} \lambda_l^{j\star}\nabla_{x_j} g_j^l(x_j^\star) = 0, \\
&0 \leq \lambda^{j\star},\ g_j(x_j^\star) \geq 0,\ g_j(x_j^\star)^T\lambda^{j\star} = 0,
\end{aligned}
\tag{3.23}
$$

where $\lambda^{j\star} \in \mathbb{R}^3$ are Lagrange multipliers, see e.g. Facchinei and Kanzow (2009). In the last part of equation (3.23), $g_j(x_j^\star)^T\lambda^{j\star} = 0$ is the complementarity equation implying that the lth constraint g_j^l is either active ($g_j^l(x_j^\star) = 0$) or inactive ($g_j^l(x_j^\star) > 0$), but $\lambda_l^{j\star} = 0$.

If $\lambda_2^{j\star} > 0$ or $\lambda_3^{j\star} > 0$, then $x_j^\star = \underline{x}$ or $\overline{x}$ respectively, then $x_j^\star$ is independent of any parameter.

If $\lambda_1^{j\star} > 0$, then the solvency constraint (3.9) is active, i.e. the premium equilibrium $x_j^\star$ verifies $g_j^1(x_j^\star) = 0$. Hence we have

$$x_j^\star = \pi_j + \frac{k_{995}\sigma(Y)\sqrt{n_j} - K_j}{n_j}. \tag{3.24}$$

Here, the implicit function theorem is not necessary since $x_j^\star$ does not depend on $x_{-j}^\star$. We deduce that $x_j^\star$ is an increasing function of $\pi_j, k_{995}, \sigma(Y)$ and a decreasing function K_j. The function $n_j \mapsto x_j^\star(n_j)$ is not necessarily monotone. Let $z = n_j$. Differentiating Equation (3.24) with respect to z, we get

$$\varphi'(z) = \frac{1}{z^{3/2}}\left(-\frac{k\sigma(Y)}{2} + \frac{K_j}{\sqrt{z}}\right),$$

whose sign depends on the value of the other parameters.

Otherwise $\lambda_1^{j\star} = \lambda_2^{j\star} = \lambda_3^{j\star} = 0$ Insurer j's premium equilibrium verifies $\nabla_{x_j}O_j(x^\star) = 0$. If the market proxy is the arithmetic mean (3.5), then

$$\frac{n_j}{N}\left(1 + \beta_j - 2\beta_j\frac{x_j^\star}{m_j(\boldsymbol{x}^\star)} + \beta_j\frac{\pi_j}{m_j(\boldsymbol{x}^\star)}\right) = 0.$$

Let $\boldsymbol{x}_y^j$ be the premium vector with the jth component equal to y, i.e. $\boldsymbol{x}_y^j = (x_1, \dots, x_{j-1}, y, x_{j+1}, \dots, x_J)$. Given a parameter z on which we want to investigate the sensitivity, we define the bivariate function $F_{\boldsymbol{x}}^j$ as

$$F_{\boldsymbol{x}}^j(z, y) = \frac{\partial O_j}{\partial x_j}(\boldsymbol{x}_y^j, z) = \frac{n_j}{N}\left(1 + \beta_j - 2\beta_j\frac{x_j^\star}{m_j(\boldsymbol{x}^\star)} + \beta_j\frac{\pi_j}{m_j(\boldsymbol{x}^\star)}\right).$$

Hence, the KKT condition can be simply rewritten as $F_{\boldsymbol{x}^\star}^j(z, x_j^\star) = 0$. By the continuous differentiability of F with respect to z and y and the fact that $F_{\boldsymbol{x}}^j(z, y) = 0$ has at least one solution (z_0, y_0), we can invoke the implicit function theorem, see Appendix 3.8.6.1 for details. So, if $\frac{\partial F_{\boldsymbol{x}}^j}{\partial y}(z_0, y_0) \neq 0$, the derivative of φ is given by

$$\varphi'(z) = -\left.\frac{\frac{\partial F_{\boldsymbol{x}}^j}{\partial z}(z, y)}{\frac{\partial F_{\boldsymbol{x}}^j}{\partial y}(z, y)}\right|_{y=\varphi(z)}.$$

In our case, the sign of the denominator can be easily deduced

$$\frac{\partial F_{\boldsymbol{x}}^j}{\partial y}(z, y) = \frac{\partial^2 O_j}{\partial x_j^2}(\boldsymbol{x}_y^j, z) = -2\beta_j\frac{n_j}{Nm_j(\boldsymbol{x})} < 0 \Rightarrow \operatorname{sign}(\varphi'(z)) = \operatorname{sign}\left(\frac{\partial F_{\boldsymbol{x}}^j}{\partial z}(z, \varphi(z))\right).$$

The sign of the derivative w.r.t. π_j, β_j, n_j is given in Table 3.18.

z	$\frac{\partial}{\partial z}\frac{\partial O_j}{\partial x_j}(\boldsymbol{x}_y^j, z)(z, y)$	$\frac{\partial}{\partial z}\frac{\partial O_j}{\partial x_j}(\boldsymbol{x}_{\varphi(z)}^j, z)$	sign $\varphi'(z)$
π_j	$\frac{n_j\beta_j}{Nm_j(\boldsymbol{x})}$	$\frac{n_j\beta_j}{Nm_j(\boldsymbol{x}^\star)}$	> 0
β_j	$\frac{n_j}{N}\left(1 - 2\beta_j\frac{y}{m_j(\boldsymbol{x})} + \frac{\pi_j}{m_j(\boldsymbol{x})}\right)$	$\frac{-n_j}{zN}$	< 0
n_j	$1 + \beta_j - 2\beta_j\frac{x_j^\star}{m_j(\boldsymbol{x}^\star)} + \beta_j\frac{\pi_j}{m_j(\boldsymbol{x}^\star)}$	0	0

Table 3.18 – Sensitivity analysis of premium $x_j^\star$ when market proxy is (3.5).

If the market proxy is the weighted mean (3.6), then

$$\frac{n_j}{N}\left(1+\beta_j-2\beta_j\frac{x_j^\star}{m_j(\boldsymbol{x}^\star,\boldsymbol{n})}+\beta_j\frac{\pi_j}{m_j(\boldsymbol{x}^\star,\boldsymbol{n})}\right)=0.$$

Here, we define the bivariate function $G_{\boldsymbol{x}}^j$ as

$$G_{\boldsymbol{x}}^j(z,y)=\frac{\partial O_j}{\partial x_j}(\boldsymbol{x}_y^j,z)=\frac{n_j}{N}\left(1+\beta_j-2\beta_j\frac{x_j^\star}{m_j(\boldsymbol{x}^\star)}+\beta_j\frac{\pi_j}{m_j(\boldsymbol{x}^\star)}\right).$$

Hence, the KKT condition can be simply rewritten as $G_{\boldsymbol{x}^\star}^j(z,x_j^\star)=0$. Again, we apply the implicit function theorem. In our case, the sign of the denominator can be easily deduced

$$\frac{\partial G_{\boldsymbol{x}}^j}{\partial y}(z,y)=\frac{\partial^2 O_j}{\partial x_j^2}(\boldsymbol{x}_y^j,z)=-2\beta_j\frac{n_j}{Nm_j(\boldsymbol{x},\boldsymbol{n})}<0\Rightarrow \text{sign}(\varphi'(z))=\text{sign}\left(\frac{\partial G_{\boldsymbol{x}}^j}{\partial z}(z,\varphi(z))\right).$$

The sign of the derivative w.r.t. π_j,β_j,n_j is given in Table 3.19. □

z	$\frac{\partial}{\partial z}\frac{\partial O_j}{\partial x_j}(\boldsymbol{x}_y^j,z)(z,y)$	$\frac{\partial}{\partial z}\frac{\partial O_j}{\partial x_j}(\boldsymbol{x}_{\varphi(z)}^j,z)$	sign $\varphi'(z)$
π_j	$\frac{n_j\beta_j}{Nm_j(\boldsymbol{x},\boldsymbol{n})}$	$\frac{n_j\beta_j}{Nm_j(\boldsymbol{x}^\star,\boldsymbol{n})}$	>0
β_j	$\frac{n_j}{N}\left(1-2\beta_j\frac{y}{m_j(\boldsymbol{x},\boldsymbol{n})}+\frac{\pi_j}{m_j(\boldsymbol{x},\boldsymbol{n})}\right)$	$\frac{-n_j}{zN}$	<0
n_j	$1+\beta_j-\frac{2\beta_j y}{m_j(\boldsymbol{x},\boldsymbol{n})}+\frac{\beta_j\pi_j}{m_j(\boldsymbol{x},\boldsymbol{n})}+\frac{(\pi_j-2)z\beta_j}{N(N-z)m_j(\boldsymbol{x},\boldsymbol{n})}$	$\frac{(\pi_j-2)z\beta_j}{N(N-z)m_j(\boldsymbol{x}^\star,\boldsymbol{n})}$	sign(π_j-2)

Table 3.19 – Sensitivity analysis of premium $x_j^\star$ when market proxy is (3.6).

3.8.2.4 A necessary and sufficient condition

Using the proposition of Appendix 3.8.2.2, we get the determinant of M_1

$$\begin{aligned}\det(M_1) &= (-1)^{J+1}\frac{2b_1}{w_1}\prod_{k\neq 1}(-a_k-2b_k/w_k)+(-1)^{J+1}\sum_{j=2}^{J}(-a_j)\prod_{k\neq j}(-a_k-2b_k/w_k)\\ &= \frac{2b_1}{w_1}\prod_{k\neq 1}(a_k+2b_k/w_k)-\sum_{j=2}^{J}a_j\prod_{k\neq j}(a_k+2b_k/w_k).\end{aligned}$$

Let $a_j=1+\beta_j$ and $b_j=\beta_j$. Thus, $\det(M)=0$ is equivalent to $\det(M_1)=0$. Using $w_{-j}^\Sigma=\sum_{k\neq j}w_k$, that is

$$\frac{2\beta_1 w_{-1}^\Sigma}{w_1}\prod_{k\neq 1}(1+\beta_k+2\beta_k w_{-k}^\Sigma/w_k)=\sum_{j=2}^{J}(1+\beta_j)\prod_{k\neq j}(1+\beta_k+2\beta_k w_{-k}^\Sigma/w_k)$$

Using $\tilde\beta_j=1+\beta_j+2\beta_j w_{-j}^\Sigma/w_j$ and $\tilde\beta_{-j}^\Pi=\prod_{k\neq j}\tilde\beta_k$,

$$\det(M_1)=0\Leftrightarrow 2\beta_1 w_{-1}^\Sigma\tilde\beta_1=w_1\sum_{j=2}^{J}(1+\beta_j)\tilde\beta_j.$$

There are many solutions so that β_j's solve this equation. But there is a unique solution when $w_j=w$ is constant and β_j's are all identical. Say $\beta_j=\beta$ leading to $\tilde\beta_j=1+\beta+2\beta(J-1)$. Hence $\det(M_1)=0$ yields to $\beta=1$

$$\Leftrightarrow 2\beta w(J-1)(1+\beta+2\beta(J-1))=w\sum_{j=2}^{J}(1+\beta)(1+\beta+2\beta(J-1))$$

$$\Leftrightarrow 2\beta(J-1)(1+\beta+2\beta(J-1))=(1+\beta)(J-1)(1+\beta+2\beta(J-1))\Leftrightarrow 2\beta=1+\beta\Leftrightarrow\beta=1$$

3.8.2.5 The two-insurer case

The objection function is the following generic for $j = 1, 2$

$$O_j(\boldsymbol{x}) = \left(a_j - b_j \frac{x_j}{x_{-j}}\right)(x_j - c_j), \overline{x} \geq x_j \geq \underline{x},$$

Note that a_i, b_i, c_i are always positive. We recall that $g_1(x_j)$ is the solvency constraint, $g_2(x_j) = x_j - \underline{x}$ and $g_3(x_j) = \overline{x} - x_j$.

In the following, we use in red the (strict) positivity Lagrange multipliers (enforcing $\underline{x}$ or $\overline{x}$) and in orange the (strict) premium bound inequalities.

The premium equilibrium $\boldsymbol{x}^\star$ for each Insurer j solves the necessary Karush-Kuhn-Tucker conditions:

$$\begin{aligned} &\nabla_{x_j} O_j(x^\star) + \sum_{1 \leq l \leq 3} \lambda_l^{j\star} \nabla_{x_j} g_j^l(x_j^\star) = 0, \\ &0 \leq \lambda^{j\star}, \; g_j(x_j^\star) \geq 0, \; g_j(x_j^\star)^T \lambda^{j\star} = 0, \end{aligned} \tag{3.25}$$

where $\lambda^{j\star} \in \mathbb{R}^3$ are Lagrange multipliers, see e.g. Facchinei and Kanzow (2009). In the last part of equation (3.23), $g_j(x_j^\star)^T \lambda^{j\star} = 0$ is the complementarity equation implying that the lth constraint g_j^l is either active ($g_j^l(x_j^\star) = 0$) or inactive ($g_j^l(x_j^\star) > 0$), but $\lambda_l^{j\star} = 0$.

We suppose that no solvency constraint. Hence, $\lambda_1^{j\star} = 0$. (3.23) becomes for $j = 1, 2$ (only 2 insurers)

$$\begin{cases} b_j c_j \frac{1}{x_{-j}} + a_j - 2b_j \frac{x_j}{x_{-j}} + \lambda_2^j - \lambda_3^j = 0 \\ \lambda_2^j (x_j - \underline{x}) = 0 \\ \lambda_3^j (\overline{x} - x_j) = 0 \\ x_j \in [\underline{x}, \overline{x}], \lambda_i^j \geq 0 \end{cases} \tag{3.26}$$

We consider 9 cases for the premium equilibrium $(x_1^\star, x_2^\star)$ depending on Lagrange multipliers λ_j^l, see Table 3.20.

	$(x_1^\star, x_2^\star)$	Player 1: $\lambda_2^1 > 0, \lambda_3^1 = 0$	Player 1: $\lambda_2^1 = 0, \lambda_3^1 = 0$	Player 1: $\lambda_2^1 = 0, \lambda_3^1 > 0$
Player 2	$\lambda_2^2 > 0, \lambda_3^2 = 0$	$(\underline{x}, \underline{x})$	$(\tilde{x}_a, \underline{x})$	$(\overline{x}, \underline{x})$
	$\lambda_2^2 = 0, \lambda_3^2 = 0$	$(\underline{x}, \tilde{x}_b)$	(x_1^{NC}, x_2^{NC})	$(\overline{x}, \tilde{x}_c)$
	$\lambda_2^2 = 0, \lambda_3^2 > 0$	$(\underline{x}, \overline{x})$	$(\tilde{x}_d, \overline{x})$	$(\overline{x}, \overline{x})$

Table 3.20 – 9 cases for Lagrange multipliers $(x_1^\star, x_2^\star)$ with $\underline{x} < \tilde{x}_. < \overline{x}$

3.8.2.5.1 Case (x_1^{NC}, x_2^{NC}) (3.26) becomes

$$\begin{cases} b_1 c_1 \frac{1}{x_2} + a_1 - 2b_1 \frac{x_1}{x_2} = 0 \\ b_2 c_2 \frac{1}{x_1} + a_2 - 2b_2 \frac{x_2}{x_1} = 0 \\ \lambda_2^j = 0, \lambda_3^j = 0 \end{cases} \Leftrightarrow \begin{cases} x_2 = \frac{2b_1 b_2 c_2 + a_2 b_1 c_1}{4b_1 b_2 - a_1 a_2} \\ x_1 = \frac{2b_1 b_2 c_1 + a_1 b_2 c_2}{4b_1 b_2 - a_1 a_2} \end{cases}$$

This case is possible iff $4b_1 b_2 - a_1 a_2 \neq 0$ and prem. bound constraints are

$$\underline{x} < x_1^{NC} = \frac{2b_1 b_2 c_1 + a_1 b_2 c_2}{4b_1 b_2 - a_1 a_2} < \overline{x}, \; \underline{x} < x_2^{NC} = \frac{2b_1 b_2 c_2 + a_2 b_1 c_1}{4b_1 b_2 - a_1 a_2} < \overline{x}.$$

As the numerator is positive, $4b_1 b_2 - a_1 a_2 < 0$ makes impossible these inequalities.

In the case of objective function (3.7) and the market proxy is the arithmetic mean (3.5), we choose $a_j = 1 + \beta_j$, $b_j = \beta_j(J-1) = \beta_j$ and $c_j = \pi_j$. The constraints become

$$\underline{x} < x_1^{NC} = \frac{2\beta_1\beta_2\pi_1 + (1+\beta_1)\beta_2\pi_2}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)} < \overline{x},\ \underline{x} < x_2^{NC} = \frac{2\beta_1\beta_2\pi_2 + (1+\beta_2)\beta_1\pi_1}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)} < \overline{x}$$

In the case of positive determinant $4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2) > 0$, we get

$$\begin{cases} \underline{x}(4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)) < 2\beta_1\beta_2\pi_1 + (1+\beta_1)\beta_2\pi_2 < \overline{x}(4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)) \\ \underline{x}(4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)) < 2\beta_1\beta_2\pi_2 + (1+\beta_2)\beta_1\pi_1 < \overline{x}(4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)) \end{cases}$$

$$\Leftrightarrow \begin{cases} \underline{x}(4 - \frac{(1+\beta_1)(1+\beta_2)}{\beta_1\beta_2}) < 2\pi_1 + \frac{(1+\beta_1)}{\beta_1}\pi_2 < \overline{x}(4 - \frac{(1+\beta_1)(1+\beta_2)}{\beta_1\beta_2}) \\ \underline{x}(4 - \frac{(1+\beta_1)(1+\beta_2)}{\beta_1\beta_2}) < 2\pi_2 + \frac{(1+\beta_2)}{\beta_2}\pi_1 < \overline{x}(4 - \frac{(1+\beta_1)(1+\beta_2)}{\beta_1\beta_2}) \end{cases}$$

$$\Leftrightarrow \begin{cases} \underline{x}(4 - \frac{(1+\beta_1)(1+\beta_2)}{\beta_1\beta_2}) < 2\pi_1 + \frac{(1+\beta_1)}{\beta_1}\pi_2 < \overline{x}(4 - \frac{(1+\beta_1)(1+\beta_2)}{\beta_1\beta_2}) \\ -\frac{(1+\beta_1)}{\beta_1}\underline{x}(2 - \frac{(1+\beta_1)(1+\beta_2)}{2\beta_1\beta_2}) > -\frac{(1+\beta_1)}{\beta_1}\pi_2 - \frac{(1+\beta_1)}{\beta_1}\frac{(1+\beta_2)}{2\beta_2}\pi_1 > -\frac{(1+\beta_1)}{\beta_1}\overline{x}(2 - \frac{(1+\beta_1)(1+\beta_2)}{2\beta_1\beta_2}) \end{cases}$$

$$\Rightarrow \left(2\underline{x} - \frac{(1+\beta_1)}{\beta_1}\overline{x}\right)(2 - \frac{(1+\beta_1)(1+\beta_2)}{2\beta_1\beta_2}) < \pi_1\left(2 - \frac{(1+\beta_1)(1+\beta_2)}{2\beta_1\beta_2}\right)$$

$$< \left(2\overline{x} - \frac{(1+\beta_1)}{\beta_1}\underline{x}\right)(2 - \frac{(1+\beta_1)(1+\beta_2)}{2\beta_1\beta_2})$$

$$\Rightarrow 2\underline{x} - \frac{(1+\beta_1)}{\beta_1}\overline{x} < \pi_1 < 2\overline{x} - \frac{(1+\beta_1)}{\beta_1}\underline{x}$$

In the case of negative determinant, there are two times a sign change in inequalities leading to the same result. Similarly for π_2, we get

$$2\underline{x} - \frac{(1+\beta_2)}{\beta_2}\overline{x} < \pi_2 < 2\overline{x} - \frac{(1+\beta_2)}{\beta_2}\underline{x}.$$

3.8.2.5.2 Case $(\underline{x}, \underline{x})$ (3.26) becomes

$$\begin{cases} b_jc_j\frac{1}{x_{-j}} + a_j - 2b_j\frac{x_j}{x_{-j}} + \lambda_2^j = 0 \\ x_j - \underline{x} = 0 \\ \lambda_3^j = 0 \end{cases} \Leftrightarrow \begin{cases} b_1c_1\frac{1}{\underline{x}} + a_1 - 2b_1 + \lambda_2^1 = 0 \\ b_2c_2\frac{1}{\underline{x}} + a_2 - 2b_2 + \lambda_2^2 = 0 \\ x_1 = x_2 = \underline{x}, \lambda_3^1 = \lambda_3^2 = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_2^1 = 2b_1 - a_1 - \frac{b_1c_1}{\underline{x}} \\ \lambda_2^2 = 2b_2 - a_2 - \frac{b_2c_2}{\underline{x}} \end{cases}$$

This case is possible iff (positivity of Lagrange multipliers) $2b_1 - a_1 - \frac{b_1c_1}{\underline{x}} > 0$ and $2b_2 - a_2 - \frac{b_2c_2}{\underline{x}} > 0$. The objective value at $(\underline{x}, \underline{x})$ is $O_j(\underline{x}, \underline{x}) = (a_j - b_j)(\underline{x} - c_j)$.

In the case of objective function (3.7) and the market proxy is the arithmetic mean (3.5), we choose $a_j = 1 + \beta_j$, $b_j = \beta_j$ and $c_j = \pi_j$. The positivity of Lagrange multipliers leads to

$$\forall j, \underline{x}\frac{\beta_j - 1}{\beta_j} > \pi_j.$$

If $0 < \beta_j \leq 1$ then this condition is impossible.

3.8.2.5.3 Case $(\overline{x}, \overline{x})$ (3.26) becomes

$$\begin{cases} b_jc_j\frac{1}{x_{-j}} + a_j - 2b_j\frac{x_j}{x_{-j}} - \lambda_3^j = 0 \\ \overline{x} - x_j = 0 \\ \lambda_2^j = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_3^j = -2b_j + b_jc_j\frac{1}{\overline{x}} + a_j \\ x_j = \overline{x} \\ \lambda_2^j = 0 \end{cases}$$

This case is possible iff (positivity of Lagrange multipliers) $-2b_j + b_jc_j\frac{1}{\overline{x}} + a_j > 0$. The objective value at $(\underline{x}, \underline{x})$ is $O_j(\overline{x}, \overline{x}) = (a_j - b_j)(\underline{x} - c_j) > O_j(\underline{x}, \underline{x})$.

In the case of objective function (3.7) and the market proxy is the arithmetic mean (3.5), we choose $a_j = 1+\beta_j$, $b_j = \beta_j(J-1) = \beta_j$ and $c_j = \pi_j$. The positivity of Lagrange multipliers leads to

$$\forall j, \pi_j > \frac{\beta_j - 1}{\beta_j}\overline{x}.$$

If $0 < \beta_j \leq 1$ then this condition is impossible.

3.8.2.5.4 Case $(\tilde{x}_a, \underline{x})$ (3.26) becomes

$$\left\{\begin{array}{l} b_1c_1\frac{1}{\underline{x}} + a_1 - 2b_1\frac{x_1}{\underline{x}} = 0 \\ b_2c_2\frac{1}{x_1} + a_2 - 2b_2\frac{\underline{x}}{x_1} + \lambda_2^2 = 0 \\ x_2 = \underline{x}, \lambda_2^1 = \lambda_3^1 = \lambda_3^2 = 0 \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} x_1 = \frac{b_1c_1+a_1\underline{x}}{2b_1} \\ \lambda_2^2 = \frac{-2b_1b_2c_2+4b_1b_2\underline{x}}{b_1c_1+a_1\underline{x}} - a_2 \\ x_2 = \underline{x}, \lambda_3^1 = \lambda_3^2 = 0 \end{array}\right.$$

This case is possible iff $b_1c_1 + a_1\underline{x} \neq 0$ (always true because positive) and the prem. bound inequality and the lagr. pos. constraint are resp.

$$\underline{x} < \tilde{x}_a = \frac{b_1c_1 + a_1\underline{x}}{2b_1} < \overline{x}, \quad \frac{-2b_1b_2c_2 + 4b_1b_2\underline{x}}{b_1c_1 + a_1\underline{x}} > a_2.$$

In the case of objective function (3.7) and the market proxy is the arithmetic mean (3.5), we choose $a_j = 1+\beta_j$, $b_j = \beta_j$ and $c_j = \pi_j$. The premium is

$$\tilde{x}_a = \frac{\beta_1\pi_1 + (1+\beta_1)\underline{x}}{2\beta_1}.$$

We know the sign of the denominator $\beta_1\pi_1 + (1+\beta_1)\underline{x} > 0$. The prem. bound constraint is

$$\underline{x}\frac{\beta_1 - 1}{\beta_1} < \pi_1 < 2\overline{x} - \frac{1+\beta_1}{\beta_1}\underline{x}.$$

The Lagr. pos. constraint becomes

$$\begin{aligned} & \frac{-2\beta_1\beta_2\pi_2 + 4\beta_1\beta_2\underline{x}}{\beta_1\pi_1 + (1+\beta_1)\underline{x}} > 1 + \beta_2. \\ \Leftrightarrow \quad & -2\beta_1\beta_2\pi_2 + 4\beta_1\beta_2\underline{x} > (1+\beta_2)(\beta_1\pi_1 + (1+\beta_1)\underline{x}) \\ \Leftrightarrow \quad & (4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2))\underline{x} > (1+\beta_2)\beta_1\pi_1 + 2\beta_1\beta_2\pi_2 \\ \Leftrightarrow \quad & \underline{x} > \frac{(1+\beta_2)\beta_1\pi_1 + 2\beta_1\beta_2\pi_2}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)} = x_2^{NC} \end{aligned}$$

Using the prem. bound ineq., this implies

$$4\underline{x} - \frac{(1+\beta_2)(\beta_1 - 1)}{\beta_1\beta_2}\underline{x} > 2\pi_2 + \frac{(1+\beta_2)(1+\beta_1)}{\beta_2\beta_1}\underline{x} \Leftrightarrow 4\underline{x} - 2\frac{(1+\beta_2)}{\beta_2}\underline{x} > 2\pi_2 \Leftrightarrow \frac{\beta_2 - 1}{\beta_2}\underline{x} > \pi_2$$

3.8.2.5.5 Case $(\overline{x}, \underline{x})$ (3.26) becomes

$$\left\{\begin{array}{l} b_1c_1\frac{1}{\underline{x}} + a_1 - 2b_1\frac{\overline{x}}{\underline{x}} - \lambda_3^1 = 0 \\ b_2c_2\frac{1}{\overline{x}} + a_2 - 2b_2\frac{\underline{x}}{\overline{x}} + \lambda_2^2 = 0 \\ \lambda_2^1 = 0, x_2 = \underline{x} \\ \lambda_3^2 = 0, x_1 = \overline{x} \end{array}\right. \Leftrightarrow \left\{\begin{array}{l} \lambda_3^1 = b_1c_1\frac{1}{\underline{x}} + a_1 - 2b_1\frac{\overline{x}}{\underline{x}} \\ \lambda_2^2 = -b_2c_2\frac{1}{\overline{x}} - a_2 + 2b_2\frac{\underline{x}}{\overline{x}} \\ \lambda_2^1 = 0, x_2 = \underline{x} \\ \lambda_3^2 = 0, x_1 = \overline{x} \end{array}\right.$$

This case is possible iff the Lagr. multipliers are positive

$$b_1c_1 + a_1\underline{x} - 2b_1\overline{x} > 0, -b_2c_2 - a_2\overline{x} + 2b_2\underline{x} > 0$$

In the case of objective function (3.7) and the market proxy is the arithmetic mean (3.5), we choose $a_j = 1+\beta_j$, $b_j = \beta_j$ and $c_j = \pi_j$. The constraints become

$$\pi_1 > 2\overline{x} - \frac{1+\beta_1}{\beta_1}\underline{x}, 2\underline{x} - \frac{1+\beta_2}{\beta_2}\overline{x} > \pi_2$$

Equivalently the positivity of multipliers are

$$\begin{cases} \beta_1\pi_1 + (1+\beta_1)\underline{x} > 2\beta_1\overline{x} \\ 2\beta_2\underline{x} > \beta_2\pi_2 + (1+\beta_2)\overline{x} \end{cases} \Leftrightarrow \begin{cases} \frac{\pi_1}{2\underline{x}} + \frac{1}{2\beta_1} + 1/2 > \frac{\overline{x}}{\underline{x}} \\ \frac{\underline{x}}{\overline{x}} > \frac{\pi_2}{2\overline{x}} + \frac{1}{2\beta_2} + 1/2 \end{cases}$$

Or $\frac{\overline{x}}{\underline{x}} > 1 > \frac{\underline{x}}{\overline{x}}$ So $\lambda_2^2 > 0$ is impossible when $\beta_2 \leq 1$ but $\lambda_3^1 > 0$ is always true when $\beta_1 \leq 1$.

3.8.2.5.6 Case $(\overline{x}, \tilde{x}_c)$ (3.26) becomes

$$\begin{cases} b_1c_1\frac{1}{x_2} + a_1 - 2b_1\frac{\overline{x}}{x_2} - \lambda_3^1 = 0 \\ b_2c_2\frac{1}{\overline{x}} + a_2 - 2b_2\frac{x_2}{\overline{x}} = 0 \\ x_1 = \overline{x}, \lambda_2^1 = \lambda_2^2 = \lambda_3^2 = 0 \end{cases} \Leftrightarrow \begin{cases} \lambda_3^1 = \frac{2b_2b_1c_1}{b_2c_2+a_2\overline{x}} + a_1 - \frac{4b_1b_2\overline{x}}{b_2c_2+a_2\overline{x}} \\ x_2 = c_2/2 + a_2\overline{x}/(2b_2) \\ x_1 = \overline{x}, \lambda_2^1 = \lambda_2^2 = \lambda_3^2 = 0 \end{cases}$$

This case is possible iff $b_2c_2 + a_2\overline{x} \neq 0$ (always true because positive) and the lagr. pos. and the prem. bound constraints are resp.

$$\frac{2b_2b_1c_1 - 4b_1b_2\overline{x}}{b_2c_2 + a_2\overline{x}} + a_1 > 0, \ \underline{x} < c_2/2 + a_2\overline{x}/(2b_2) < \overline{x}$$

In the case of objective function (3.7) and the market proxy is the arithmetic mean (3.5), we choose $a_j = 1+\beta_j$, $b_j = \beta_j(J-1) = \beta_j$ and $c_j = \pi_j$. We have

$$\tilde{x}_c = \frac{\beta_2\pi_2 + (1+\beta_2)\overline{x}}{2\beta_2}.$$

The Lagr. pos. constraint becomes

$$\begin{aligned} & \frac{2\beta_2\beta_1\pi_1 - 4\beta_1\beta_2\overline{x}}{\beta_2\pi_2 + (1+\beta_2)\overline{x}} + 1 + \beta_1 > 0 \\ \Leftrightarrow\ & 2\beta_2\beta_1\pi_1 - 4\beta_1\beta_2\overline{x} + (1+\beta_1)\beta_2\pi_2 + (1+\beta_1)(1+\beta_2)\overline{x} > 0 \\ \Leftrightarrow\ & 2\beta_2\beta_1\pi_1 + (1+\beta_1)\beta_2\pi_2 > 4\beta_1\beta_2\overline{x} - (1+\beta_1)(1+\beta_2)\overline{x} \\ \Leftrightarrow\ & \frac{2\beta_2\beta_1\pi_1 + (1+\beta_1)\beta_2\pi_2}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)} > \overline{x} \\ \Leftrightarrow\ & x_1^{NC} > \overline{x}. \end{aligned}$$

The prem. bound inequalities rewrite

$$\underline{x} < \tilde{x}_c < \overline{x} \Leftrightarrow 2\underline{x} - \frac{1+\beta_2}{\beta_2}\overline{x} < \pi_2 < \overline{x}\frac{\beta_2 - 1}{\beta_2}.$$

Using the last inequality and $\tilde{x}_1^{NC} > \overline{x}$ we arrive to

$$\pi_1 > 2\overline{x} - \overline{x}\frac{1+\beta_1}{\beta_1} = \overline{x}\frac{\beta_1 - 1}{\beta_1}$$

3.8.2.5.7 Summary with respect to Lagrange multipliers

We plot the 4 bounds for the break-even premium in Figure 3.13. We summarize the constraints either lagr. pos. or prem. bound ineq. in Table 3.21 for the value of KKT system solution (3.26).

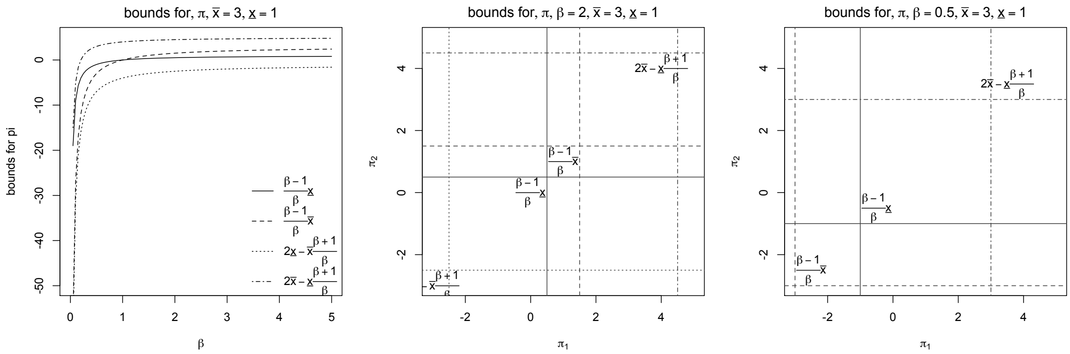


Figure 3.13 – Bound examples for π with solid line $\underline{x}\frac{\beta-1}{\beta}$, dashed line $\overline{x}\frac{\beta-1}{\beta}$, dotted line $2\underline{x} - \frac{1+\beta}{\beta}\overline{x}$, dash-dotted line $2\overline{x} - \frac{1+\beta}{\beta}\underline{x}$

	prem. eq.	Player 1 $\lambda_2^1 > 0, \lambda_3^1 = 0$	$\lambda_2^1 = 0, \lambda_3^1 = 0$	$\lambda_2^1 = 0, \lambda_3^1 > 0$
	$\lambda_2^2 > 0$ $\lambda_3^2 = 0$	$\pi_1 < \underline{x}\frac{\beta_1-1}{\beta_1}$ $\pi_2 < \underline{x}\frac{\beta_2-1}{\beta_2}$ no pos. lagr. if $\beta_j \leq 1$ $\boldsymbol{x}^\star = (\underline{x}, \underline{x})$	$\underline{x}\frac{\beta_1-1}{\beta_1} < \pi_1 < 2\overline{x} - \frac{1+\beta_1}{\beta_1}\underline{x}$ $x_2^{NC} < \underline{x} \Rightarrow \pi_2 < \frac{\beta_2-1}{\beta_2}\underline{x}$ no pos. lagr. if $\beta_2 \leq 1$ $\boldsymbol{x}^\star = (\tilde{x}_a, \underline{x})$	$2\overline{x} - \frac{1+\beta_1}{\beta_1}\underline{x} < \pi_1$ $\pi_2 < 2\underline{x} - \frac{1+\beta_2}{\beta_2}\overline{x}$ no pos. lagr. if $\beta_2 \leq 1$ $\boldsymbol{x}^\star = (\overline{x}, \underline{x})$
P2	$\lambda_2^2 = 0$ $\lambda_3^2 = 0$	$x_1^{NC} < \underline{x} \Rightarrow \pi_1 < \frac{\beta_1-1}{\beta_1}\underline{x}$ $\underline{x}\frac{\beta_2-1}{\beta_2} < \pi_2 < 2\overline{x} - \frac{1+\beta_2}{\beta_2}\underline{x}$ no pos. lagr. if $\beta_1 \leq 1$ $\boldsymbol{x}^\star = (\underline{x}, \tilde{x}_b)$	$2\underline{x} - \frac{1+\beta_1}{\beta_1}\overline{x} < \pi_1 < 2\overline{x} - \frac{1+\beta_1}{\beta_1}\underline{x}$ $2\underline{x} - \frac{(1+\beta_2)}{\beta_2}\overline{x} < \pi_2 < 2\overline{x} - \frac{(1+\beta_2)}{\beta_2}\underline{x}$ need positive det. $\boldsymbol{x}^\star = (x_1^{NC}, x_2^{NC})$	$x_1^{NC} > \overline{x} \Rightarrow \pi_1 > \overline{x}\frac{\beta_1-1}{\beta_1}$ $2\underline{x} - \frac{1+\beta_2}{\beta_2}\overline{x} < \pi_2 < \overline{x}\frac{\beta_2-1}{\beta_2}$ no prem. bound if $\beta_2 \leq 1$ $\boldsymbol{x}^\star = (\overline{x}, \tilde{x}_c)$
	$\lambda_2^2 = 0$ $\lambda_3^2 > 0$	$\pi_1 < 2\underline{x} - \frac{1+\beta_1}{\beta_1}\overline{x}$ $2\overline{x} - \frac{1+\beta_2}{\beta_2}\underline{x} < \pi_2$ no pos. lagr. if $\beta_1 \leq 1$ $\boldsymbol{x}^\star = (\underline{x}, \overline{x})$	$2\underline{x} - \frac{1+\beta_1}{\beta_1}\overline{x} < \pi_1 < \overline{x}\frac{\beta_1-1}{\beta_1}$ $x_2^{NC} > \overline{x} \Rightarrow \pi_2 > \overline{x}\frac{\beta_2-1}{\beta_2}$ no prem. bound if $\beta_1 \leq 1$ $\boldsymbol{x}^\star = (\tilde{x}_d, \overline{x})$	$\frac{\beta_1-1}{\beta_1}\overline{x} < \pi_1$ $\frac{\beta_2-1}{\beta_2}\overline{x} < \pi_2$ $\boldsymbol{x}^\star = (\overline{x}, \overline{x})$

Table 3.21 – Summary of 9 cases for the KKT system (3.26) $(x_1^\star, x_2^\star)$: in red the pos. Lagr. constraints (enforcing $\underline{x}$ or $\overline{x}$) and in orange the prem. bound inequalities.

3.8.2.5.8 Computation of the Unique Equilibrium for each π conditions

According to 3.4.1, there exists a unique equilibrium for each π condition. Table 3.22 summarizes the different solutions of the KKT system, but we should determine the unique equilibrium for each condition on π.

Let us denote the 4 bounds for π by

$$l_1(\beta) = 2\underline{x} - \frac{1+\beta}{\beta}\overline{x},\ l_2(\beta) = \frac{\beta-1}{\beta}\underline{x},\ l_3(\beta) = \frac{\beta-1}{\beta}\overline{x},\ l_4(\beta) = 2\overline{x} - \frac{1+\beta}{\beta}\underline{x}.$$

There are all plotted in Figure 3.13. Note that $l_1(\beta) < l_2(\beta), l_3(\beta) < l_4(\beta)$ for all β and $l_2(\beta) < l_3(\beta)$ for $\beta > 1$, $l_2(\beta) > l_3(\beta)$ for $\beta >= 1$.

As a reminder

$$x_1^{NC} = \frac{2\beta_1\beta_2\pi_1 + (1+\beta_1)\beta_2\pi_2}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)}, x_2^{NC} = \frac{2\beta_1\beta_2\pi_2 + (1+\beta_2)\beta_1\pi_1}{4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2)}, \tilde{x}_a = \frac{\beta_1\pi_1 + (1+\beta_1)\underline{x}}{2\beta_1},$$

$$\tilde{x}_b = \frac{\beta_2\pi_2 + (1+\beta_2)\underline{x}}{2\beta_2}, \tilde{x}_c = \frac{\beta_2\pi_2 + (1+\beta_2)\overline{x}}{2\beta_2}, \tilde{x}_d = \frac{\beta_1\pi_1 + (1+\beta_1)\overline{x}}{2\beta_1}.$$

When solving KKT system (3.26) we previously use some implications instead of equivalences in order to surround π. Therefore, and because of unicity of equilibrium, we have to clarify which is the unique equilibrium for each π condition.

Firstly, for $l_1(\beta_1) < \pi_1 < l_2(\beta_1)$ and $l_1(\beta_2) < \pi_2 < l_2(\beta_2)$, we get two possible conditions $(\underline{x}, \underline{x}), (x_1^{NC}, x_2^{NC})$. Using condition of solution $(\underline{x}, \underline{x})$ under equivalence, we get

$$\begin{cases} 2\underline{x} - \frac{1+\beta_1}{\beta_1}\overline{x} < \pi_1 < \frac{\beta_1 - 1}{\beta_1}\underline{x} \\ 2\underline{x} - \frac{(1+\beta_2)}{\beta_2}\overline{x} < \pi_2 < \frac{\beta_2 - 1}{\beta_2}\underline{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} 2\underline{x}\beta_1\beta_2 - (1+\beta_1)\beta_2\overline{x} < \pi_1\beta_1\beta_2 < (\beta_1 - 1)\beta_2\underline{x} \\ 2\underline{x}\beta_2(1+\beta_1) - (1+\beta_2)(1+\beta_1)\overline{x} < \pi_2\beta_2(1+\beta_1) < (\beta_2 - 1)(1+\beta_1)\underline{x} \end{cases}$$

$$\Rightarrow \pi_1\beta_1\beta_2 + \pi_2\beta_2(1+\beta_1) < (\beta_1 - 1)\beta_2\underline{x} + (\beta_2 - 1)(1+\beta_1)\underline{x} = (3\beta_1\beta_2 - 1 - \beta_1 - \beta_2)\underline{x}$$

And when $4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2) > 0$ (i.e. $det(M) > 0$), then $x_1^{NC} < \underline{x}$. We thus eliminate (x_1^{NC}, x_2^{NC}) when $l_1(\beta_2) < \pi_2 < l_2(\beta_2)$ and $l_1(\beta_2) < \pi_1 < l_2(\beta_2)$.

Secondly, using same reasoning for the case $l_3(\beta_2) < \pi_2 < l_4(\beta_2)$ and $l_3(\beta_2) < \pi_1 < l_4(\beta_2)$, we can also eliminate (x_1^{NC}, x_2^{NC}) solution.

Thirdly, for $l_2(\beta_2) < \pi_2 < l_3(\beta_2)$ and $l_1(\beta_2) < \pi_1 < l_2(\beta_2)$, we get two possible conditions $(\underline{x}, \tilde{x}_b), (x_1^{NC}, x_2^{NC})$. However, $(\underline{x}, \tilde{x}_b)$ is a solution and imply $x_1^{NC} < \underline{x}$. Thus (x_1^{NC}, x_2^{NC}) is not an equilibrium. Using same reasoning, we can also eliminate the unconstrained solution (x_1^{NC}, x_2^{NC}) cases for

— $l_2(\beta_1) < \pi_1 < l_3(\beta_1)$ and $l_1(\beta_2) < \pi_2 < l_2(\beta_2)$,
— $l_3(\beta_1) < \pi_1 < l_4(\beta_1)$ and $l_1(\beta_2) < \pi_2 < l_2(\beta_2)$,
— $l_1(\beta_1) < \pi_1 < l_2(\beta_1)$ and $l_2(\beta_2) < \pi_2 < l_3(\beta_2)$,
— $l_1(\beta_1) < \pi_1 < l_2(\beta_1)$ and $l_3(\beta_2) < \pi_2 < l_4(\beta_2)$,
— $l_2(\beta_1) < \pi_1 < l_3(\beta_1)$ and $l_3(\beta_2) < \pi_2 < l_4(\beta_2)$,
— $l_3(\beta_1) < \pi_1 < l_4(\beta_1)$ and $l_2(\beta_2) < \pi_2 < l_3(\beta_2)$.

Furthermore

$$O_1(\tilde{x}_a, \underline{x}) = -\frac{\beta_1\pi_1}{2\underline{x}}(-\pi_1/2 + \frac{1+\beta_1}{\beta_1}\underline{x}); O_1(\overline{x}, \tilde{x}_c) = (1 - \beta_1(\overline{x}/\tilde{x}_c - 1))(\overline{x} - \pi_1)$$

How to order?

This leads to Table 3.22.

	π_1				
$\pi_2 \vert \pi_1$	$. < l_1(\beta_1)$	$l_1(\beta_1) < . < l_2(\beta_1)$	$l_2(\beta_1) < . < l_3(\beta_1)$	$l_3(\beta_1) < . < l_4(\beta_1)$	$l_4(\beta_1) < .$
$. < l_1(\beta_2)$	$(\underline{x}, \underline{x})$	$(\underline{x}, \underline{x})$	$(\tilde{x}_a, \underline{x})$	$(\tilde{x}_a, \underline{x})$	$(\underline{x}, \overline{x})$
$l_1(\beta_2) < . < l_2(\beta_2)$	$(\underline{x}, \underline{x})$	$(\underline{x}, \underline{x})$,	$(\tilde{x}_a, \underline{x})$,	$(\tilde{x}_a, \underline{x}), , (\overline{x}, \tilde{x}_c)$	$(\overline{x}, \tilde{x}_c)$
$l_2(\beta_2) < . < l_3(\beta_2)$	$(\underline{x}, \tilde{x}_b)$	$(\underline{x}, \tilde{x}_b)$,	(x_1^{NC}, x_2^{NC})	$, (\overline{x}, \tilde{x}_c)$	$(\overline{x}, \tilde{x}_c)$
$l_3(\beta_2) < . < l_4(\beta_2)$	$(\underline{x}, \tilde{x}_b)$	$(\underline{x}, \tilde{x}_b), (\tilde{x}_d, \overline{x})$	$, (\tilde{x}_d, \overline{x})$	$(\overline{x}, \overline{x})$,	$(\overline{x}, \overline{x})$
$l_4(\beta_2) < .$	$(\underline{x}, \overline{x})$	$(\tilde{x}_d, \overline{x})$	$(\tilde{x}_d, \overline{x})$	$(\overline{x}, \overline{x})$	$(\overline{x}, \overline{x})$

Table 3.22 – Summary of all cases for Unique Equilibrium $(x_1^\star, x_2^\star)$ (only valid for $\beta_j > 1$)

3.8.2.6 The two-insurer case without constraint

Let use Proposition 3.4.2.

$$M = \begin{pmatrix} 2\beta_1 & -1-\beta_1 \\ -1-\beta_2 & 2\beta_2 \end{pmatrix}$$

$$\Rightarrow \det(M) = 4\beta_1\beta_2 - (1+\beta_1)(1+\beta_2) = 3\beta_1\beta_2 - 1 - \beta_1 - \beta_2.$$

The value of the determinant cancels when $\beta_2 = (1+\beta_1)/(3\beta_1 - 1)$ for $\beta_1 \neq 1/3$. The determinant is positive when

	$\beta_1 > 1/3$	$\beta_1 < 1/3$	$\beta_1 = 1/3$
$\beta_2 > (1+\beta_1)/(3\beta_1 - 1)$	$\det(M) > 0$	$\det(M) < 0$	$\det(M) < 0$
$\beta_2 = (1+\beta_1)/(3\beta_1 - 1)$	$\det(M) = 0$	$\det(M) = 0$	.
$\beta_2 < (1+\beta_1)/(3\beta_1 - 1)$	$\det(M) < 0$	$\det(M) > 0$	$\det(M) < 0$

In particular, the determinant is positive when $\beta_1, \beta_2 > (1+\beta_1)/(3\beta_1-1)$. since $\det(M) = 3(\beta_1-1)(\beta_2-1)+2(\beta_1+\beta_2)-4$. We focus here our analysis on the case $\beta_1 > 1/3$ and $\beta_2 > (1+\beta_1)/(3\beta_1-1)$. Indeed, we are interested on the strictly concave problem conditioned by $det(M) > 0$.

In the case of non-zero determinant and positive $\beta_i > 0$, we have

$$M^{-1} = \frac{1}{3\beta_1\beta_2 - 1 - \beta_1 - \beta_2}\begin{pmatrix} 2\beta_2 & 1+\beta_2 \\ 1+\beta_1 & 2\beta_1 \end{pmatrix}$$

$$\Rightarrow \boldsymbol{x}^\star = \frac{1}{3\beta_1\beta_2 - 1 - \beta_1 - \beta_2}\begin{pmatrix} 2\beta_2 & 1+\beta_2 \\ 1+\beta_1 & 2\beta_1 \end{pmatrix}\begin{pmatrix} \beta_1\pi_1 \\ \beta_2\pi_2 \end{pmatrix} = \begin{pmatrix} \frac{2\beta_1\beta_2\pi_1 + (1+\beta_2)\beta_2\pi_2}{3\beta_1\beta_2 - 1 - \beta_1 - \beta_2} \\ \frac{(1+\beta_1)\beta_1\pi_1 + 2\beta_1\beta_2\pi_2}{3\beta_1\beta_2 - 1 - \beta_1 - \beta_2} \end{pmatrix}$$

This result is in line with Proposition 3.4.4. Furthermore

$$\frac{\partial \boldsymbol{x}_1^\star}{\partial \beta_2} = \frac{Num}{(3\beta_1\beta_2 - 1 - \beta_1 - \beta_2)^2}$$

The numerator is

$$\begin{aligned} Num &= (2\beta_1\pi_1 + \beta_2\pi_2 + (1+\beta_2)\pi_2)(3\beta_1\beta_2 - 1 - \beta_1 - \beta_2) - (2\beta_1\beta_2\pi_1 + (1+\beta_2)\beta_2\pi_2)(3\beta_1 - 1) \\ &= (2\beta_1\pi_1 + (1+2\beta_2)\pi_2)\beta_2(3\beta_1 - 1) + (2\beta_1\pi_1 + (1+2\beta_2)\pi_2)(-1-\beta_1) \\ &\quad -(2\beta_1\beta_2\pi_1 + (1+\beta_2)\beta_2\pi_2)(3\beta_1 - 1) \\ &= \beta_2^2\pi_2(3\beta_1 - 1) - (2\beta_1\pi_1 + (1+2\beta_2)\pi_2)(1+\beta_1) \\ &= \beta_2^2\pi_2(3\beta_1 - 1) - 2\beta_1\pi_1(1+\beta_1) - (1+2\beta_2)\pi_2(1+\beta_1) \\ &= \beta_2^2\pi_2(3\beta_1 - 1) - 2\beta_2\pi_2(1+\beta_1) - 2\beta_1\pi_1(1+\beta_1) - \pi_2(1+\beta_1) \end{aligned}$$

We want to determinate the derivative sign. We thus compute the roots of the polynom

$$\Delta = (-2\pi_2(1+\beta_1))^2 + 4(2\beta_1\pi_1 + \pi_2)(1+\beta_1)\pi_2(3\beta_1 - 1)$$

We have $\Delta > 0$, $\forall \beta_1 > 1/3$. Thus the polynom admits two real roots: $\tilde{\beta_2^1}$, $\tilde{\beta_2^2}$.

$$\tilde{\beta_2^1} = \frac{2\pi_2(1+\beta_1) - \sqrt{\Delta}}{2\pi_2(3\beta_1 - 1)}$$

However, we have $\forall \beta_1 > 1/3$, $\tilde{\beta_2^1} < 0 \Leftrightarrow 2\pi_2(1+\beta_1) - \sqrt{\Delta} < 0$. Indeed,

$$\sqrt{\Delta} = 2\pi_2(1+\beta_1)\sqrt{1 + \frac{(2\beta_1\pi_1 + \pi_2)(3\beta_1 - 1)}{\pi_2(1+\beta_1)}}.$$

We previously show that to admit a maximum equilibrium we should have $\beta_1 > 1/3$, $\beta_2 > (1+\beta_1)/(3\beta_1-1) > 0$. Thus $\tilde{\beta_2^1}$ is never considered here. We now must determine if $\tilde{\beta_2^2} > (1+\beta_1)/(3\beta_1-1)$ in order to know if there exist a threshold for which the sign of the derivative of price equilibrium of i with respect to β_{-i} is changing. We have:

$$\begin{aligned} \tilde{\beta_2^2} > \frac{(1+\beta_1)}{(3\beta_1 - 1)} &\Leftrightarrow \frac{2\pi_2(1+\beta_1)}{2\pi_2(3\beta_1 - 1)}(1 + \sqrt{1 + \frac{(2\beta_1\pi_1 + \pi_2)(3\beta_1 - 1)}{\pi_2(1+\beta_1)}}) > \frac{(1+\beta_1)}{(3\beta_1 - 1)} \\ &\Leftrightarrow \sqrt{1 + \frac{(2\beta_1\pi_1 + \pi_2)(3\beta_1 - 1)}{\pi_2(1+\beta_1)}} > 0 \end{aligned}$$

Thus, there always exist a threshold of $\tilde{\beta_2}^2$ for which the derivative of firm i price equilibrium change sign with respect to the price sensitivity of competitor $-i$. This mean that when $(1+\beta_1)/(3\beta_1-1) <$

$\beta_2 < \tilde{\beta_2}^2$, an increase of price sensitivity of consumers with respect to competitor $-i$ lead to a decrease of price equilibrium for both insurers. Intuitively, the decrease of price competitor $-i$ over pass the price elasticity advantage of i. However, when $\beta_2 > \tilde{\beta_2}^2$, an increase of β_2 lead to a decrease of $x_2^\star$ and an increase of $x_1^\star$. Hence, when β_2 is high enough a marginal increase of this parameter lead to a important comparative advantage for firm 1.

3.8.3 Proofs of Section 3.5

3.8.3.1 Properties of the multinomial logit function

We recall that the choice probability function is defined as

$$p_{j\to k}(x) = p_{j\to j}(x)\left(\delta_{jk} + (1-\delta_{jk})e^{f_j(x_j,x_k)}\right), \text{ and } p_{j\to j}(x) = \frac{1}{1+\sum\limits_{l\neq j} e^{f_j(x_j,x_l)}},$$

where the summation is over $l \in \{1,\dots,J\} - \{j\}$ and f_j is the price function. The price function f_j goes from $(t,u) \in \mathbb{R}^2 \mapsto f_j(t,u) \in \mathbb{R}$. Partial derivatives are denoted by

$$\frac{\partial f_j(t,u)}{\partial t} = f'_{j1}(t,u) \text{ and } \frac{\partial f_j(t,u)}{\partial u} = f'_{j2}(t,u).$$

Derivatives of higher order use the same notation principle.

The lg function has the good property to be infinitely differentiable. We have

$$\frac{\partial p_{j\to j}(x)}{\partial x_i} = -\frac{\partial}{\partial x_i}\left(\sum_{l\neq j} e^{f_j(x_j,x_l)}\right)\frac{1}{\left(1+\sum\limits_{l\neq j} e^{f_j(x_j,x_l)}\right)^2}.$$

Since we have

$$\frac{\partial}{\partial x_i}\sum_{l\neq j} e^{f_j(x_j,x_l)} = \delta_{ji}\sum_{l\neq j} f'_{j1}(x_j,x_l)e^{f_j(x_j,x_l)} + (1-\delta_{ji})f'_{j2}(x_j,x_l)e^{f_j(x_j,x_i)},$$

we deduce

$$\frac{\partial p_{j\to j}(x)}{\partial x_i} = -\delta_{ji}\sum_{l\neq j}\frac{f'_{j1}(x_j,x_l)e^{f_j(x_j,x_l)}}{\left(1+\sum\limits_{l\neq j} e^{f_j(x_j,x_l)}\right)^2} - (1-\delta_{ji})f'_{j2}(x_j,x_l)\frac{f'_{j1}(x_j,x_i)}{\left(1+\sum\limits_{l\neq j} e^{f'_{j1}(x_j,x_l)}\right)^2}.$$

This is equivalent to

$$\frac{\partial p_{j\to j}(x)}{\partial x_i} = -\left(\sum_{l\neq j} f'_{j1}(x_j,x_l)\,\mathrm{lg}_j^l(x)\right)p_{i\to j}(x)\delta_{ij} - f'_{j2}(x_j,x_l)p_{j\to i}(x)p_{j\to j}(x)(1-\delta_{ij}).$$

Furthermore,

$$\frac{\partial p_{j\to j}(x)}{\partial x_i}\left(\delta_{jk} + (1-\delta_{jk})e^{f_j(x_j,x_k)}\right) =$$

$$-\left(\sum_{l\neq j} f'_{j1}(x_j,x_l)p_{j\to l}(x)\right)p_{j\to k}(x)\delta_{ij} - f'_{j2}(x_j,x_i)p_{j\to i}(x)p_{j\to k}(x)(1-\delta_{ij}).$$

and also

$$p_{j\to j}(x)\frac{\partial}{\partial x_i}\left(\delta_{jk} + (1-\delta_{jk})e^{f_j(x_j,x_k)}\right) = p_{j\to j}(x)(1-\delta_{jk})\left(\delta_{ik}f'_{j2}(x_j,x_k)e^{f_j(x_j,x_k)} + \delta_{ij}f'_{j1}(x_j,x_k)e^{f_j(x_j,x_k)}\right).$$

Hence, we get

$$\frac{\partial p_{j\to k}(x)}{\partial x_i} = -\delta_{ij}\left(\sum_{l\neq j} f'_{j1}(x_j,x_l)p_{j\to l}(x)\right)p_{j\to k}(x) - (1-\delta_{ij})f'_{j2}(x_j,x_i)p_{j\to i}(x)p_{j\to k}(x)$$
$$+ (1-\delta_{jk})\left[\delta_{ij}f'_{j1}(x_j,x_k)p_{j\to k}(x) + \delta_{ik}f'_{j2}(x_j,x_k)p_{j\to k}(x)\right].$$

Similarly, the second order derivative is given by *

$$\frac{\partial^2 p_{j\to k}(x)}{\partial x_m \partial x_i} = -\delta_{ij}\left(\delta_{jm}\sum_{l\neq j} f''_{j11}(x_j,x_l)p_{j\to l} + (1-\delta_{jm})f''_{j12}(x_j,x_m)p_{j\to m} + \sum_{l\neq j} f'_{j1}(x_j,x_l)\frac{\partial p_{j\to l}}{\partial x_m}\right)p_{j\to k}$$
$$-\delta_{ij}\left(\sum_{l\neq j} f'_{j1}(x_j,x_l)p_{j\to l}\right)\frac{\partial p_{j\to k}}{\partial x_m}$$
$$-(1-\delta_{ij})\left(\left(\delta_{jm}f''_{j21}(x_j,x_i) + \delta_{im}f''_{j22}(x_j,x_i)\right)p_{j\to i}p_{j\to k} + f'_{j2}(x_j,x_i)\frac{\partial p_{j\to i}}{\partial x_m}p_{j\to k} + f'_{j2}(x_j,x_i)p_{j\to i}\frac{\partial p_{j\to k}}{\partial x_m}\right)$$
$$+(1-\delta_{jk})\delta_{ij}\left(\left(f''_{j11}(x_j,x_k)\delta_{jm} + f''_{j12}(x_j,x_k)\delta_{km}\right)p_{j\to k} + f'_{j1}(x_j,x_k)\frac{\partial p_{j\to k}}{\partial x_m}\right)$$
$$+(1-\delta_{jk})\delta_{ik}\left(\left(f''_{j21}(x_j,x_k)\delta_{jm} + f''_{j22}(x_j,x_k)\delta_{im}\right)p_{j\to k} + f'_{j2}(x_j,x_k)\frac{\partial p_{j\to k}}{\partial x_m}\right).$$

In particular

$$\frac{\partial^2 p_{j\to j}(x)}{\partial x_j^2} = -\operatorname{lg}_j^j\sum_{l\neq j}\left(f'_{j1}(x_j,x_l)\right)^2 p_{j\to l} + 2\left(\sum_{l\neq j} f'_{j1}(x_j,x_l)p_{j\to l}\right)^2 p_{j\to j}.$$

3.8.3.2 Properties of the transition probability

Proposition 3.8.2. *Transition probability $p_{l\to j}(\boldsymbol{x})$ is a strictly decreasing function of x_j given $\boldsymbol{x}_{-j}$ and verifies $0 < p_{l\to j}(\boldsymbol{x}) < 1$.*

Proof. Differentiating the transition probability yields to the following result for $i, j, k \in \{1,\dots,J\}$

$$\frac{\partial p_{j\to k}(\boldsymbol{x})}{\partial x_i} = -\delta_{ij}\left(\sum_{l\neq j} f'_{j1}(x_j,x_l)p_{j\to l}(\boldsymbol{x})\right)p_{j\to k}(\boldsymbol{x}) - (1-\delta_{ij})f'_{j2}(x_j,x_i)p_{j\to i}(\boldsymbol{x})p_{j\to k}(\boldsymbol{x})$$
$$+ (1-\delta_{jk})\left[\delta_{ij}f'_{j1}(x_j,x_k)p_{j\to k}(\boldsymbol{x}) + \delta_{ik}f'_{j2}(x_j,x_k)p_{j\to k}(\boldsymbol{x})\right].$$

Let ϕ_l be the family function $x_j \mapsto p_{l\to j}(\boldsymbol{x})$ for $l = 1,\dots,J$. ϕ_j has the following derivative

$$\phi'_j(x_j) = -\left(\sum_{l\neq j} f'_{j1}(x_j,x_l)p_{j\to l}(\boldsymbol{x})\right)p_{j\to j}(\boldsymbol{x}),$$

where $f'_{j1}(t,u) = \frac{\partial f_j(t,u)}{\partial t}$. Since for the two considered price function, we have $\bar{f}'_{j1}(x_j,x_l) = \alpha_j/x_l > 0$ and $\tilde{f}'_{j1}(x_j,x_l) = \tilde{\alpha}_j > 0$, then the function ϕ_j is strictly decreasing. For $l \neq j$, the function ϕ_l has the following derivative

$$\phi'_l(x_j) = f'_{j2}(x_l,x_j)p_{l\to j}(\boldsymbol{x})(1-p_{l\to j}(\boldsymbol{x})),$$

*. We remove the variable x when possible.

where $f'_{j2}(t,u) = \frac{\partial f_j(t,u)}{\partial u}$. Again, for the two considered price function, we have $f'_{j2}(x_j, x_l) = -\alpha_j \frac{x_j}{x_l^2} < 0$ and $\tilde{f}'_{j2}(x_j, x_l) = -\tilde{\alpha}_j < 0$. So, the function ϕ_l is strictly decreasing.

Futhermore, the function ϕ_l decreases from 1 to 0 such that $\phi_l(x_j) \to 1$ (resp. $\phi_l(x_j) \to 0$) when $x_j \to -\infty$ (resp. $x_j \to -\infty$). When $i \neq j$, functions $x_i \mapsto p_{l\to j}(x)$ are also strictly increasing. Let $\underline{\boldsymbol{x}}^j = (\underline{x}, \ldots, \underline{x}, \overline{x}, \underline{x}, \ldots, \underline{x})$ and $\overline{\boldsymbol{x}}^j = (\overline{x}, \ldots, \overline{x}, \underline{x}, \overline{x}, \ldots, \overline{x})$. We have

$$\forall \boldsymbol{x} \in [\underline{x}, \overline{x}]^J, 0 < p_{l\to j}(\underline{x}^{j-}) < p_{l\to j}(\boldsymbol{x}) < p_{l\to j}(\overline{x}^j_-) < 1.$$

□

3.8.3.3 Expected portfolio size function

The portfolio size $x_j \mapsto \hat{N}_j(x)$ function has the following derivative

$$\frac{\partial E(N_j(x))}{\partial x_j} = -n_j \left(\sum_{l\neq j} f'_{j1}(x_j, x_l) \operatorname{lg}_j^l(x) \right) \operatorname{lg}_j^j(x) + \sum_{l\neq j} n_l f'_{j2}(x_l, x_j) \operatorname{lg}_l^j(x)(1 - \operatorname{lg}_l^j(x)).$$

Hence, it is decreasing from the total market size $\sum_l n_l$ to 0. So the function $x_j \mapsto \hat{N}_j$ is both a quasiconcave and a quasiconvex function. Therefore, using the C^2 characterization of quasiconcave and quasiconvex functions, we have that

$$\frac{\partial E(N_j(x))}{\partial x_j} = 0 \Rightarrow \frac{\partial^2 E(N_j(x))}{\partial x_j^2} = 0.$$

Note that the function $E(N_j(x))$ has two asymptotes when either $x_j \to 0$ or $x_j \to +\infty$ for fixed x_{-j}.

3.8.3.4 Partial derivatives of the objective function

Consider a general objective function

$$O_j(x) = \left(a_j - b_j \frac{x_j}{m_j(x)} \right) (x_j - c_j),$$

where $m_j(x) = \frac{1}{J-1} \sum_{i\neq j} x_i$. Simple manipulations yield to

$$\frac{\partial}{\partial x_i} \left(\frac{x_j}{\sum\limits_{k\neq j} x_k} \right) = \frac{\delta_{ij}}{m_j(x)} + (1 - \delta_{ij}) \frac{-x_j}{m_j(x)^2 (J-1)},$$

$$\frac{\partial^2}{\partial x_i \partial x_m} \left(\frac{x_j}{\sum\limits_{k\neq j} x_k} \right) = \frac{-\delta_{ij}(1-\delta_{jm})}{m_j(x)^2(J-1)} + \frac{-(1-\delta_{ij})\delta_{jm}}{m_j(x)^2(J-1)} + \frac{2x_j(1-\delta_{ij})(1-\delta_{jm})}{m_j(x)^3(J-1)^2}.$$

So, we have

$$\frac{\partial O_j}{\partial x_i}(x) = -b_j \left(\frac{\delta_{ij}}{m_j(x)} + (1-\delta_{ij}) \frac{-x_j}{m_j(x)^2(J-1)} \right) (x_j - c_j) + \left(a_j - b_j \frac{x_j}{m_j(x)} \right) \delta_{ij}.$$

$$\begin{aligned}
\frac{\partial^2 O_j}{\partial x_m \partial x_i}(x) &= -b_j\left(\frac{-\delta_{ij}(1-\delta_{jm})}{m_j(x)^2(J-1)} + \frac{-(1-\delta_{ij})\delta_{jm}}{m_j(x)^2(J-1)} + \frac{2x_j(1-\delta_{ij})(1-\delta_{jm})}{m_j(x)^3(J-1)^2}\right)(x_j - c_j) \\
&\quad -b_j\left(\frac{\delta_{ij}}{m_j(x)} + (1-\delta_{ij})\frac{-x_j}{m_j(x)^2(J-1)}\right)\delta_{jm} + \left(-b_j\frac{\delta_{jm}}{m_j(x)} - b_j\frac{-x_j(1-\delta_{jm})}{(J-1)m_j(x)^2}\right)\delta_{ij} \\
&= \left(b_j\frac{\delta_{ij}(1-\delta_{jm}) + (1-\delta_{ij})\delta_{jm}}{(J-1)m_j(x)^2} - b_j\frac{2x_j(1-\delta_{ij})(1-\delta_{jm})}{m_j(x)^3(J-1)^2}\right)(x_j - c_j) - b_j\frac{2\delta_{ij}\delta_{jm}}{m_j(x)} \\
&\quad + b_j x_j\frac{(1-\delta_{ij})\delta_{jm} + (1-\delta_{jm})\delta_{ij}}{(J-1)m_j(x)^2} \\
&= b_j(2x_j - c_j)\frac{(1-\delta_{ij})\delta_{jm} + (1-\delta_{jm})\delta_{ij}}{(J-1)m_j(x)^2} - \frac{2b_j x_j(1-\delta_{ij})(1-\delta_{jm})}{m_j(x)^3(J-1)^2}(x_j - c_j) - b_j\frac{2\delta_{ij}\delta_{jm}}{m_j(x)}.
\end{aligned}$$

3.8.3.5 Proofs of the repeated game

Proof of Prop. 3.5.1. As reported in Appendix 3.8.3.3, insurer choice probability functions $x_j \mapsto p_{l\to j}(x)$ are (strictly) decreasing functions from 1 to 0. Note that $p_{l\to j}(x) = 0$ (respectively $p_{l\to j}(x) = 1$) is only attained when x_j tends to $+\infty$ ($-\infty$). When $i \neq j$ functions $x_i \mapsto p_{l\to j}(x)$ are strictly increasing. Let $\underline{x}^{j-} = (\underline{x}, \dots, \underline{x}, \overline{x}, \underline{x}, \dots, \underline{x})$ and $\overline{x}^j_- = (\overline{x}, \dots, \overline{x}, \underline{x}, \overline{x}, \dots, \overline{x})$. We have

$$0 < p_{l\to j}(\underline{x}^{j-}) < p_{l\to j}(x) < p_{l\to j}(\overline{x}^j_-) < 1,$$

for all $x \in [\underline{x}, \overline{x}]^J$. Taking supremum and infimum on player j, we get

$$0 < \underline{p}_l = \inf_j p_{l\to j}(\underline{x}^{j-}) \text{ and } \sup_j p_{l\to j}(\overline{x}^j_-) = \overline{p}_l < 1.$$

Using the definition of portfolio size $N_{j,t}(x)$ given in Subsection 3.2.2.1 as a sum of binomial random variables $B_{lj,t}(x)$, we have

$$\begin{aligned}
& P\left(N_{j,t}(x) = m_j | N_{j,t-1} > 0, \mathrm{Card}(J_{t-1}) > 1\right) \\
&= P\left(\sum_{l\in J_{t-1}} B_{lj,t}(x) = m_j \,\middle|\, N_{j,t-1} > 0, \mathrm{Card}(J_{t-1}) > 1\right) \\
&= \sum_{\substack{\tilde m_1,\dots,\tilde m_{J_{t-1}} \geq 0 \\ \text{s.t. } \sum_l \tilde m_l = m_j}} \prod_{l\in J_{t-1}} P(B_{lj,t}(x) = \tilde m_l) \\
&= \sum_{\substack{\tilde m_1,\dots,\tilde m_{J_{t-1}} \geq 0 \\ \text{s.t. } \sum_l \tilde m_l = m_j}} \prod_{l\in J_{t-1}} \binom{n_{l,t-1}}{\tilde m_l} p_{l\to j}(x)^{\tilde m_l}\left(1 - p_{l\to j}(x)\right)^{n_{l,t-1}-\tilde m_j} \\
&> \sum_{\substack{\tilde m_1,\dots,\tilde m_{J_{t-1}} \geq 0 \\ \text{s.t. } \sum_l \tilde m_l = m_j}} \prod_{l\in J_{t-1}} \binom{n_{l,t-1}}{\tilde m_l} \underline{p}_l^{\tilde m_l}(1 - \overline{p}_l)^{n_{l,t-1}-\tilde m_j} = \xi > 0.
\end{aligned}$$

Therefore,

$$
\begin{aligned}
& P(\mathrm{Card}(J_t) = 0 |\, \mathrm{Card}(J_{t-1}) > 1) \\
= \; & P\left(\forall j \in J_{t-1}, N_{j,t}(x) \geq 0, K_{j,t-1} + N_{j,t}(x) x^\star_{j,t}(1-e_j) < \sum_{i=1}^{N_{j,t}(x)} Y_i |\, \mathrm{Card}(J_{t-1}) > 1\right) \\
\geq \; & P\left(\forall j \in J_{t-1}, N_{j,t}(x) > 0, K_{j,t-1} + N_{j,t}(x) x^\star_{j,t}(1-e_j) < \sum_{i=1}^{N_{j,t}(x)} Y_i |\, \mathrm{Card}(J_{t-1}) > 1\right) \\
\geq \; & \sum_{m_j=1}^{N} P_t\left(N_{j,t}(x) = m_j |\, \mathrm{Card}(J_{t-1}) > 1\right) P\left(K_{j,t-1} + m_j x^\star_{j,t}(1-e_j) < \sum_{i=1}^{m_j} Y_i\right) \\
> \; & \sum_{m_j=1}^{N} \xi P\left(K_{j,t-1} + m_j x^\star_{j,t}(1-e_j) < \sum_{i=1}^{m_j} Y_i\right) = \bar{\xi} > 0.
\end{aligned}
$$

Thus, we have

$$
\begin{aligned}
& P(\mathrm{Card}(J_t) > 1 |\, \mathrm{Card}(J_{t-1}) > 1) \\
= \; & 1 - P(\mathrm{Card}(J_t) = 0 |\, \mathrm{Card}(J_{t-1}) > 1) - P(\mathrm{Card}(J_t) = 1 |\, \mathrm{Card}(J_{t-1}) > 1) \\
\leq \; & 1 - P(\mathrm{Card}(J_t) = 0 |\, \mathrm{Card}(J_{t-1}) > 1) \\
< \; & 1 - \bar{\xi} < 1.
\end{aligned}
$$

By successive conditioning, we get

$$
P(\mathrm{Card}(J_t) > 1) = P(\mathrm{Card}(J_0) > 1) \prod_{s=1}^{t} P(\mathrm{Card}(J_s) > 1 |\, \mathrm{Card}(J_{s-1}) > 1) < \left(1 - \bar{\xi}\right)^t .
$$

So, the probability $P(\mathrm{Card}(J_t) > 1)$ decreases geometrically as t increases. □

Proof of Prop. 3.5.2. Let us consider a price vector x such that $x_j < x_k$ for all $k \neq j$. Since the change probability $p_{k\to j}$ (for $k \neq j$) is a decreasing function (see Appendix 3.8.3.1), $p_{k\to j}(x) > p_{k\to l}(x)$ for $l \neq j$ given the initial portfolio sizes n_j's are constant.

Below we use the stochastic orders ($\leq_{\mathrm{st}}$, $\leq_{\mathrm{cx}}$) and the majorization order ($\leq_{\mathrm{m}}$) whose definitions and main properties are recalled in the Appendices 3.8.1.7 and 3.8.1.8 respectively. Using the convolution property of the stochastic order J times, we can show a stochastic order of the portfolio size

$$
N_k(\boldsymbol{x}) \leq_{\mathrm{st}} N_j(\boldsymbol{x}), \forall k \neq j.
$$

Let us consider the underwriting result per policy

$$
uw_j(\boldsymbol{x}, n) = \frac{1}{n}\left(n x_j(1-e_j) - \sum_{i=1}^{n} Y_i\right) = x_j(1-e_j) - \sum_{i=1}^{n} \frac{1}{n} Y_i,
$$

for insurer j having n policies, where Y_i denotes the total claim amount per policy.

Let $n < \tilde{n}$ be two policy numbers and $\boldsymbol{a}_{\tilde{n}}, \boldsymbol{a}_n \in \mathbb{R}^{\tilde{n}}$ be defined as

$$
\boldsymbol{a}_{\tilde{n}} = \left(\frac{1}{\tilde{n}}, \ldots, \frac{1}{\tilde{n}}\right) \quad \text{and} \quad \boldsymbol{a}_n = \left(\underbrace{\frac{1}{n}, \ldots, \frac{1}{n}}_{\text{size } n}, \underbrace{0, \ldots, 0}_{\text{size } \tilde{n}-n}\right).
$$

Since $\boldsymbol{a}_{\tilde{n}} \leq_{\mathrm{m}} \boldsymbol{a}_n$ and $(Y_i)_i$'s are i.i.d. random variables, we have $\sum_i a_{\tilde{n},i} Y_i \leq_{\mathrm{cx}} \sum_i a_{n,i} Y_i$ i.e.

$$
\sum_{i=1}^{\tilde{n}} \frac{1}{\tilde{n}} Y_i \leq_{\mathrm{cx}} \sum_{i=1}^{n} \frac{1}{n} Y_i.
$$

For all increasing convex functions ϕ, the function $x \mapsto \phi(x+a)$ is still increasing and convex. Thus for all random variables X, Y such that $X \leq_{\text{icx}} Y$ and real numbers a, b, $a \leq b$, we have

$$E(\phi(X+a)) \leq E(\phi(X+b)) \leq E(\phi(Y+b)) \Leftrightarrow a + X \leq_{\text{icx}} b + Y.$$

As $x_j(1-e_j) \leq x_k(1-e_k)$ and using the fact that $X \leq_{\text{cx}} Y$ is equivalent to $-X \leq_{\text{cx}} -Y$, we have

$$uw_j(\boldsymbol{x}, \tilde{n}) \leq_{\text{icx}} uw_k(\boldsymbol{x}, n), \forall k \neq j.$$

Using Theorem 3.A.23 of Shaked and Shanthikumar (2007), except that for all ϕ convex, $E(\phi(uw_j(\boldsymbol{x}, n)))$ is a decreasing function of n and $N_k(\boldsymbol{x}) \leq_{\text{st}} N_j(\boldsymbol{x})$, we can show $UW_j = uw_j(\boldsymbol{x}, N_j(\boldsymbol{x})) \leq_{\text{icx}} uw_k(\boldsymbol{x}, N_k(\boldsymbol{x})) = UW_k$. □

3.8.4 Details of Section 3.6

3.8.4.1 Computation details

Computation is based on a Karush-Kuhn-Tucker (KKT) reformulation of the generalized Nash equilibrium problem (GNEP). We present briefly the problem reformulation and refer the interested readers to e.g. Facchinei and Kanzow (2009), Dreves et al. (2011) or Dutang (2013). In our setting we have J players and three constraints for each player. For each j of the J subproblems, the KKT conditions are

$$\begin{aligned} \nabla_{x_j} O_j(x) - \sum_{1 \leq m \leq 3} \lambda^j_m \nabla_{x_j} g^m_j(x) = 0, \\ 0 \leq \lambda^j \perp g_j(x) \geq 0. \end{aligned}$$

The inequality part is called the complementarity constraint. The reformulation proposed uses a complementarity function $\phi(a, b)$ to reformulate the inequality constraints $\lambda^j, g_j(x) \geq 0$ and $\lambda^{jT} g_j(x) = 0$.

A point satisfying the KKT conditions is also a generalized Nash equilibrium if the objective functions are pseudoconcave and a constraint qualification holds. We have seen that objective functions are either strictly concave or pseudoconcave. Whereas constraint qualifications are always verified for linear constraints, or strictly monotone functions, see Theorem 2 of Arrow and Enthoven (1961), which is also verified.

By definition, a complementarity function is such that $\phi(a, b) = 0$ is equivalent to $a, b \geq 0$ and $ab = 0$. A typical example is $\phi(a, b) = \min(a, b)$ or $\phi(a, b) = \sqrt{a^2 + b^2} - (a + b)$ called the Fischer-Burmeister function. With this tool, the KKT condition can be rewritten as

$$\begin{aligned} \nabla_{x_j} L_j(x, \lambda^j) = 0 \\ \phi_.(\lambda^j, g_j(x)) = 0 \end{aligned},$$

where L_j is the Lagrangian function for the subproblem j and $\phi_.$ denotes the component wise version of ϕ. So, subproblem j reduces to solving a so-called nonsmooth equation. In this paper, we use the Fischer-Burmeister complementarity function. This method is implemented in the R package **GNE** of Dutang (2015), whereas the game functions are implemented in the R package **NLIG** of Dutang and Mouminoux (2018).

3.8.5 Appendix on the loss model

3.8.5.1 Reminders from probability theory

3.8.5.2 A Poisson-Gamma mixture

Consider a random variable M. Assume that $M|\Lambda = \lambda \sim \mathcal{P}(\lambda)$ and $\Lambda \sim \mathcal{G}(r, p/(1-p))$. Let $q = p/(1-p)$. The probability generating function of M is

$$\begin{aligned} P_M(t) &= E(t^M) = E(E(t^M|\Lambda)) = E(e^{\Lambda(t-1)}) = M_\Lambda(t-1) = \left(\frac{q}{q-(t-1)}\right)^r \\ &= \left(\frac{p/(1-p)}{p/(1-p)+1-t}\right)^r = \left(\frac{p}{p+1-p-(1-p)t}\right)^r = \left(\frac{p}{1-(1-p)t}\right)^r \end{aligned}$$

Thus, we find the well known result that $M \sim \mathcal{NB}(r,p)$.

3.8.5.3 Scale parameters for usual discrete distributions

Consider two independent random variables M_1, M_2.

— if $M_i \sim \mathcal{P}(\lambda)$, then $M_1 + M_2 \sim \mathcal{P}(2\lambda)$ since

$$P_{M_1+M_2}(t) = P_{M_1}(t)P_{M_2}(t) = e^{\lambda(t-1)}e^{\lambda(t-1)} = e^{2\lambda(t-1)}.$$

— if $M_i \sim \mathcal{NB}(r,p)$, then $M_1 + M_2 \sim \mathcal{NB}(2r,p)$ since

$$P_{M_1+M_2}(t) = P_{M_1}(t)P_{M_2}(t) = \left(\frac{p}{1-(1-p)t}\right)^r \left(\frac{p}{1-(1-p)t}\right)^r = \left(\frac{p}{1-(1-p)t}\right)^{2r}.$$

— if $M_i \sim \mathcal{B}(n,p)$, then $M_1 + M_2 \sim \mathcal{B}(2n,p)$ since

$$P_{M_1+M_2}(t) = P_{M_1}(t)P_{M_2}(t) = (1-p+pt))^n (1-p+pt))^n = (1-p+pt))^{2n}.$$

3.8.5.4 Scale parameters for multinomial distributions

Consider two independent multinomial vectors M_1, M_2: $M_1 \sim \mathcal{M}_k(n, p_1, \dots, p_k)$ and $M_2 \sim \mathcal{M}_k(m, q_1, \dots, q_k)$. We have

$$E(z^{M_1+M_2}) = \left(\sum_i p_i z_i\right)^n \left(\sum_i q_i z_i\right)^m$$

Only when $\forall i, p_i = q_i$, we get a multinomial vector

$$E(z^{M_1+M_2}) = \left(\sum_i p_i z_i\right)^{n+m}.$$

3.8.5.5 Fréchet copula

Consider the multivariate Fréchet copula defined as

$$C_a^F(u_1, \dots, u_J) = (1-a)C^\perp(u_1, \dots, u_J) + aC^+(u_1, \dots, u_J).$$

where $C^\perp$ is the independence copula and C^+ is the Fréchet upper bound, i.e.

$$C^\perp(u_1, \dots, u_J) = \prod_{i=1}^J u_i, \quad C^+(u_1, \dots, u_J) = \min_{i=1,\dots,J} u_i.$$

Setting $a = 1$, we get the Fréchet upper bound, whereas setting $a = -1$, we get the independence copula.

A simple algorithm to simulate is simply

1. simulate $V \sim \mathcal{U}(0,1)$
2. if $V \leq a$, then simulate $U_a \sim \mathcal{U}(0,1)$, $\forall i, U_i \leftarrow U_a$.
3. otherwise, simulate $(U_i)_i \overset{\text{i.i.d.}}{\sim} \mathcal{U}(0,1)$.

In other words, $U_i = \mathbb{1}_{V\leq a}U_a + \mathbb{1}_{V>a}V_i$, where $V, U_a, (V_i)_i \overset{\text{i.i.d.}}{\sim} \mathcal{U}(0,1)$. Indeed,

$$\begin{aligned}
P(U_1 \leq u_1, \ldots, U_J \leq u_J) &= P(U_1 \leq u_1, \ldots, U_J \leq u_J | V \leq a)P(V \leq a) \\
&\quad + P(U_1 \leq u_1, \ldots, U_J \leq u_J | V > a)P(V > a) \\
&= P(U_a \leq u_1, \ldots, U_a \leq u_J)a + P(V_1 \leq u_1, \ldots, V_J \leq u_J)(1-a) \\
&= aC^+(u_1, \ldots, u_J) + (1-a)C^\perp(u_1, \ldots, u_J).
\end{aligned}$$

3.8.6 On the sensitivity analysis

3.8.6.1 Implicit function theorem

Below the implicit function theorem, see, e.g., (Zorich, 2000, Chap. 8).

Theorem. *Let F be a bivariate C^1 function on some open disk with center in (a,b), such that $F(a,b) = 0$. If $\frac{\partial F}{\partial y}(a,b) \neq 0$, then there exists an $h > 0$, and a unique function φ defined for $]a-h, a+h[$, such that*

$$\varphi(a) = b \quad \text{and} \quad \forall |x-a| < h, F(x, \varphi(x)) = 0.$$

Moreover on $|x-a| < h$, the function φ is C^1 and

$$\varphi'(x) = - \left. \frac{\frac{\partial F}{\partial x}(x,y)}{\frac{\partial F}{\partial y}(x,y)} \right|_{y=\varphi(x)}.$$

3.8.6.2 On the sensitivity analysis

Using the implicit function theorem see Appendix 3.8.6.1, we define $F : \mathbb{R}^J \times \mathbb{R}^J \mapsto \mathbb{R}^J$

$$F(x,y) = M(y)x - v(y),$$

where y is a parameter of interest. The following hypothesis are verified

— H1: $\exists x^\star, y^\star, F(x^\star, y^\star) = 0$.

— H2/H3: F has C^1 component in the neighborhood of $(x^\star, y^\star)$.

— H4: $\text{Jac}_x F(x,y) = M(y)$ has non-zero determinant.

Thus, there exists $g : \mathbb{R}^J \mapsto \mathbb{R}^J$ such that $x^\star = g(y^\star)$, g is C^1 around $(x^\star, y^\star)$ and

$$\text{Jac}\, g(y) = -(\text{Jac}_x F(x,y))^{-1} \text{Jac}_y F(x,y)|_{x=g(y)} = -(M(y))^{-1} \text{Jac}_y F(x,y)|_{x=g(y)}.$$

Using Appendix 3.8.6.2, there exists $g : \mathbb{R}^J \mapsto \mathbb{R}^J$ such that $\boldsymbol{x}^\star = g(\boldsymbol{y}^\star)$, g is C^1 around $(\boldsymbol{x}^\star, \boldsymbol{y}^\star)$ and

$$\text{Jac}\, g(\boldsymbol{y}) = -(\text{Jac}_{\boldsymbol{x}} F(\boldsymbol{x},\boldsymbol{y}))^{-1} \text{Jac}_{\boldsymbol{y}} F(\boldsymbol{x},\boldsymbol{y})|_{\boldsymbol{x}=g(\boldsymbol{y})} = -(M(\boldsymbol{y}))^{-1} \text{Jac}_{\boldsymbol{y}} F(\boldsymbol{x},\boldsymbol{y})|_{\boldsymbol{x}=g(\boldsymbol{y})}.$$

1. Let $\boldsymbol{y} = (\beta_1, \ldots, \beta_J)$.

$$F(\boldsymbol{x},\boldsymbol{y}) = M(\boldsymbol{y})\boldsymbol{x} - v(\boldsymbol{y}), \quad M = \begin{pmatrix} 2y_1 & \frac{-1-y_1}{J-1} & \ldots & \\ \frac{-1-y_2}{J-1} & 2y_2 & \frac{-1-y_2}{J-1} & \ldots \\ & & \ddots & \\ & \ldots & \frac{-1-y_J}{J-1} & 2y_J \end{pmatrix}, v = \begin{pmatrix} y_1\pi_1 \\ \vdots \\ y_J\pi_J \end{pmatrix}.$$

We have

$$\big(F(\boldsymbol{x},\boldsymbol{y})\big)_i = M_{i,.}\boldsymbol{x} - v_i = 2y_i x_i - \frac{1+y_i}{J-1} \sum_{k\neq i} x_k - y_i\pi_i$$

$$\Rightarrow \frac{\big(F(\boldsymbol{x},\boldsymbol{y})\big)_i}{\partial y_j} = \begin{cases} 0 & \text{if } j \neq i \\ 2x_i - \frac{1}{J-1}\sum\limits_{k\neq i} x_k - \pi_i & \text{if } j = i \end{cases}$$

So

$$\mathrm{Jac}_{\,y} F(\boldsymbol{x},\boldsymbol{y}) = \begin{pmatrix} 2x_1 - \frac{1}{J-1}\sum\limits_{k\neq 1} x_k & 0 & \dots \\ & \ddots & \\ \dots & 0 & 2x_J - \frac{1}{J-1}\sum\limits_{k\neq J} x_k \end{pmatrix}$$

Hence

$$\mathrm{Jac}\, g(\boldsymbol{y}) = -(M(\boldsymbol{y}))^{-1} \begin{pmatrix} 2x_1 - \frac{1}{J-1}\sum\limits_{k\neq 1} x_k & 0 & \dots \\ & \ddots & \\ \dots & 0 & 2x_J - \frac{1}{J-1}\sum\limits_{k\neq J} x_k \end{pmatrix}_{\boldsymbol{x}=g(\boldsymbol{y})}.$$

2. Let $\boldsymbol{y} = (\pi_1, \dots, \pi_J)$.

$$F(\boldsymbol{x},\boldsymbol{y}) = M\boldsymbol{x} - v(\boldsymbol{y}), \quad M = \begin{pmatrix} 2\beta_1 & \frac{-1-\beta_1}{J-1} & \dots & \\ \frac{-1-\beta_2}{J-1} & 2\beta_2 & \frac{-1-\beta_2}{J-1} & \dots \\ & & \ddots & \\ & \dots & \frac{-1-\beta_J}{J-1} & 2\beta_J \end{pmatrix}, v = \begin{pmatrix} \beta_1 y_1 \\ \vdots \\ \beta_J y_J \end{pmatrix}.$$

We have

$$\big(F(\boldsymbol{x},\boldsymbol{y})\big)_i = M_{i,.}\boldsymbol{x} - v_i = M_{i,.}\boldsymbol{x} - y_i\beta_i.$$

So

$$\mathrm{Jac}_{\,y} F(\boldsymbol{x},\boldsymbol{y}) = \begin{pmatrix} -\beta_1 & 0 & \dots \\ & \ddots & \\ \dots & 0 & -\beta_J \end{pmatrix}$$

Hence $\mathrm{Jac}\, g(\boldsymbol{y})$ is constant:

$$\mathrm{Jac}\, g(y) = -\begin{pmatrix} 2\beta_1 & \frac{-1-\beta_1}{J-1} & \dots & \\ \frac{-1-\beta_2}{J-1} & 2\beta_2 & \frac{-1-\beta_2}{J-1} & \dots \\ & & \ddots & \\ & \dots & \frac{-1-\beta_J}{J-1} & 2\beta_J \end{pmatrix}^{-1} \begin{pmatrix} -\beta_1 & 0 & \dots \\ & \ddots & \\ \dots & 0 & -\beta_J \end{pmatrix}.$$

3.8.7 Other figures and tables of Section 3.6

3.8.7.1 Two-player market – sensitivity analysis

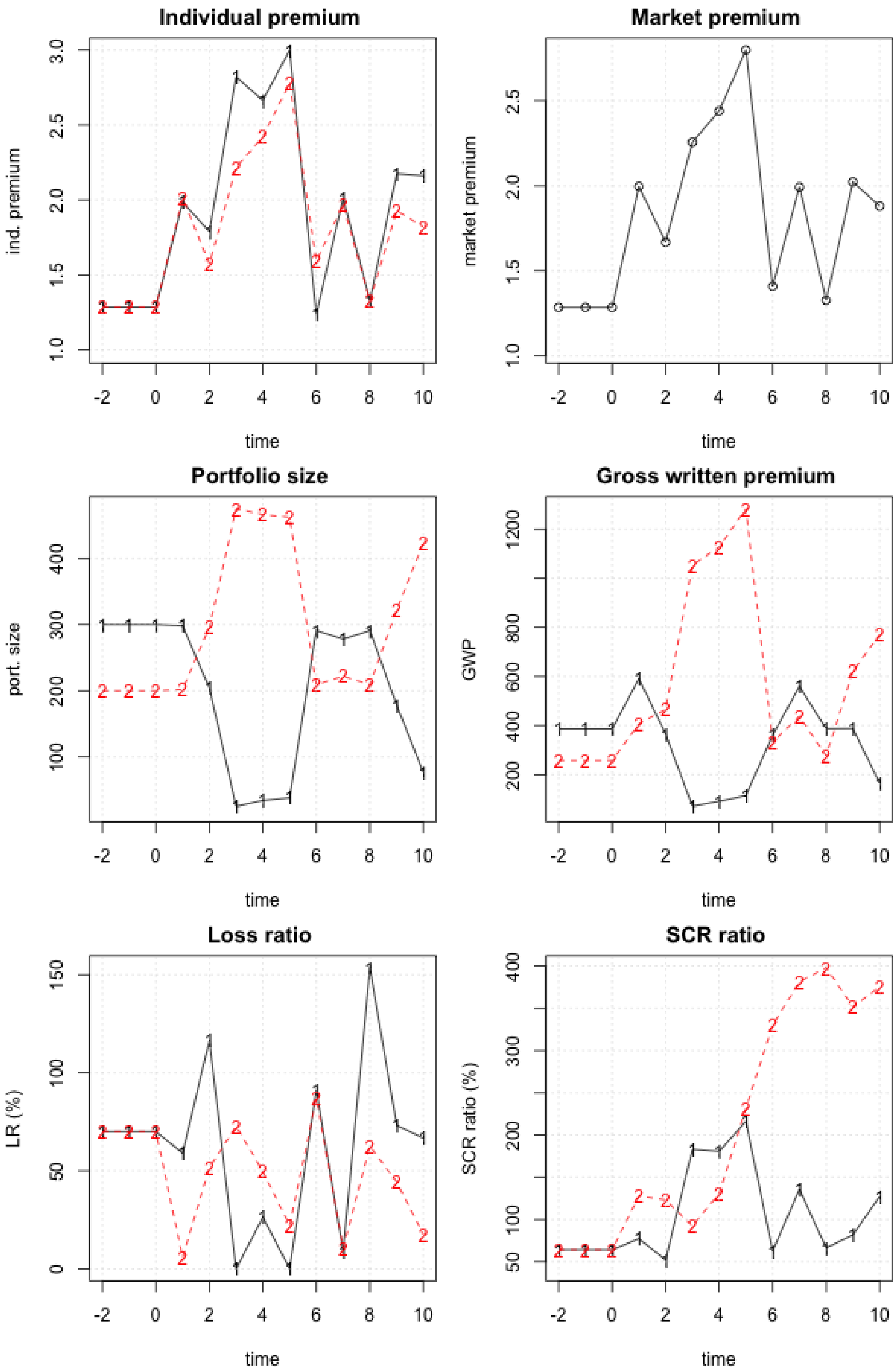


Figure 3.14 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – NBLN

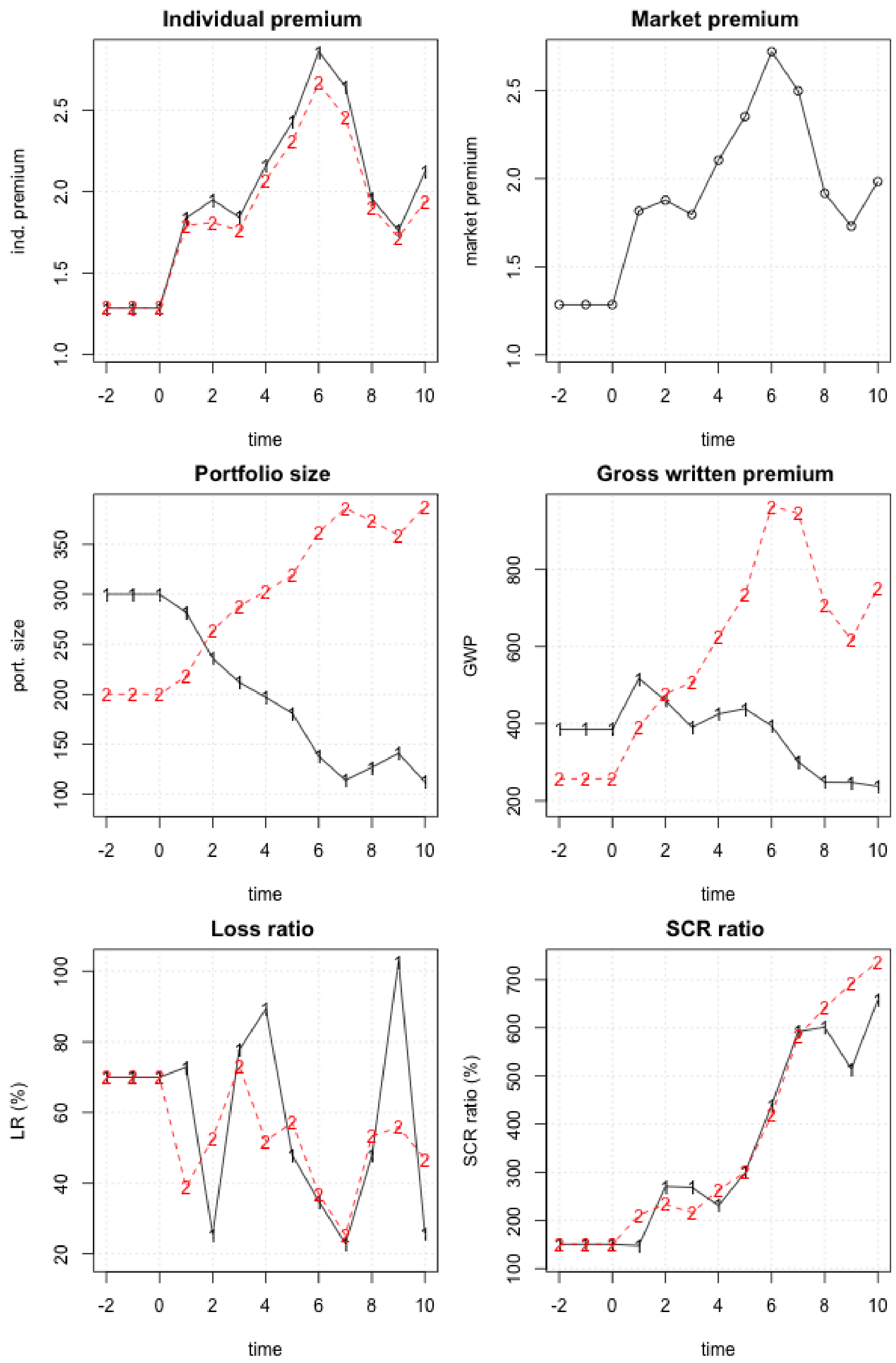


Figure 3.15 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – PG

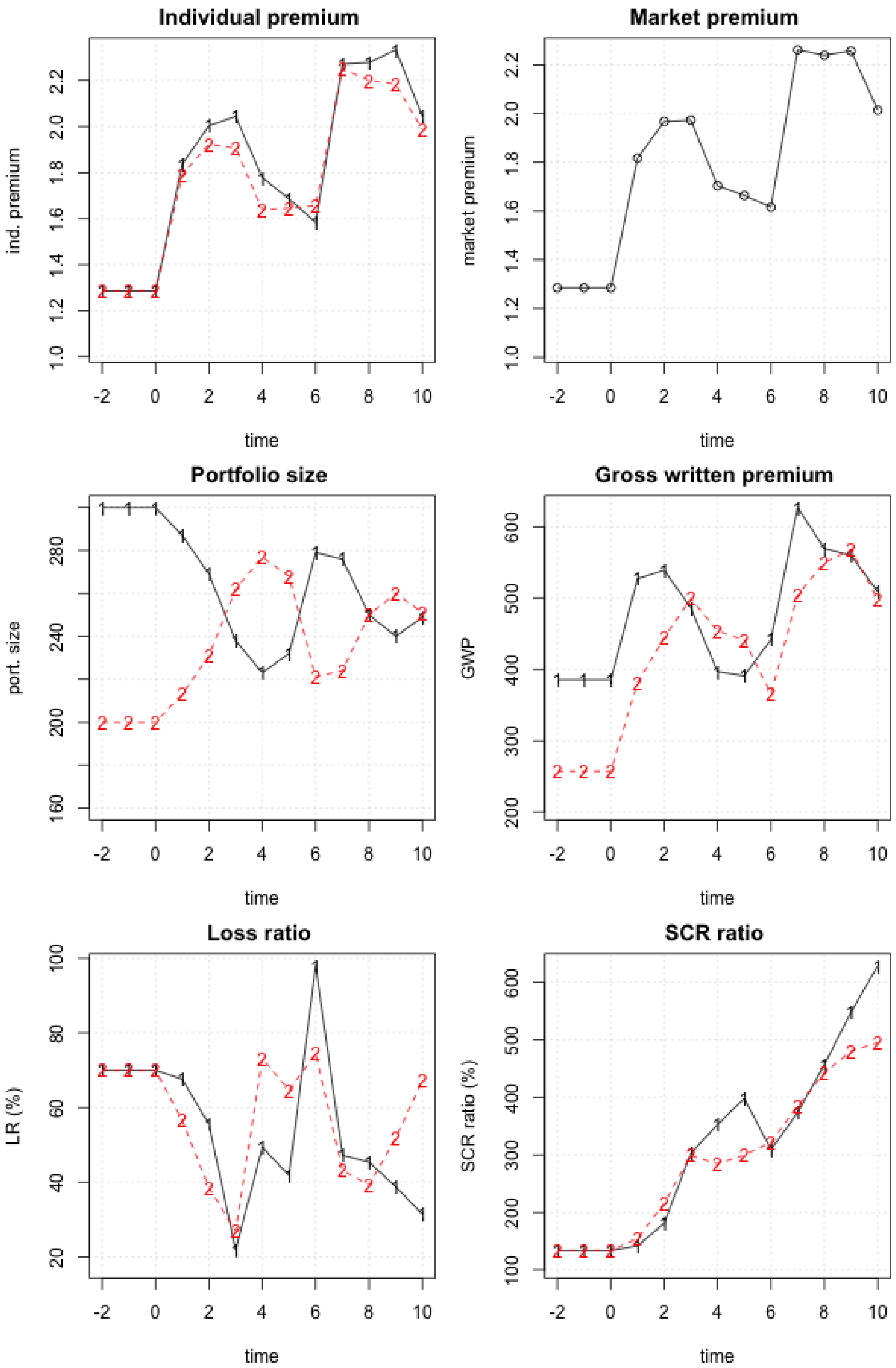


Figure 3.16 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – MLN PR

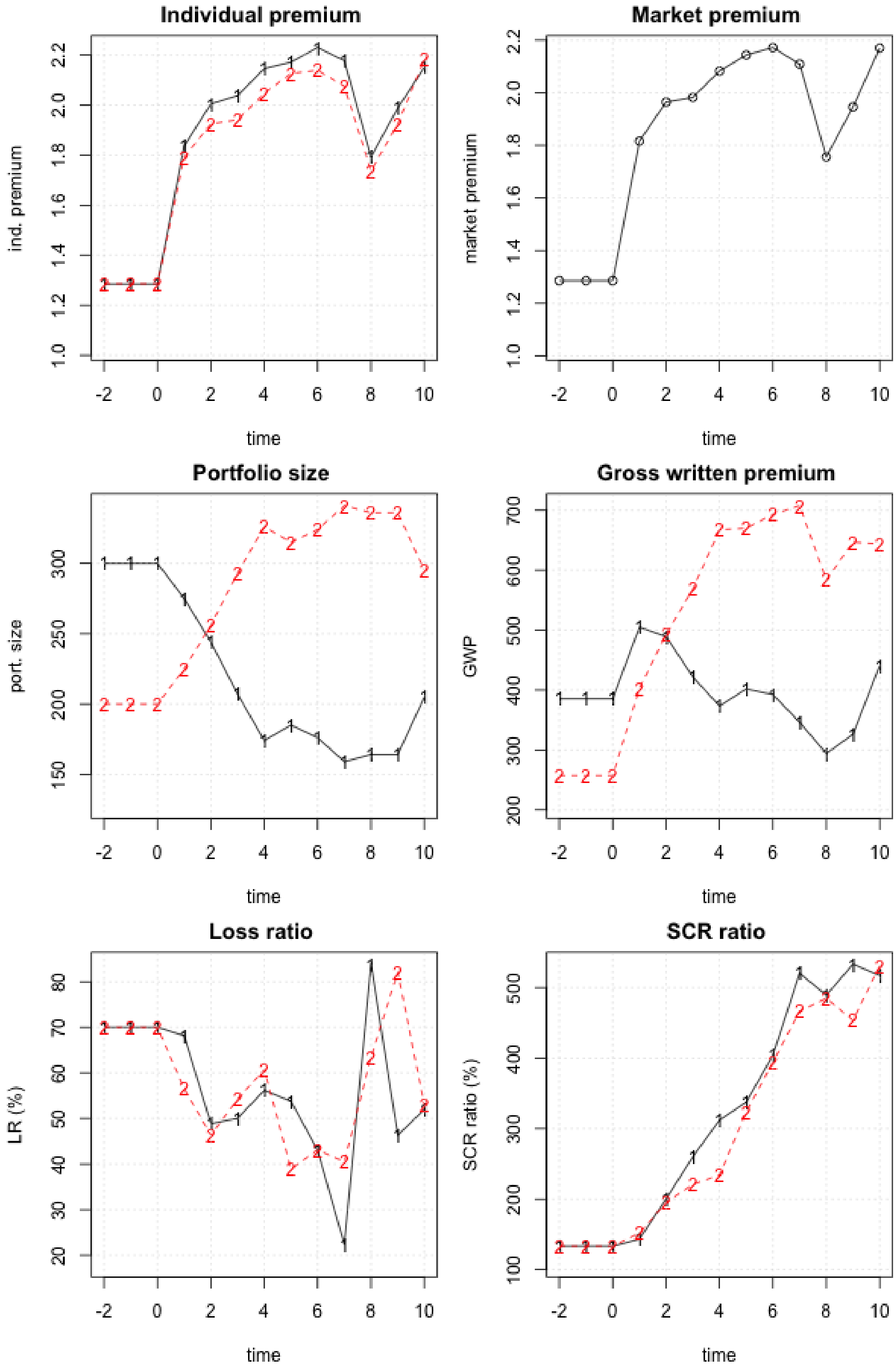


Figure 3.17 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – MLN PD 2

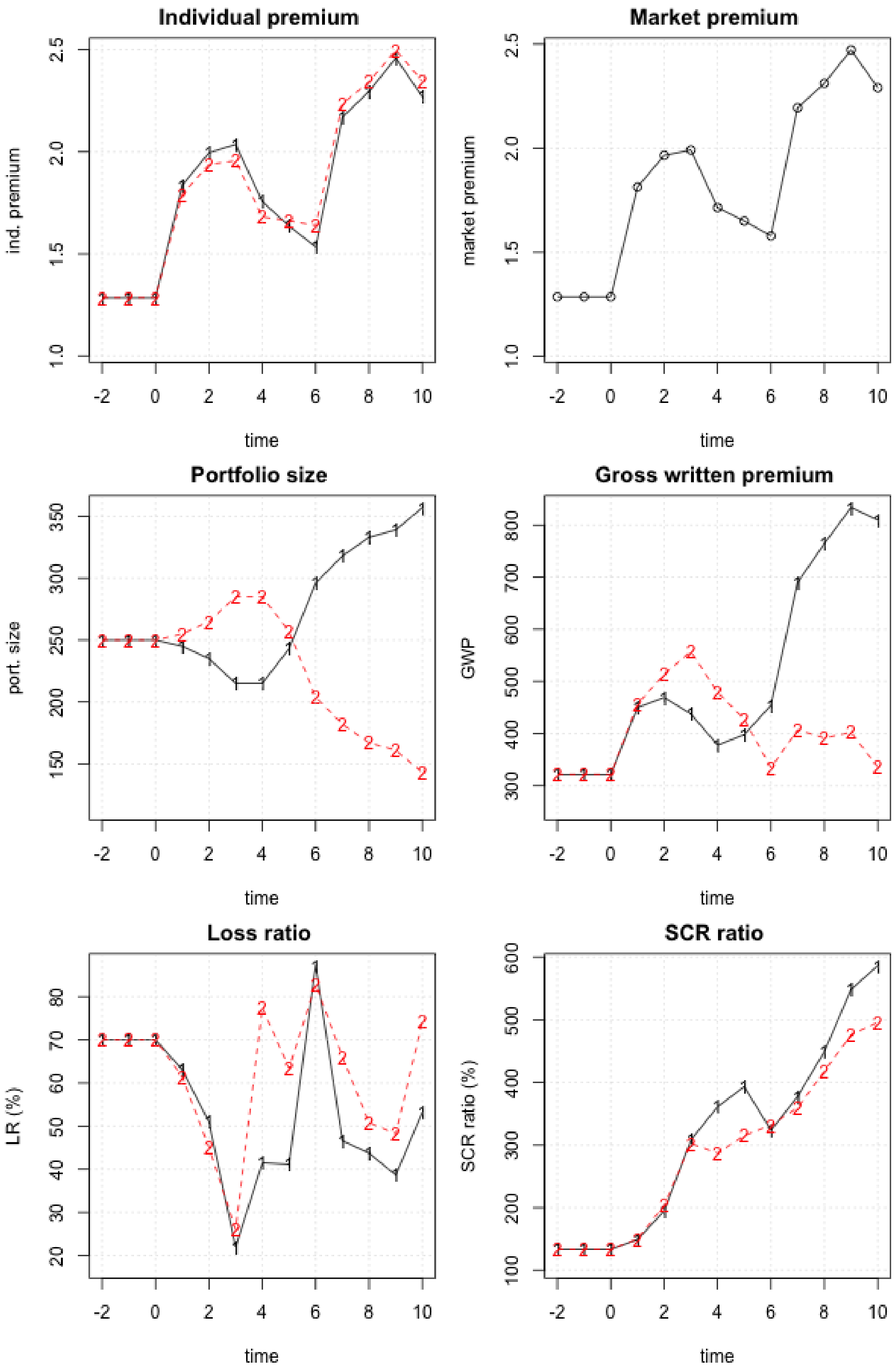


Figure 3.18 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – equalled portfolio sizes

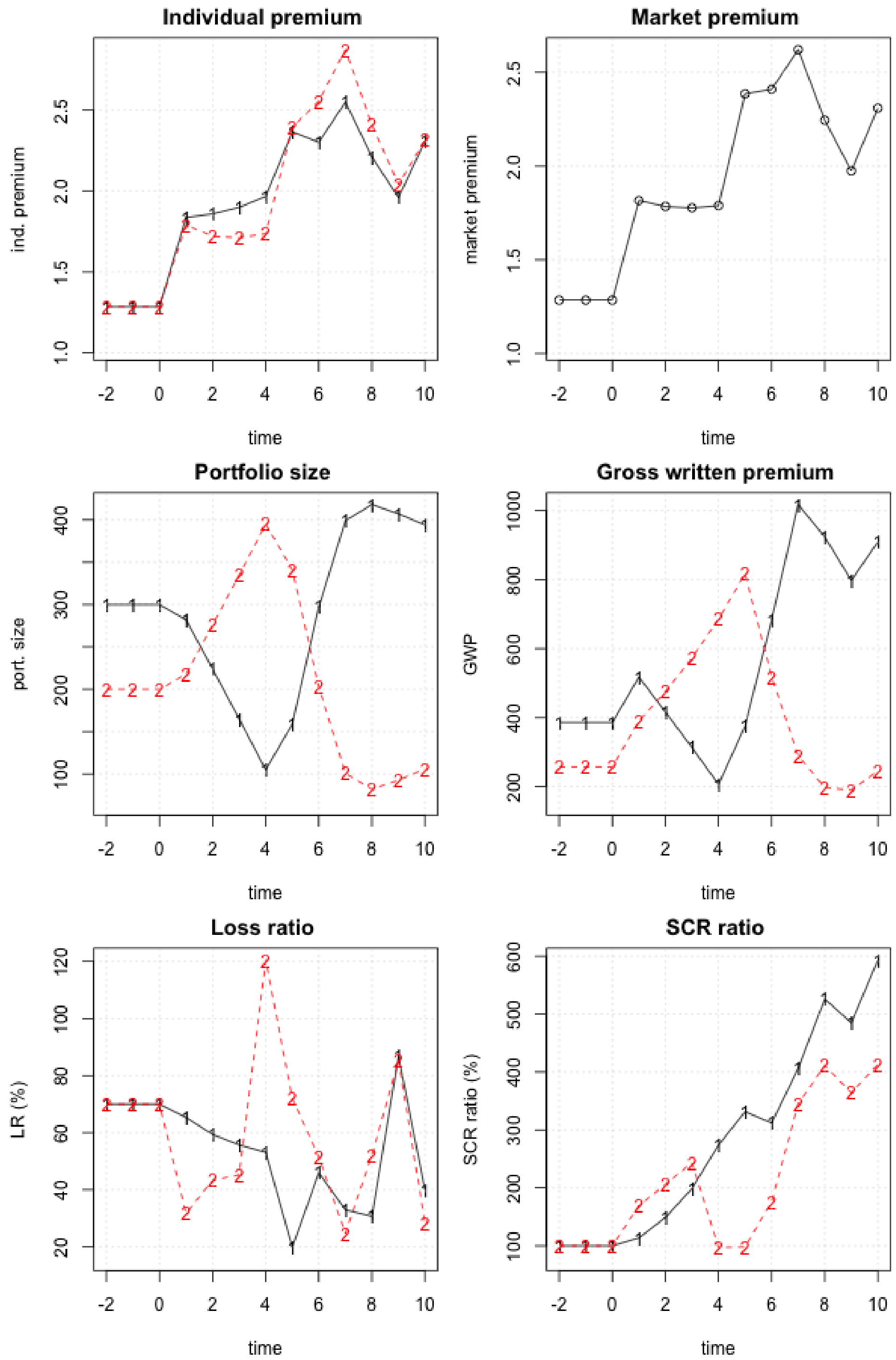


Figure 3.19 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – new capital

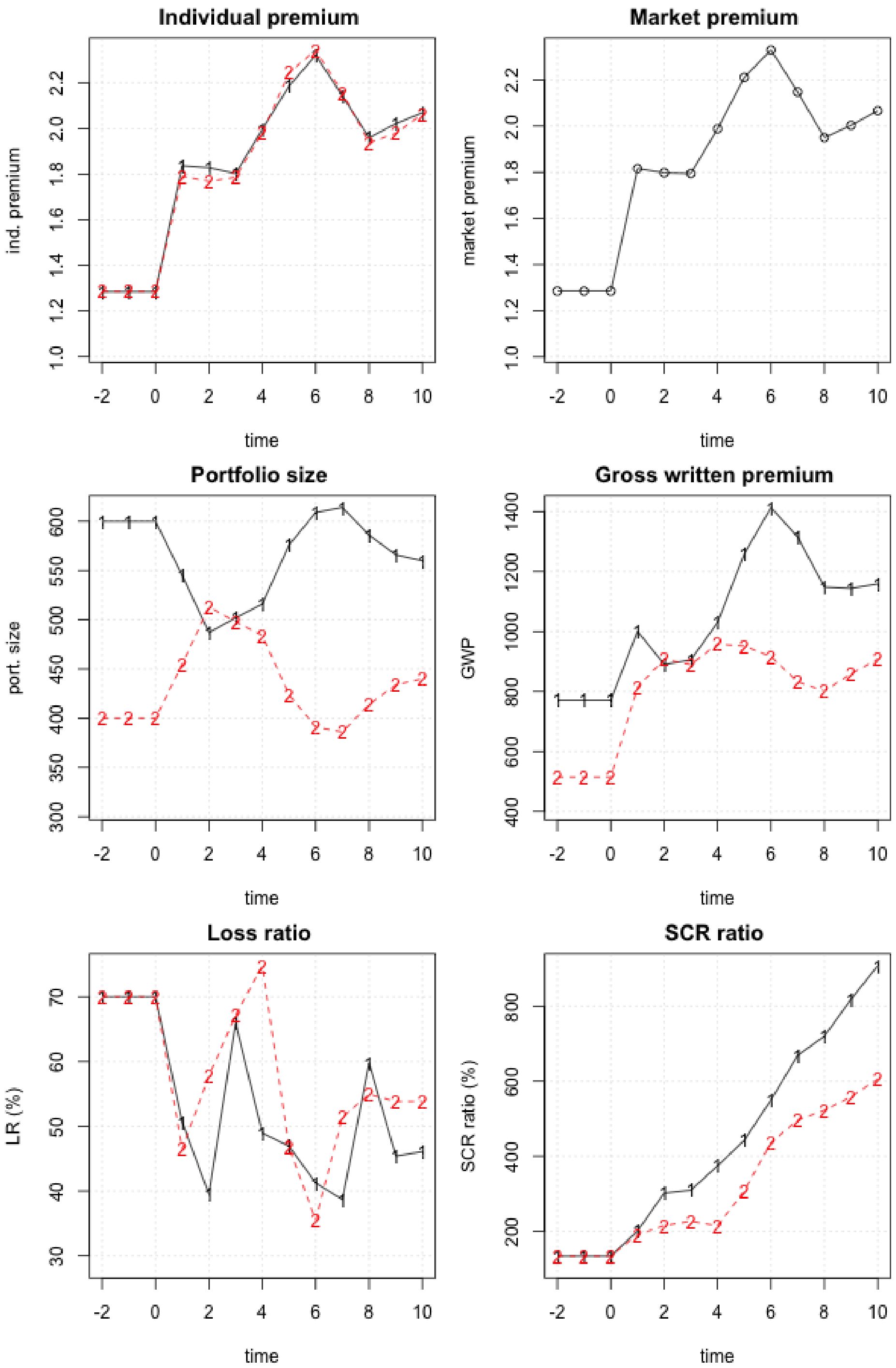


Figure 3.20 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – doubled portfolio sizes

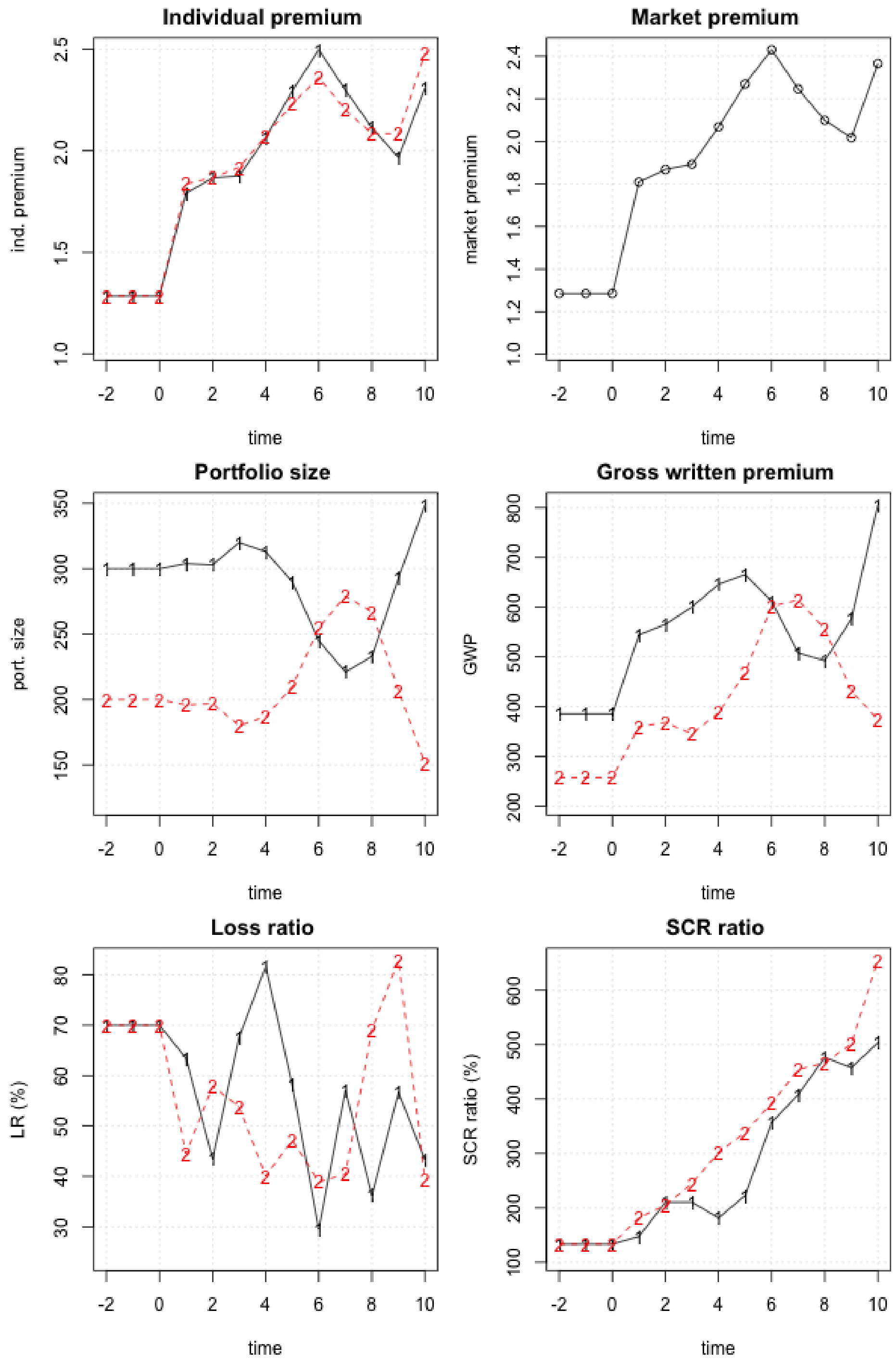


Figure 3.21 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – new price sensitivity

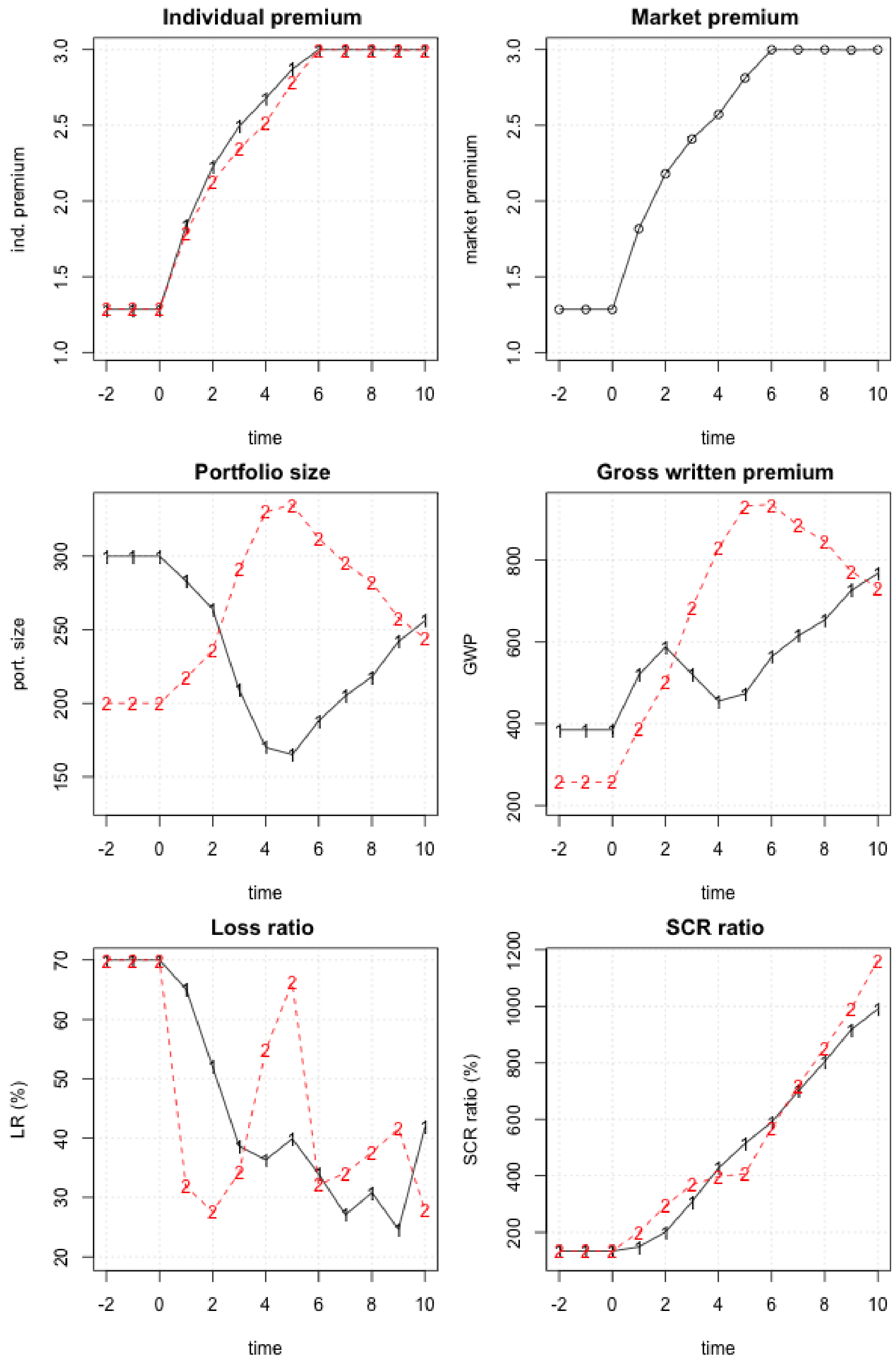


Figure 3.22 – Some indicators of the repeated game for a single run, black solid line for Insurer 1 and red dotted line for Insurer 2 – new credibility factor

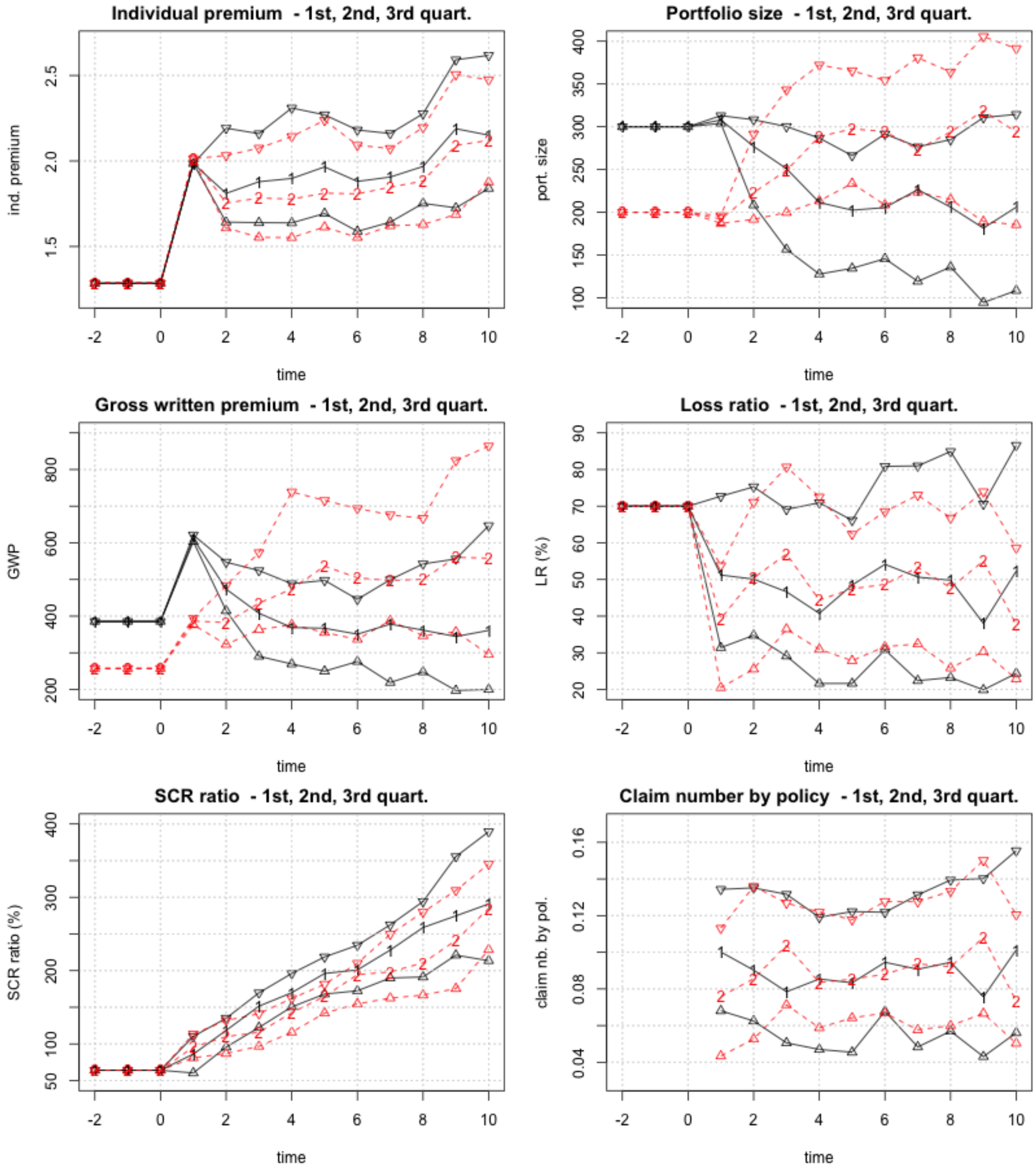


Figure 3.23 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – NBLN

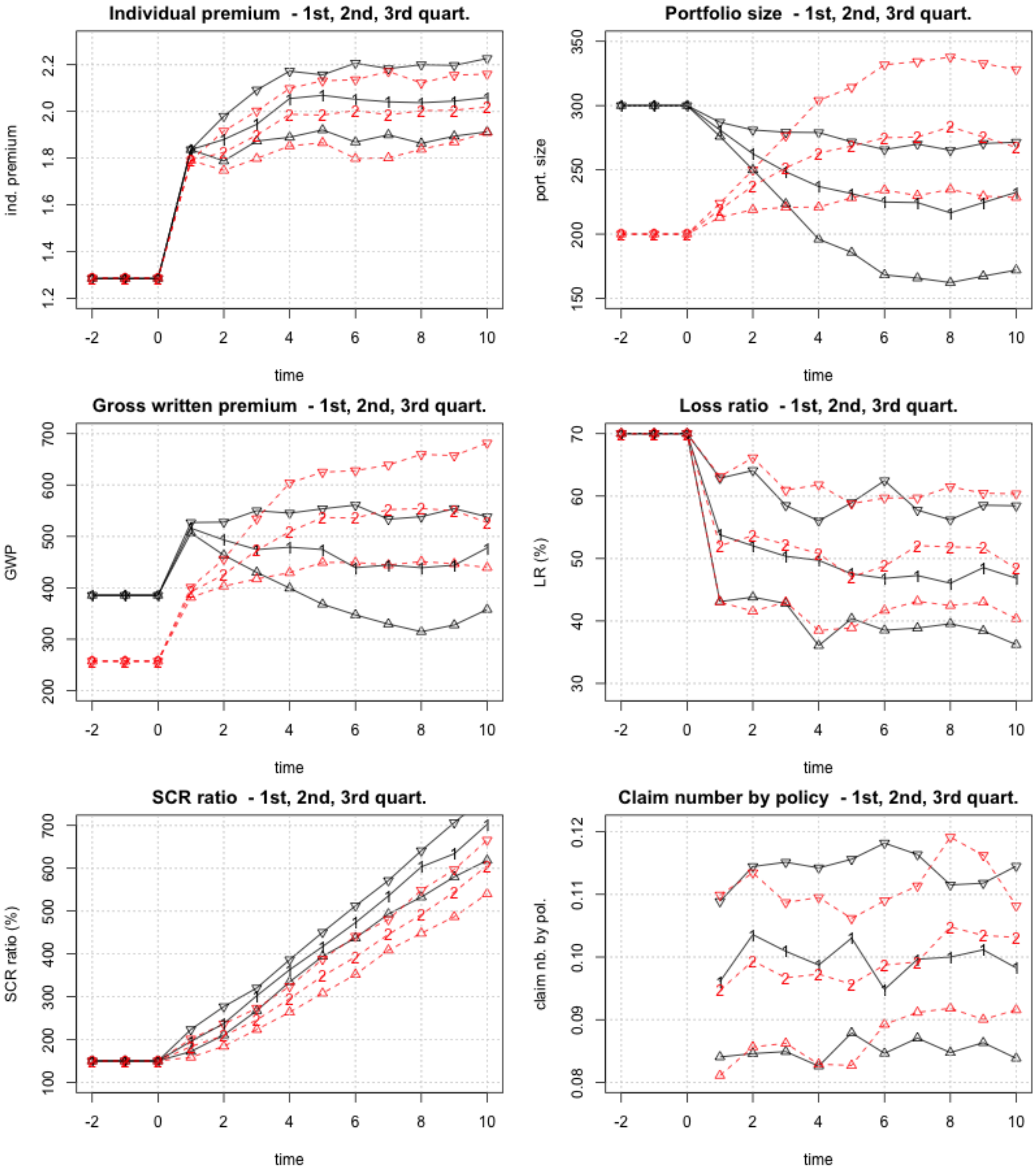


Figure 3.24 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – PG

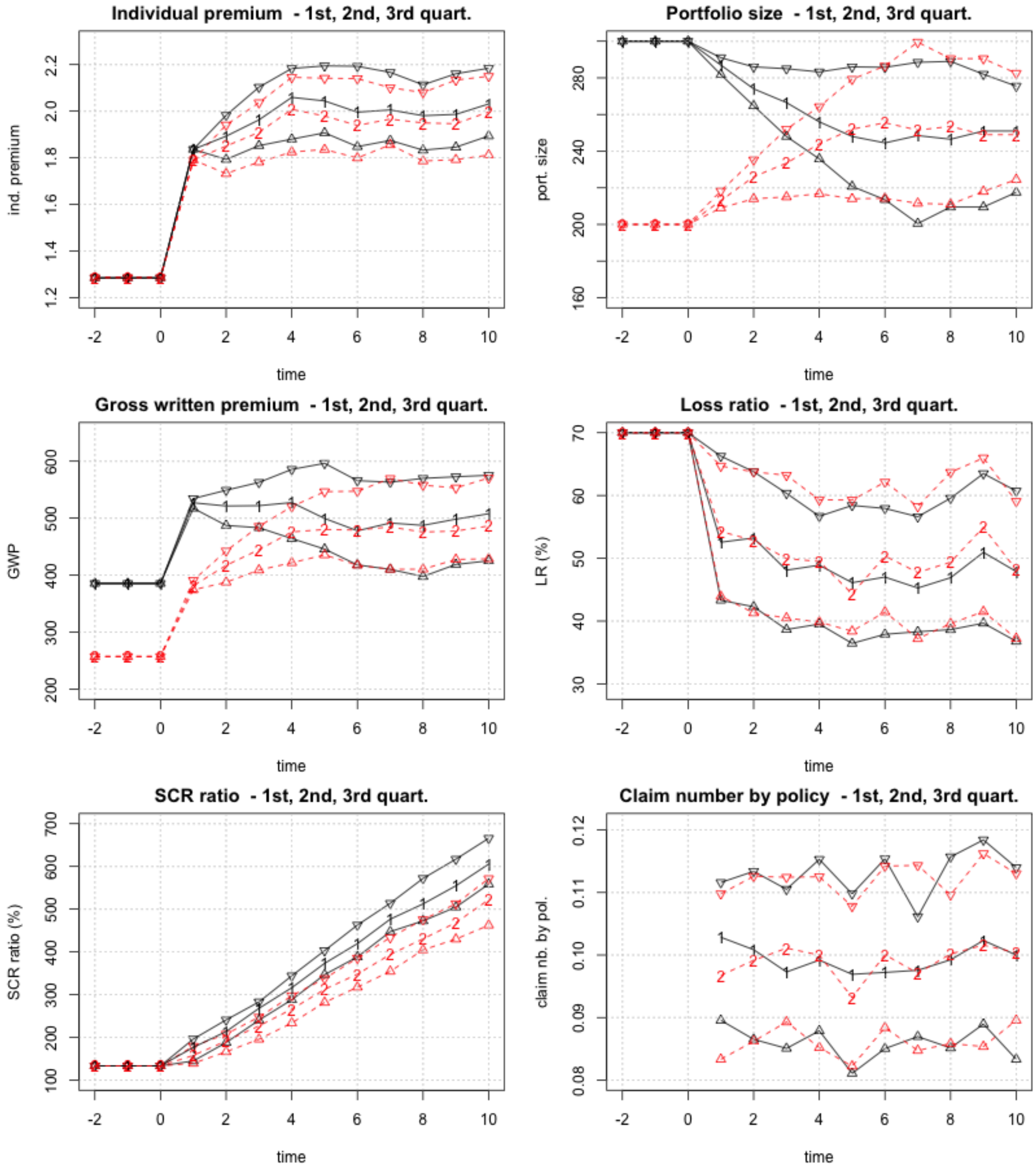


Figure 3.25 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – MLN PR

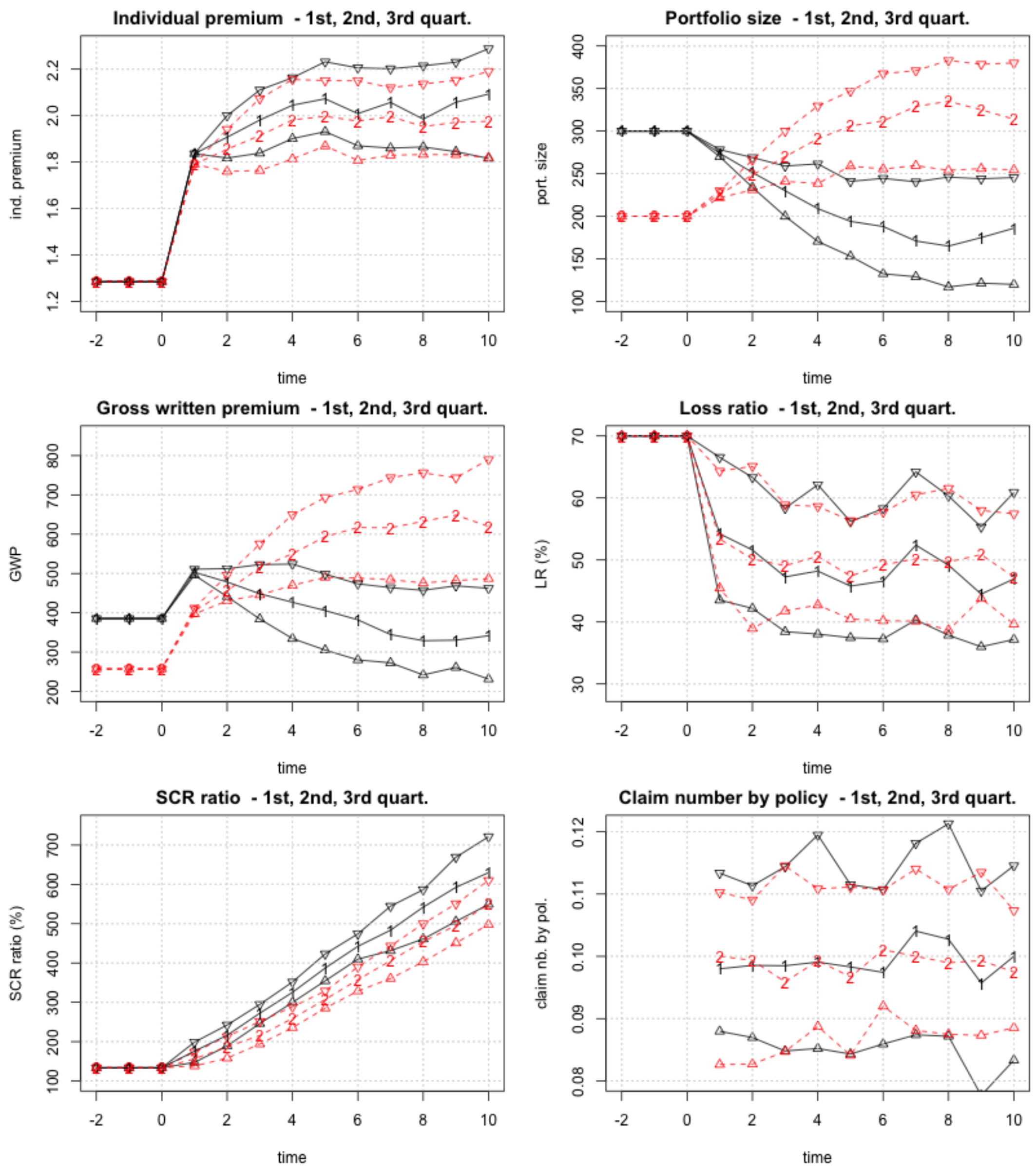


Figure 3.26 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – MLN PD 2

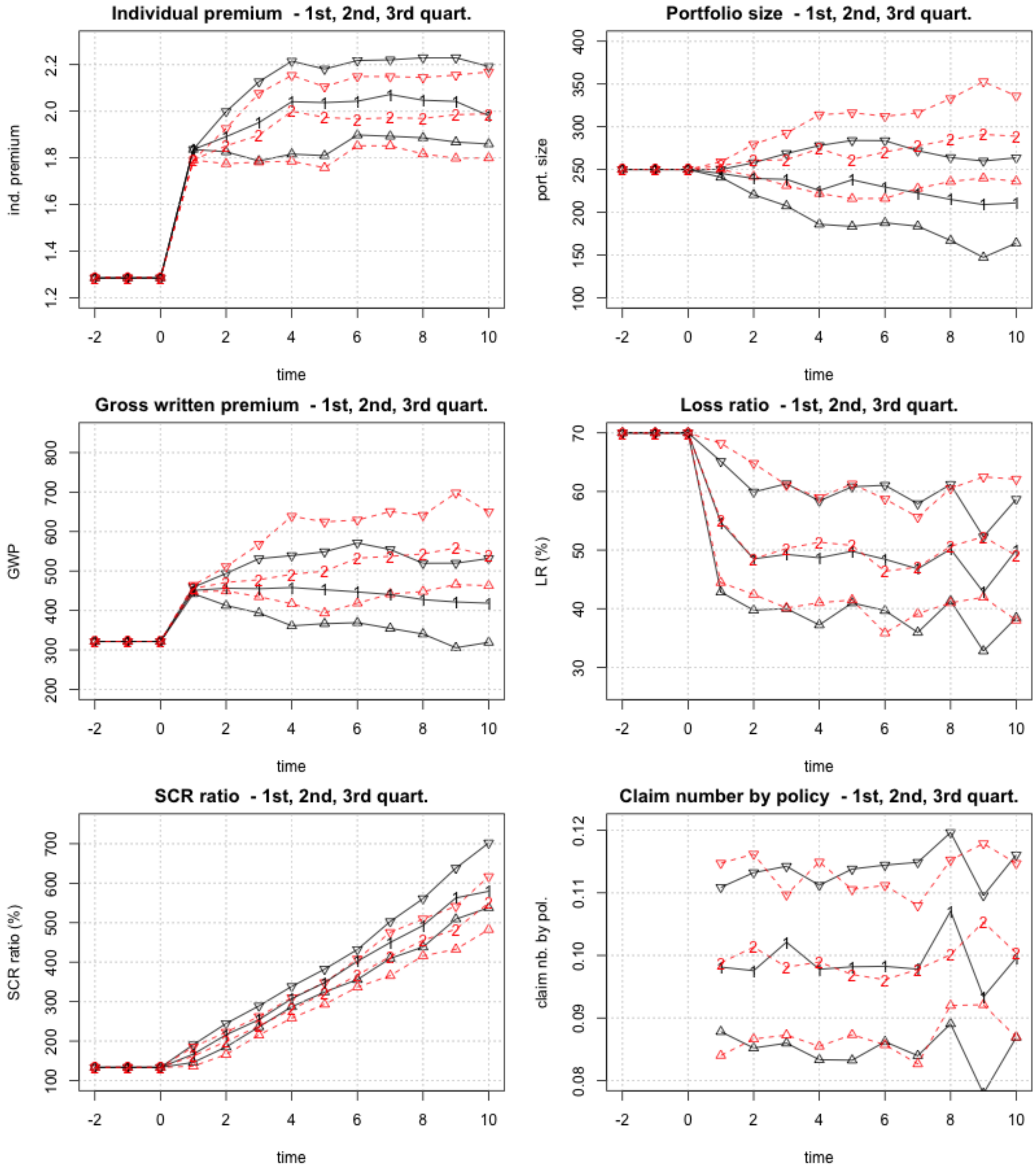


Figure 3.27 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – equalled portfolio sizes

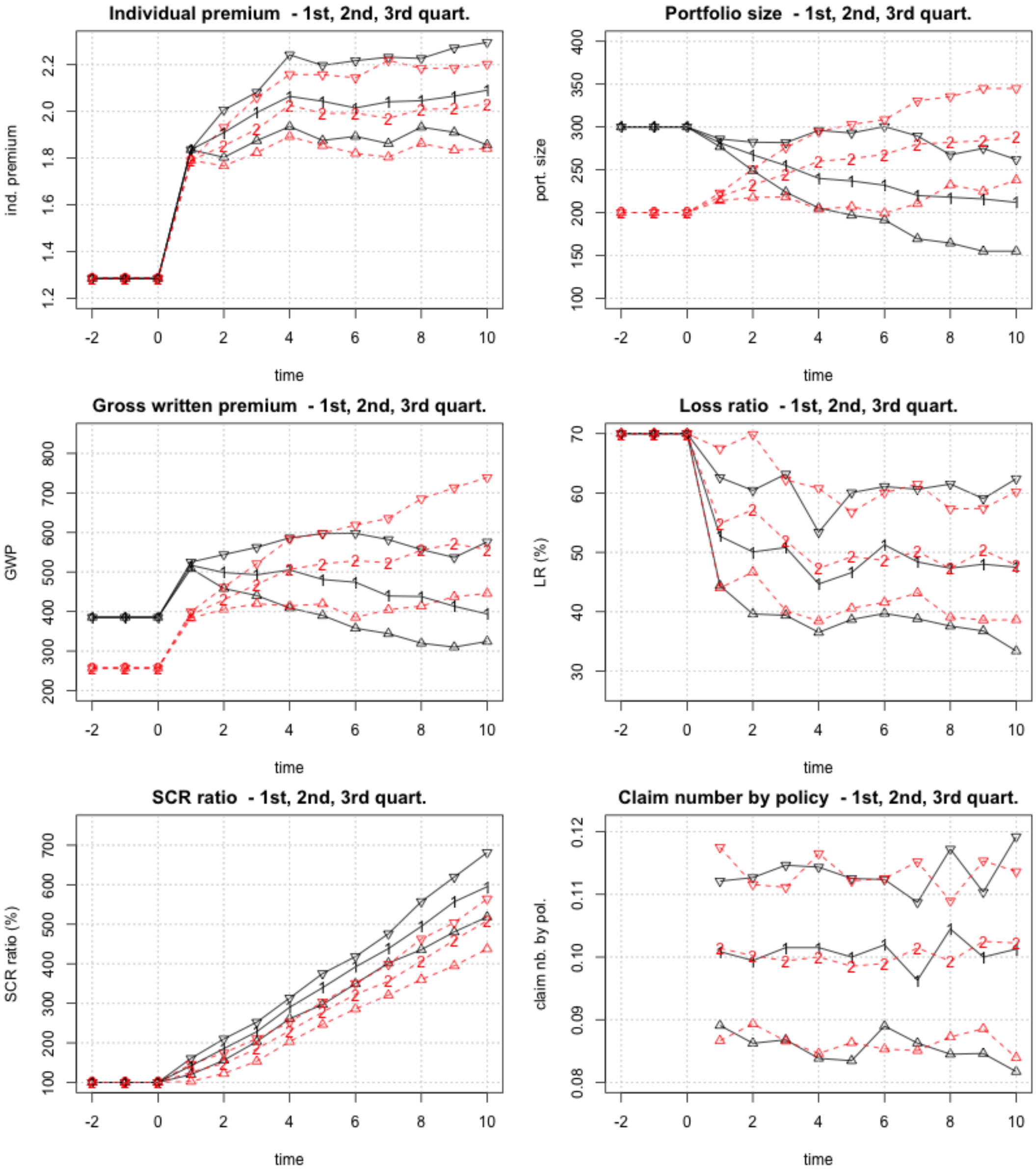


Figure 3.28 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – new capital

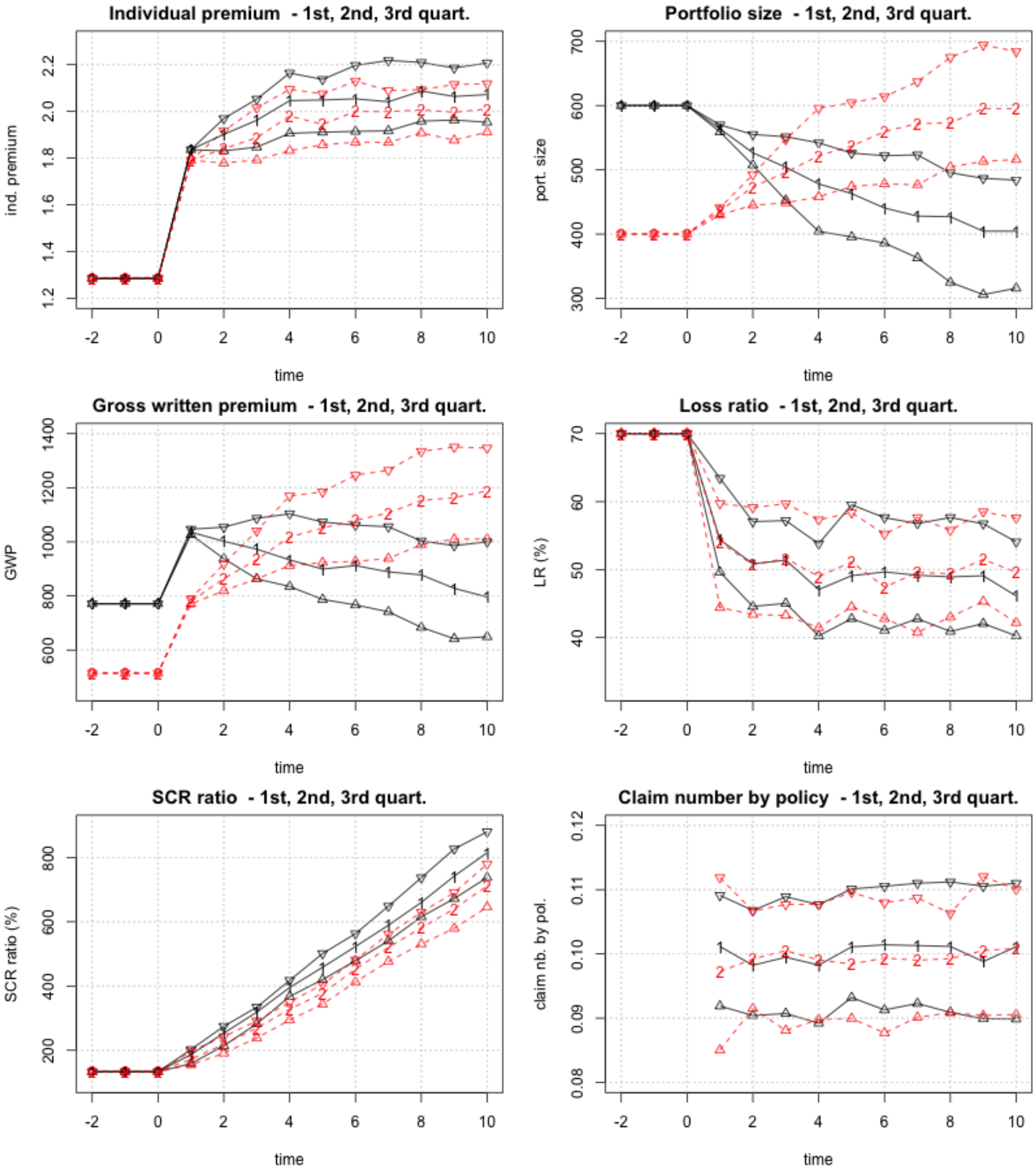


Figure 3.29 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – doubled portfolio sizes

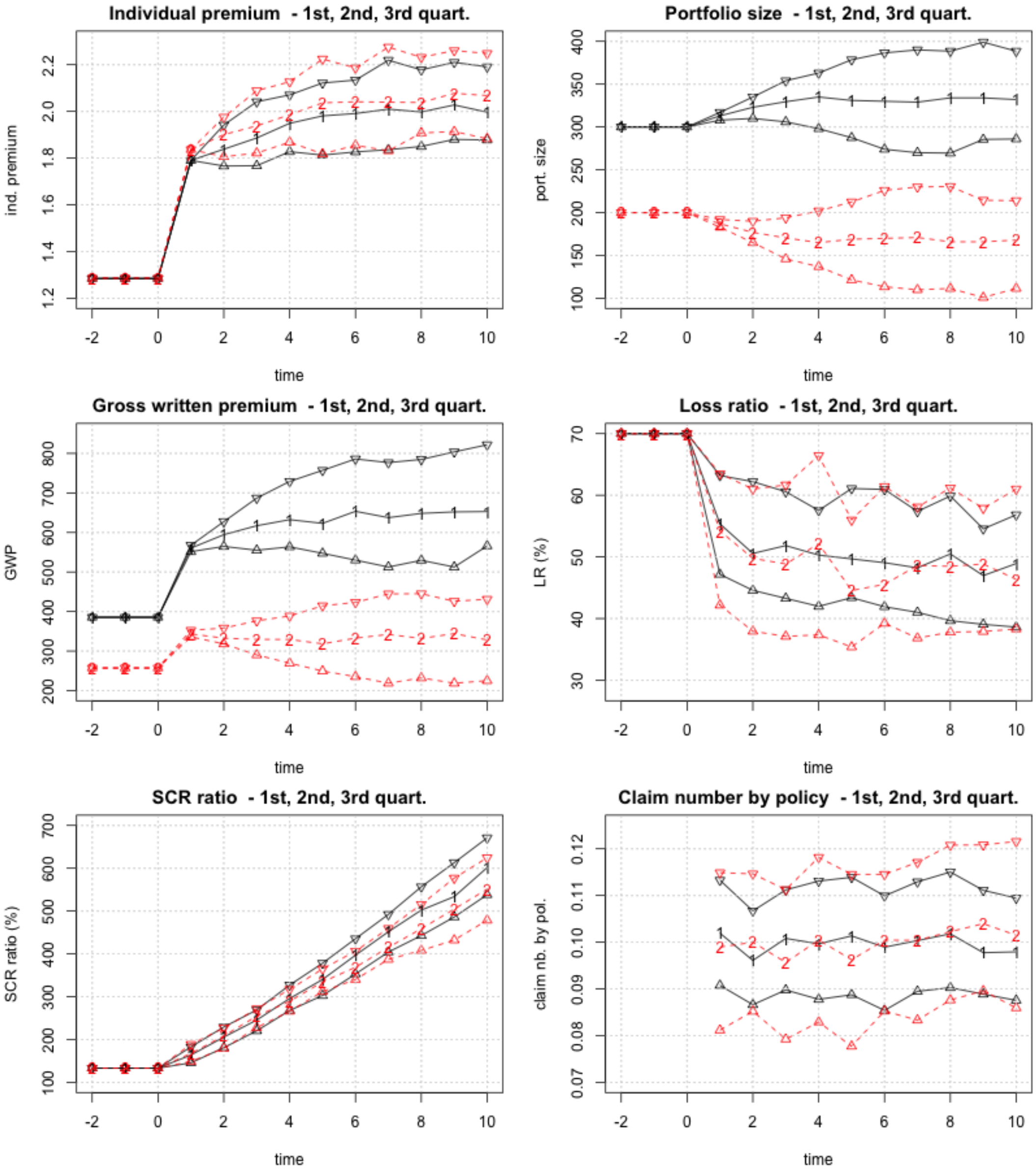


Figure 3.30 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – new price sensitivity

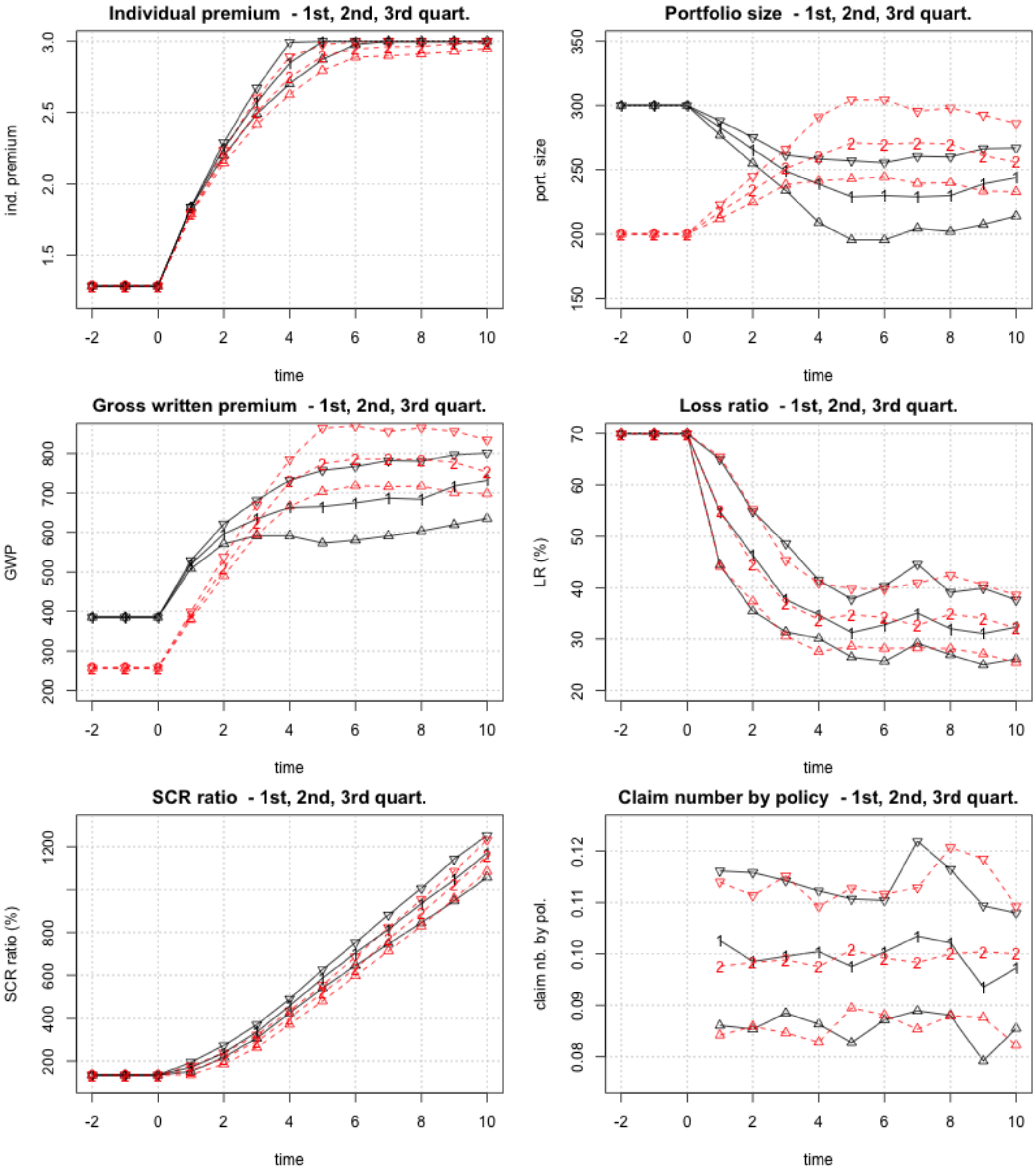


Figure 3.31 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – new credibility factor

3.8.7.2 Three-player market – sensitivity analysis

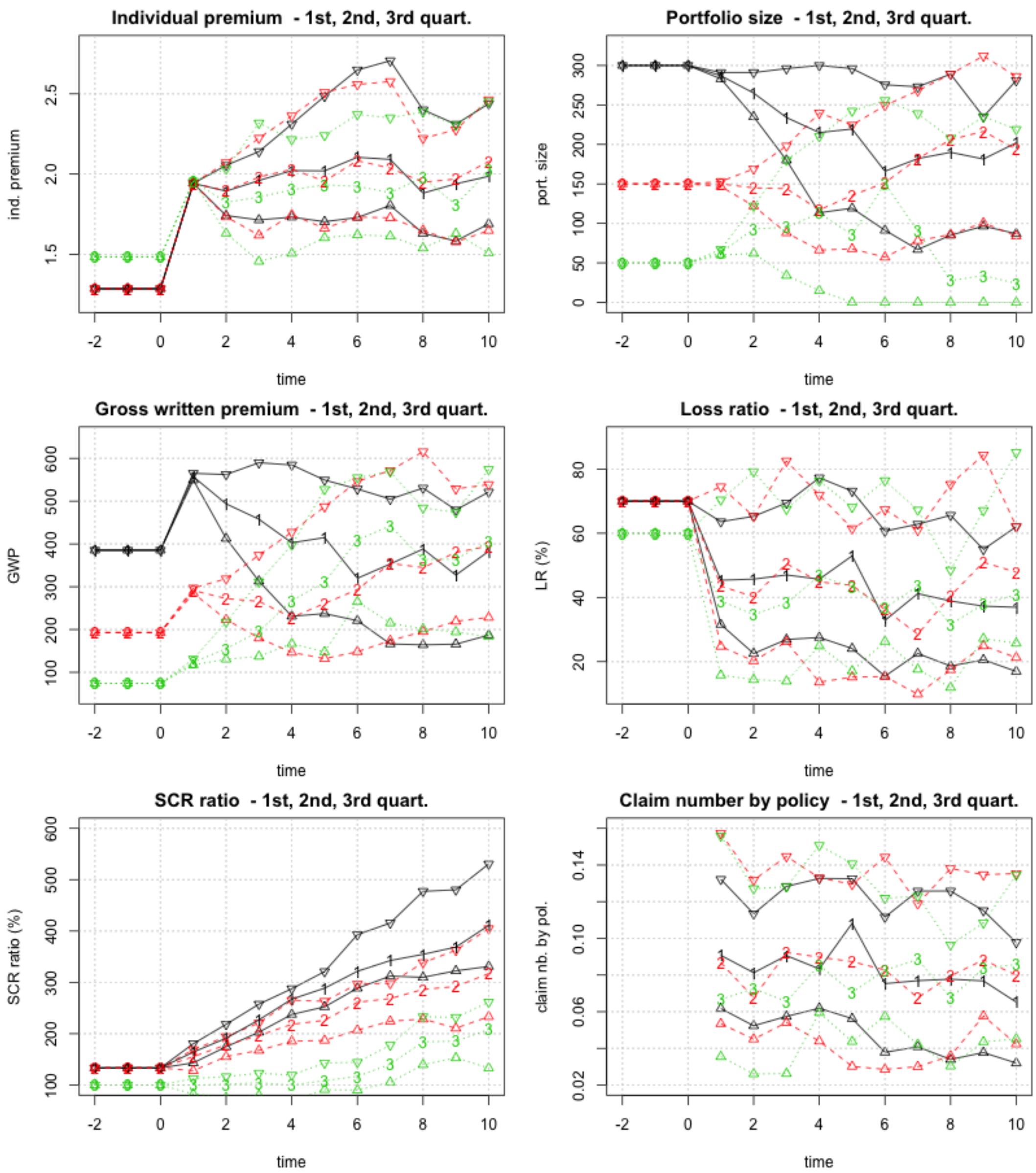


Figure 3.32 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – NBLN loss model

	Ruin probabilities			Leadership probabilities		
Period	Insurer 1	Insurer 2	Insurer 3	Insurer 1	Insurer 2	Insurer 3
1	0.01	0.00	0.10	1.00	0.00	0.00
2	0.00	0.00	0.14	0.80	0.04	0.03
3	0.00	0.00	0.19	0.45	0.08	0.26
4	0.01	0.01	0.22	0.34	0.13	0.22
5	0.01	0.01	0.24	0.31	0.17	0.17
6	0.01	0.01	0.24	0.28	0.22	0.14
7	0.01	0.01	0.25	0.30	0.23	0.10
8	0.01	0.01	0.26	0.31	0.15	0.13
9	0.01	0.01	0.24	0.27	0.15	0.13
10	0.01	0.02	0.24	0.24	0.16	0.12

Table 3.23 – Empirical probabilities of ruin and leadership over 100 runs, 3-player game with simple market proxy – NBLN loss model

	Ruin probabilities			Leadership probabilities		
Period	Insurer 1	Insurer 2	Insurer 3	Insurer 1	Insurer 2	Insurer 3
1	0.01	0.00	0.04	1.00	0.00	0.00
2	0.01	0.01	0.08	0.80	0.03	0.08
3	0.00	0.01	0.12	0.41	0.10	0.27
4	0.01	0.04	0.15	0.32	0.20	0.21
5	0.01	0.04	0.18	0.25	0.20	0.18
6	0.02	0.05	0.22	0.19	0.23	0.16
7	0.04	0.04	0.24	0.21	0.21	0.15
8	0.04	0.06	0.26	0.22	0.21	0.12
9	0.05	0.07	0.27	0.24	0.20	0.11
10	0.05	0.07	0.29	0.23	0.19	0.10

Table 3.24 – Empirical probabilities of ruin and leadership over 100 runs, 3-player game with weigthed market proxy – NBLN loss model

	Ruin probabilities			Leadership probabilities		
Period	Insurer 1	Insurer 2	Insurer 3	Insurer 1	Insurer 2	Insurer 3
1	0.00	0.00	0.00	1.00	0.00	0.00
2	0.00	0.00	0.05	0.95	0.00	0.00
3	0.00	0.00	0.06	0.87	0.01	0.00
4	0.00	0.00	0.09	0.78	0.05	0.02
5	0.00	0.00	0.11	0.71	0.08	0.06
6	0.00	0.00	0.11	0.68	0.10	0.07
7	0.00	0.00	0.11	0.64	0.14	0.05
8	0.00	0.00	0.11	0.61	0.16	0.04
9	0.00	0.00	0.10	0.60	0.13	0.08
10	0.00	0.00	0.11	0.61	0.16	0.04

Table 3.25 – Empirical probabilities of ruin and leadership over 100 runs, 3-player game with simple market proxy – MLN PR lapse

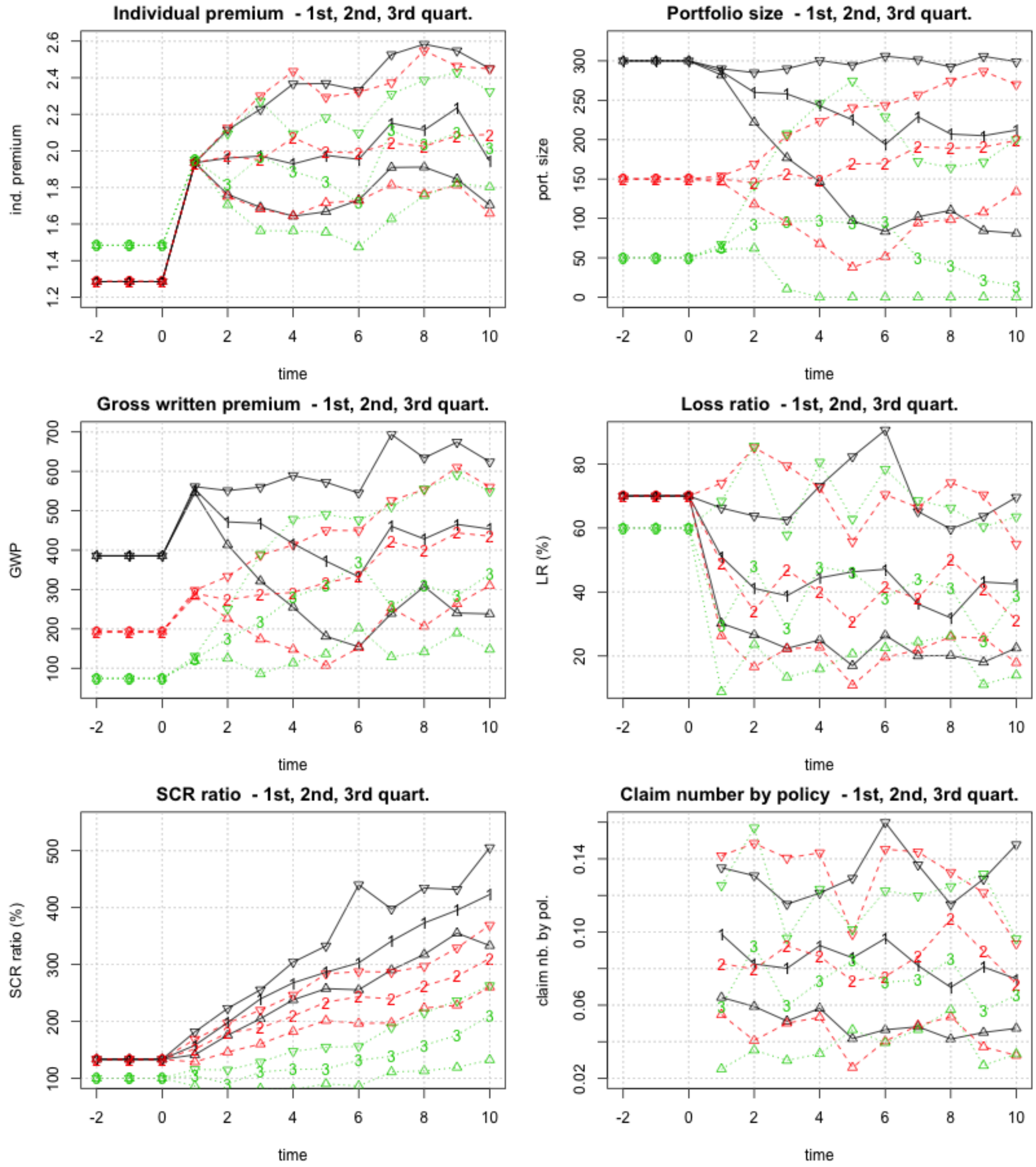


Figure 3.33 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – NBLN loss model

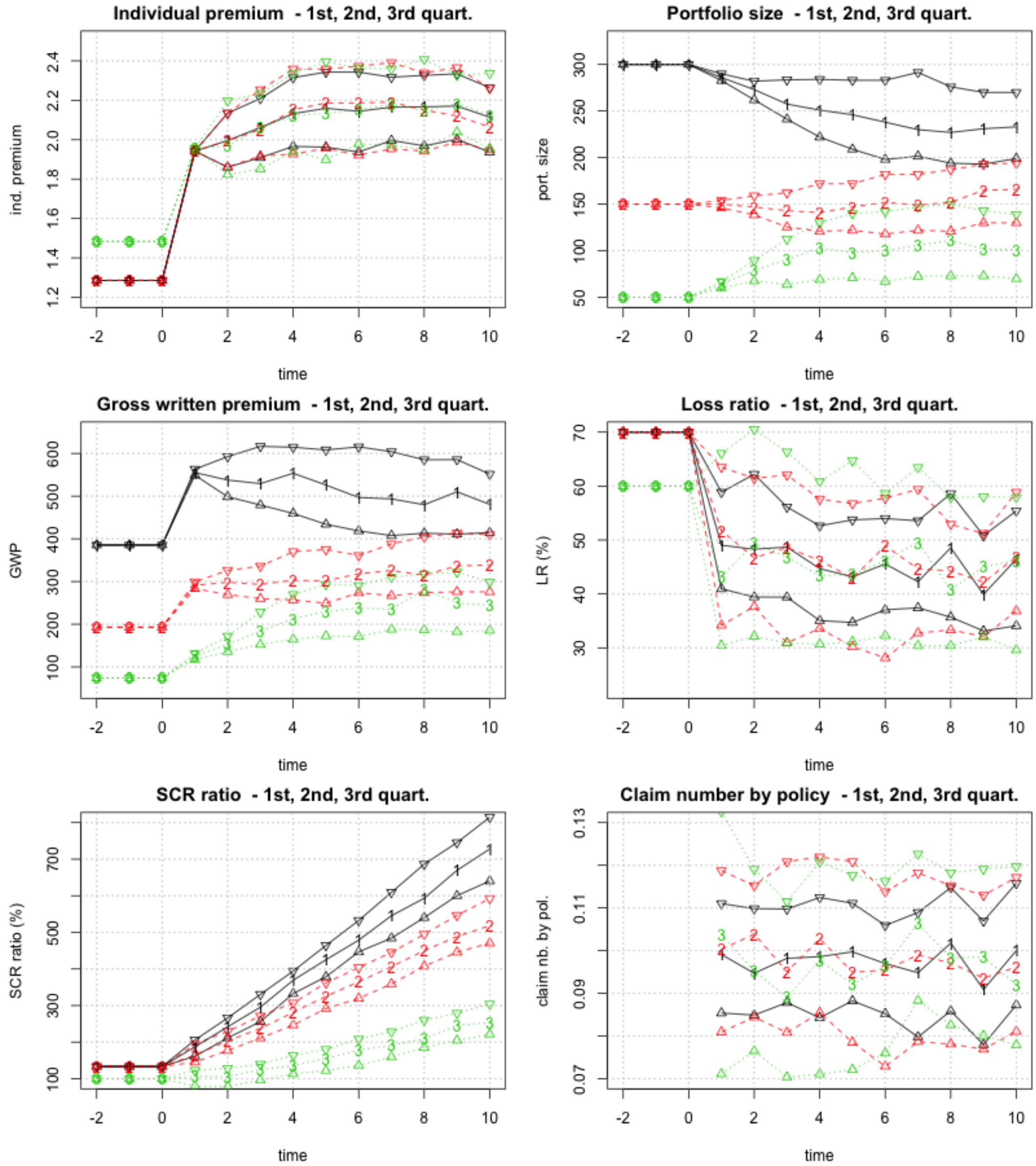


Figure 3.34 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – MLN PR lapse

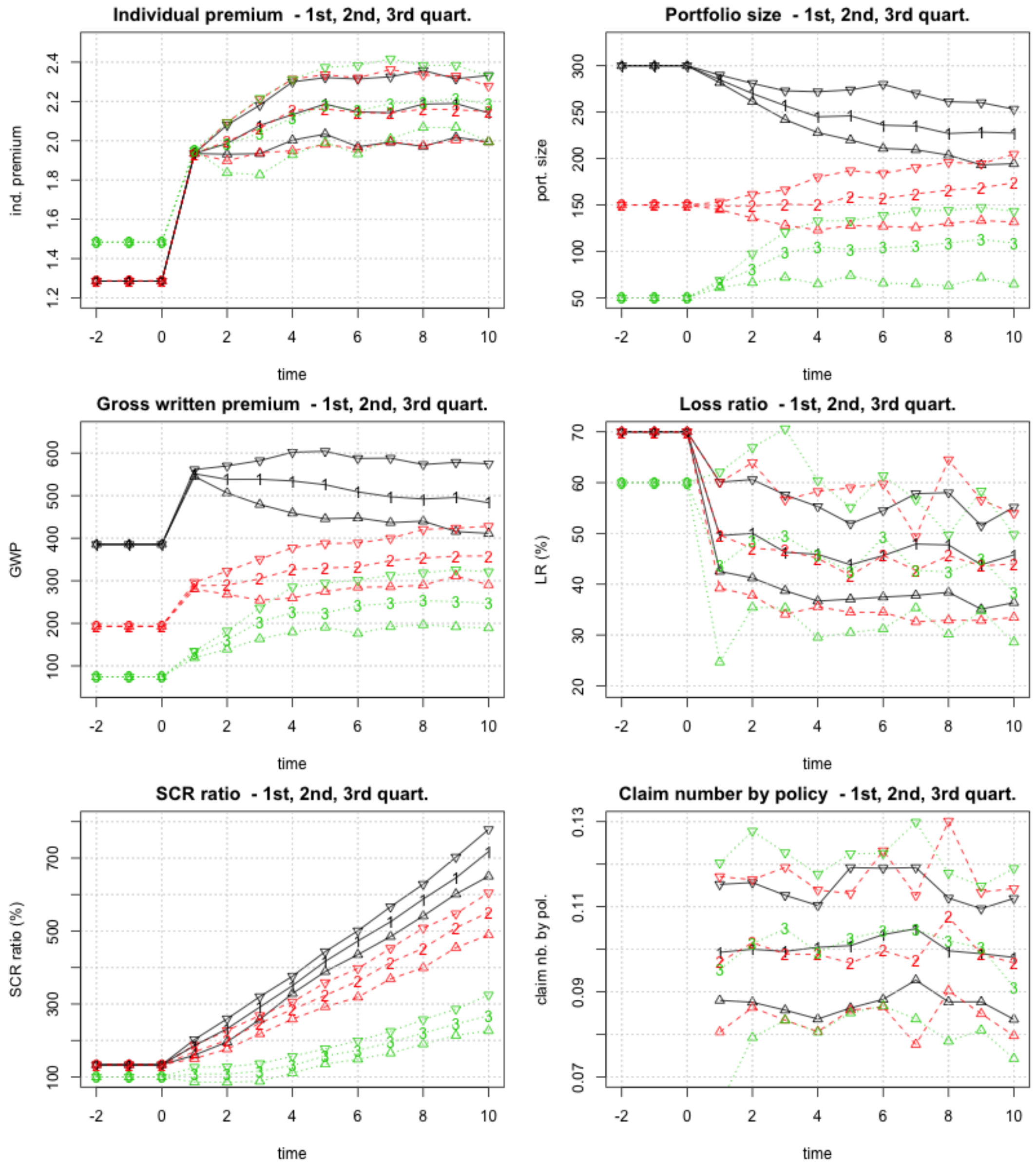


Figure 3.35 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles, 3-player game with weigthed market proxy – MLN PR lapse

	Ruin probabilities			Leadership probabilities		
Period	Insurer 1	Insurer 2	Insurer 3	Insurer 1	Insurer 2	Insurer 3
1	0.00	0.00	0.04	1.00	0.00	0.00
2	0.00	0.01	0.07	0.99	0.00	0.00
3	0.00	0.01	0.10	0.92	0.00	0.01
4	0.00	0.01	0.10	0.83	0.03	0.02
5	0.00	0.01	0.10	0.79	0.07	0.02
6	0.00	0.01	0.10	0.74	0.10	0.04
7	0.00	0.01	0.10	0.75	0.08	0.04
8	0.00	0.01	0.10	0.73	0.11	0.03
9	0.00	0.01	0.10	0.66	0.15	0.06
10	0.00	0.01	0.10	0.65	0.17	0.05

Table 3.26 – Empirical probabilities of ruin and leadership over 100 runs, 3-player game with weighted market proxy – MLN PR lapse

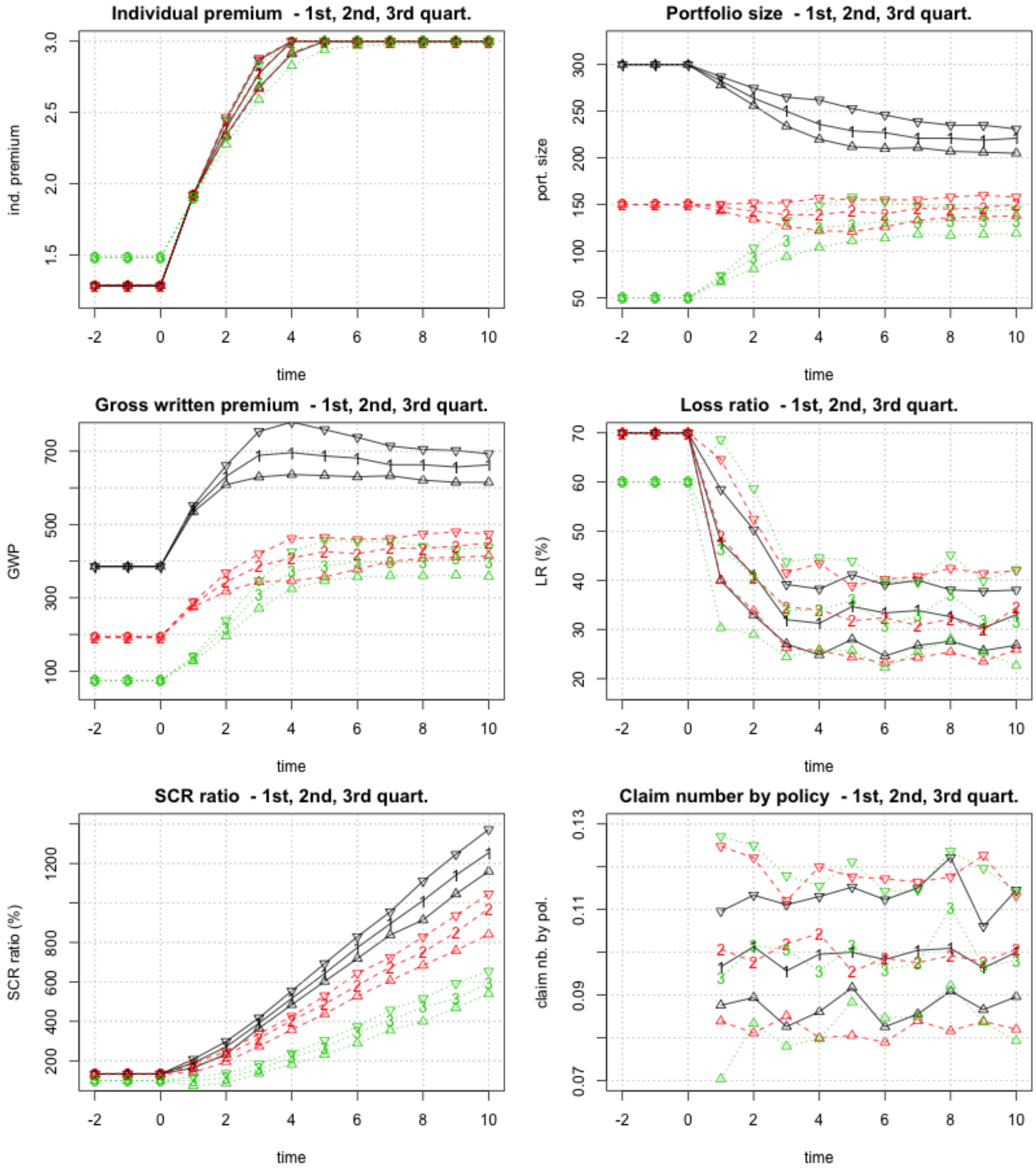


Figure 3.36 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – new credibility factor

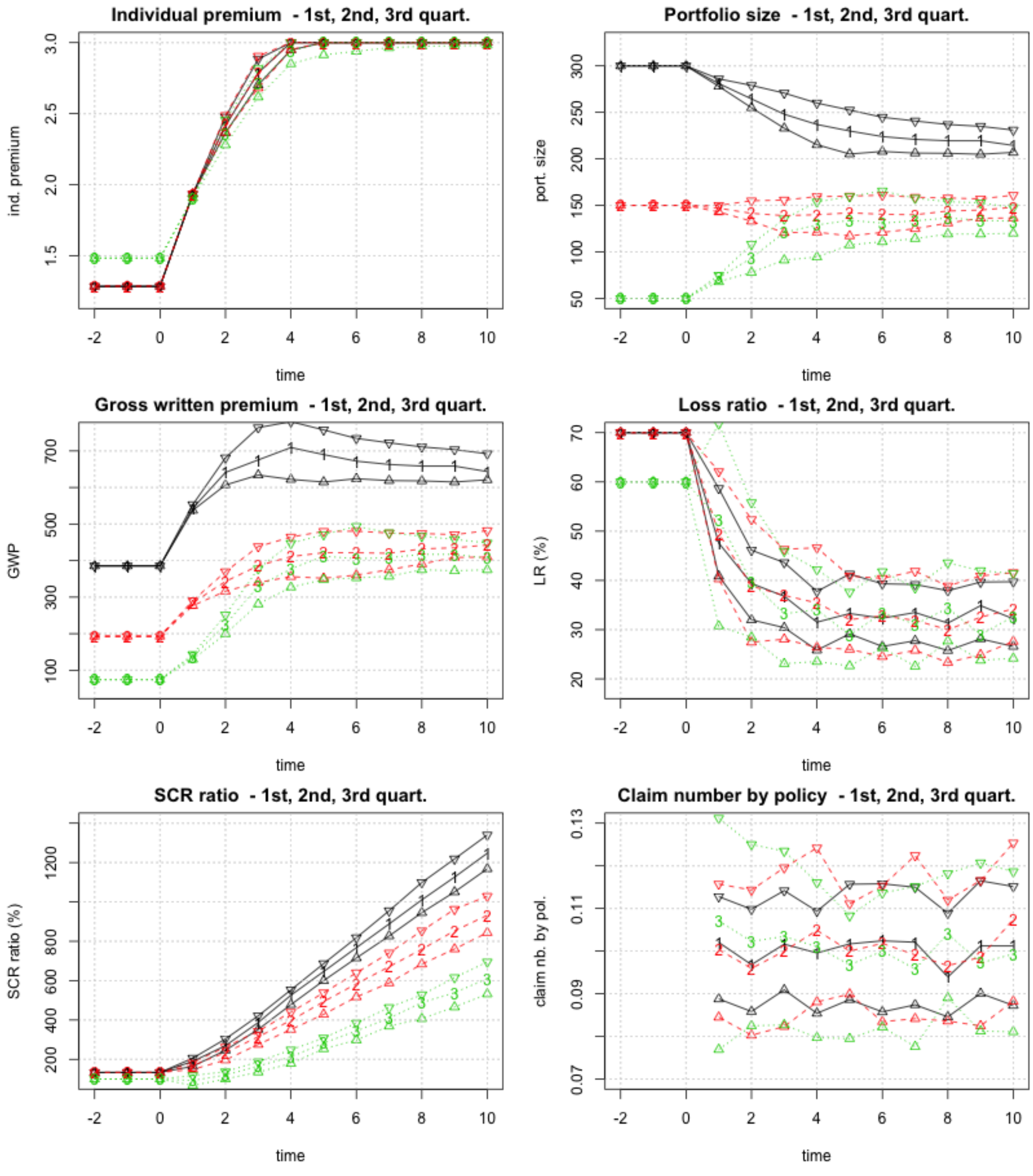


Figure 3.37 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles, 3-player game with weighted market proxy – new credibility factor

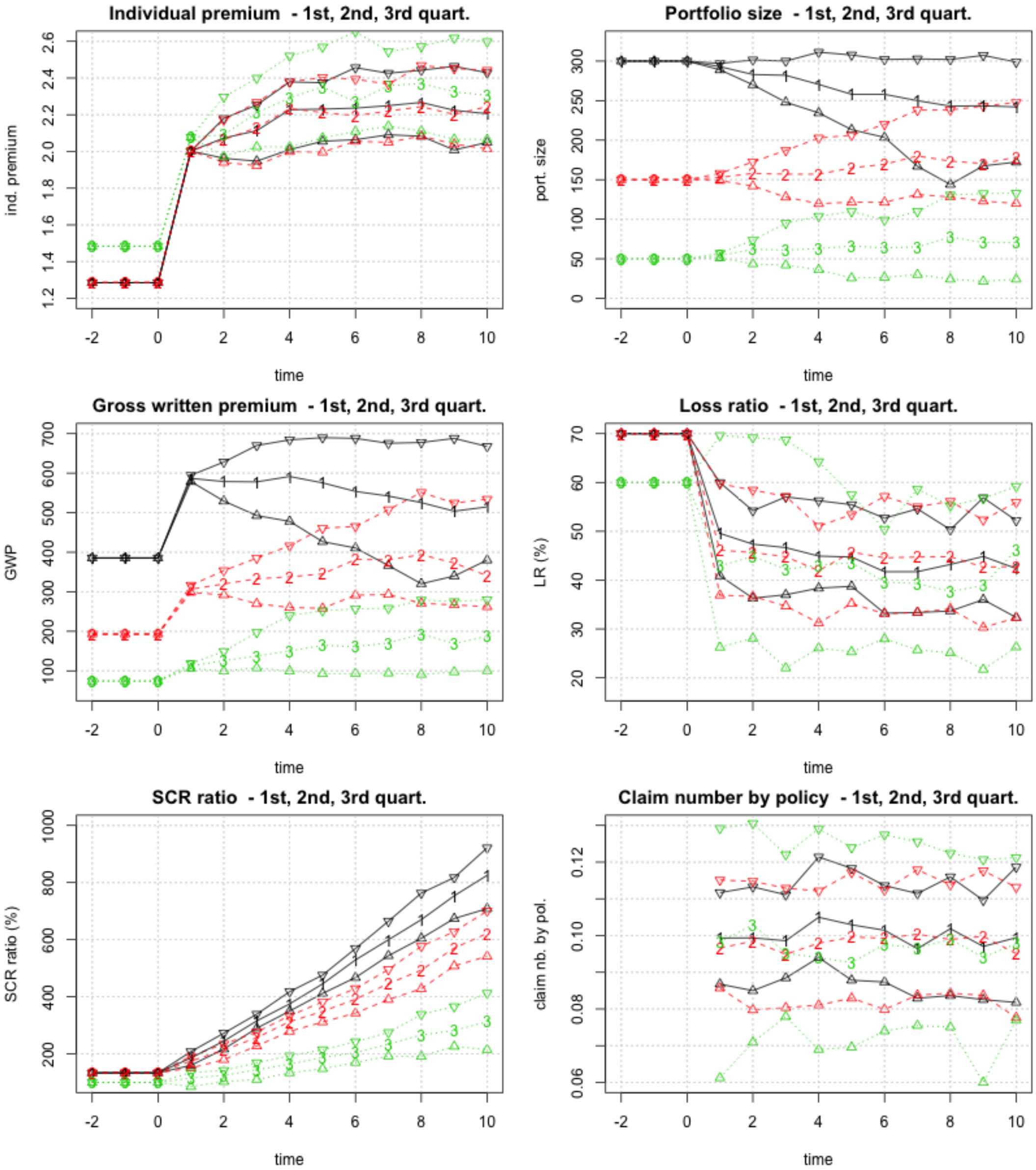


Figure 3.38 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles – new sensibility

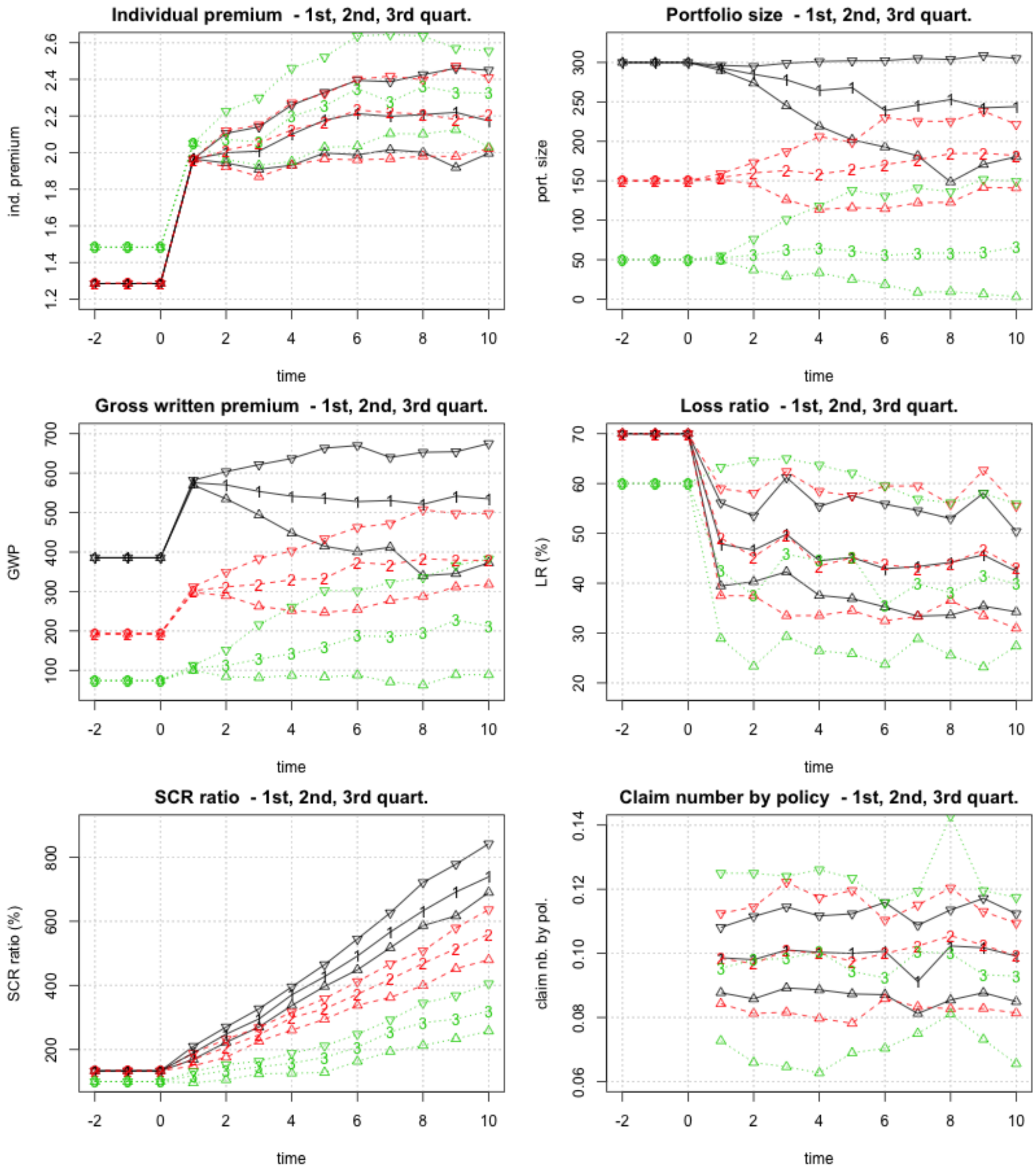


Figure 3.39 – Quartiles of some indicators of the repeated game for 100 runs, black solid line for Insurer 1 and red dotted line for Insurer 2; bottom-up triangles for 1st quartiles, numbers for medians and top-down triangles for 3rd quartiles, 3-player game with weighted market proxy – new sensibility

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Conclusion

Conclusion et perspectives

Cette thèse aborde différentes composantes du marché de l'assurance non-vie à l'aide d'approches expérimentales, théoriques et de simulations numériques. Les résultats démontrent encore une fois la complexité du marché de l'assurance. Au-delà de l'aléa inhérent aux risques, les biais cognitifs, éléments caractéristiques de la psychologie humaine, complexifient d'autant plus la rencontre de l'offre et de la demande.

D'un côté les assureurs essaient de modéliser et d'anticiper au mieux les comportements des consommateurs, de l'autre les assurés font eux-même face à leur propre complexité, les conduisant souvent à prendre des décisions non-rationnelles ou inefficientes au sens de la théorie économique. Dès lors, l'économie comportementale joue un rôle essentiel à l'explication et à la compréhension des interactions.

L'objectif de cette conclusion n'est pas d'énumérer une nouvelle fois les résultats détaillés dans cette thèse, mais de proposer et justifier l'intérêt de nouvelles perspectives de recherche qui pourraient améliorer notre compréhension du marché de l'assurance et les comportements de ses acteurs. Depuis longtemps la théorie comportementale de la décision s'est attachée à l'étude des incohérences dans les choix des agents économiques. Son objectif est de mettre en évidence les "anomalies" dans les comportements réels des agents, rarement conforme à la théorie des choix rationnels, en considérant par exemple, l'altruisme, les préférences pour le présent, l'aversion aux risques ou encore les capacités limitées de calcul et de traitements de l'information. Ce n'est pourtant que récemment que cette science, souvent expérimentale, fait l'objet d'attention particulière des entreprises, mais aussi des régulateurs, souhaitant améliorer l'anticipation des conséquences de leurs choix.

Le marché de l'assurance est un terrain idéal à l'analyse des comportements. L'aléa, souvent non exogène, donc l'asymétrie d'information en résultant, sont les raisons pour lesquelles les assureurs ont besoin d'analyser et de comprendre le comportement des assurés, tant au niveau du risque souscrit qu'au niveau de leur décision d'achat. Nous avons montrer dans cette thèse en quoi le processus de souscription est révélateur des caractéristiques des individus. Ces caractéristiques sont souvent difficiles à mesurer pour l'assureur, tel que l'aversion au risque, ou encore la confiance. Elles sont pourtant déterminantes afin d'expliquer la rencontre de l'offre et de la demande car elles permettent de comprendre les effets des biais cognitifs sur les prises de décision.

À l'aide de la mesure d'honnêteté détaillée dans le premier chapitre de cette thèse, il serait par exemple intéressant d'analyser les comportements de fraudes en assurance. En effet, il existe une réelle hétérogénéité de l'honnêteté à travers les individus. De plus, les incitations financières et le sentiment d'injustice sont déterminants dans la décision d'être plus ou moins honnête. Ces résultats font naturellement penser à l'effet de licence, où les sujets justifient un comportement négatif (la fraude) par des actes positifs effectués par le passé. En assurance automobile par exemple, nous observons que les assurés n'ayant pas déclaré de sinistres depuis longtemps ont tendance à avoir des coûts de sinistres plus élevés. L'explication pourrait porter sur l'effet de licence. Les assurés n'ayant pas eu de sinistres depuis des années, estiment que les primes payées durant toute ces années sont en fait un coût et non l'achat d'un service : la tranquillité. Ainsi, lorsque qu'il ont à déclarer un sinistre, ils pourraient être amener à sur-déclarer afin de compenser ses coûts. Sans une approche comportementale de la question, il est difficile de déterminer la vraie raison de cette augmentation des coûts de sinistres. Une nouvelle expérimentation économique, dans un environnement contrôlé, serait donc un moyen efficace pour y répondre.

Concernant le deuxième chapitre de cette thèse, je me suis intéressée à l'achat d'assurance obligatoire et l'impact de la quantité d'informations sur les prises de décision. Cependant, il serait également intéressant d'analyser les comportements face à une trop grande quantité d'information lorsque l'assurance n'est plus obligatoire. En effet, il est naturel de penser que les consommateurs, perdus en raison de l'effet d'obfuscation, soient découragés à l'achat d'un produit d'assurance optionnel. Dans ce cas, l'information rendue disponible aurait une conséquence, non seulement sur les caractéristiques du contrat choisi mais aussi, sur la décision de souscrire ou non. On peut donc imaginer de multiples implications pour les assureurs. Ces comportements pourraient expliquer d'éventuels effets de selection des risques. En effet, malgré la possibilité que l'acte d'achat soit découragé par une quantité d'information trop importante, on peut imaginer que les consommateurs souscrivant tout de même à une assurance facultative ont des caractéristiques propres à ce comportement (importante aversion aux risques, meilleure connaissances ou compréhension de l'assurance,...). Encore une fois, la théorie économique comportementale se révèle être un outil nécéssaire à l'analyse de tels biais.

Au-delà de l'utilité incontestée de la compréhension des comportements des agents pour les assureurs, il est aussi important de souligner qu'elle peut être un outil de prise de décision pour les régulateurs. Dans le dernier chapitre de cette thèse, j'ai abordé la modélisation du marché à l'aide de la théorie des jeux. Celle-ci montre une nouvelle fois l'importance des comportements, tel que l'inertie des consommateurs, sur les décisions stratégiques des assureurs. Etant donnée l'importance apportée par les régulateurs à garantir la solvabilité des assureurs, il serait également intéressant d'étendre les résultats de ce chapitre afin d'expliquer les phénomènes de ruines. En effet, bien que les chocs sur la sinistralité soient un des éléments majeurs à la survenance de ruine chez les assureurs, les parts de marché, sur le long-terme, sont aussi très dépendante de la sensibilité des consommateurs. Dans ce sens, il est donc intéréssant pour le régulateur de tenir compte de la compétition entre assureurs lors de l'étude des risques de ruine. En outre, la ruine peut survenir à cause de sinistres trop importants, elle peut également être le résultat d'un volume d'affaires trop faible pour garantir le principe de mutualisation et d'absorption des coûts fixes. Une nouvelle fois, le comportement d'achat et la sensibilité des consommateurs jouent un rôle non négligeable à la survenance de faillite sur le marché de l'assurance.

L'économie comportementale permet donc de produire des connaissances théoriques nouvelles tout en confrontant les prédictions des modèles aux observations de terrain et de laboratoire. Elle permet, grâce à son côté expérimental, de collecter des données difficilement accessibles comme par exemple l'aversion aux risques. Elle est aussi un outil pour la prise de décision publique, permettant de mesurer les impacts de politiques économiques et d'en évaluer leur efficacité. Au-delà de son intérêt pour la recherche, l'économie expérimentale est aussi un outil pédagogique pour l'enseignement de l'économie, en facilitant la compréhension des concepts et mécanismes économiques plus ou moins abstraits.

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