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Development of an exposure-based pricing approach for personal accident per risk

Masterthesis
in Actuarial Science

submitted by
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List of Abbreviations

AUB terms and conditions

CDF cumulative distribution function

CR combined ratio

DAV German Society of Actuaries

EC exposure curve

GDV German Insurance Association

ILF Increased Limit Factor

LDF loss development factor

LR loss ratio

MBBEFD Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac

PA private accident

PVP present value of the pension annuity

SI sum insured

UPR guaranteed premium refund

UW underwriting

1. Introduction

1.1. Motivation and Objective

As the primary insurer enlarged the cover of the private accident insurance policies in Germany within the last ten years according to the German Insurance Association (GDV) management reports, the pricing models for reinsurance contracts have to be adjusted[11]. In this context, the topic about developing an exposure-based pricing approach, in cooperation with SCOR Reinsurance Germany, arose. Additionally, SCORs' internal clients evaluations showed that the primary insurer try to attract more clients by offering higher disability benefits and widening their scope of cover. Furthermore, a rising trend of the claim exposure for re-/insurer is noticeable. From the modelling perspective, the actuaries face several issues with pricing private accident reinsurance contracts. So far, only an experience model is adopted that relies on the loss history of the cedents. As many primary insurer did not have to face many large losses, the current pricing does not reflect the risk appropriately. Moreover, the private accident insurance is a sum insurance with claims of the same loss amount, which makes it difficult to fit a distribution to the empirical observations. Therefore, an additional pricing approach, which also takes the exposure of the cedents into account, would lead to a more adequate assessment of the underlying risk.

First of all it is important to get an insight in the German private accident insurance. For considering the market environment within the pricing models, the increase of the claim severity and frequency has to be understood. Hence, market data is going to be analysed and reasons for the increase are worked out. Furthermore, it is going to be examined if the market trends can also be observed for the SCOR clients. Therefore, an experience model is going to be set up for the different groups of cedents. Moreover, the differences of pricing with a Long Tail versus a Short Tail model are going to be pointed out, to analyse the influence of the claim development on the amount of risk premium. Taking all the market analysis into account, a general exposure-based approach is going to be developed. Thereby, a range of exposure curve parameter and a controlling parameter should be determined, to apply the exposure pricing to all SCOR clients. Furthermore, that model is going to be adapted to the available individual clients' data, so that the risk premium for the reinsurer can be computed directly. Finally, there should be an exposure model, that in connection with the experience model, provides a more accurate pricing approach for SCOR.

1.2. Structure

In the beginning, the basics of reinsurance and pricing methods are going to be presented. It should give an overview of the possibilities for primary insurer to reinsure their business. Explicitly, the Excess of Loss reinsurance contract and its dis/-advantages are going to be explained. Furthermore, the idea of the experience pricing and exposure pricing is introduced in the second chapter, in order to understand the modelling in the fifth chapter.

The third chapter is a market study of the German private accident insurance. Besides observing the basic market measures like premium income, profitability and number of insured persons, more specific pricing key figures are analysed. In particular, reasons for the increase of the claim severity and frequency are identified. In the last part, in- and exclusions of explicit accident scenarios within the private accident insurance contracts are discussed.

The comparison of the large loss development of the SCOR clients is the topic of the fourth chapter. It is going to be an analysis of frequency and severity trends for different groups of clients. Furthermore, there is going to be a comparison of the experience pricing results with and without considering possible loss developments.

The last chapter offers a comprehensive explanation of the developed exposure-based pricing approach for private accident insurance contracts. It starts with mentioning the data basis and the general requirements the model is based on. In the next section, the idea of the approach and the implementation in R is going to be explained. Finally, the developed approach is applied on the available data and the results are going to be compared to the experience pricing.

2. Concept of Reinsurance

Reinsurance companies play a very important role for the insurance market. Their services represent a major tool for the risk management of primary insurance companies. They can provide extra capacity so that insurance companies can still fulfil the liquidity regulations under Solvency II. Also, they help to protect against extreme risks like natural disasters or terror attacks. Furthermore, primary insurance companies also profit from the expertise of the reinsurance. For this business relationship there has to be a reinsurance contract which records the risks that get transferred and the underlying conditions. As the cedents need an individual solution, there are a lot of different possibilities to set up a reinsurance agreement. The figure 2.1 gives an overview of all traditional reinsurance contracts.

For the introduction to reinsurance contracts the book of Andreas Schwepcke was used. Therefore, no individual reference per subsection will be stated.[15]

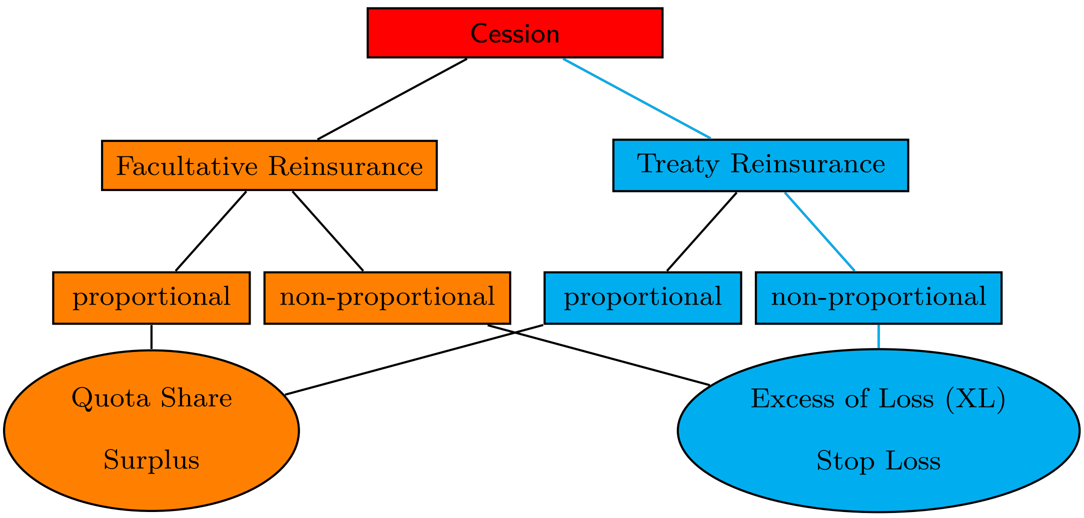

Figure 2.1.: Overview of Reinsurance Contracts

2.1. Types of Reinsurance Contracts

The primary insurer deploy **facultative** contracts to reinsure specific individual risks. In contrast, a **treaty** agreement is a mutual obligation between a primary insurer and a reinsurance company to cover a whole portfolio according to the contract terms. Usually a long term relationship between both parties is build up, but the conditions are adapted if the coverage changes. For primary insurer, it is a secure arrangement to be insured against large losses and natural catastrophes. Furthermore, treaty reinsurance allows the insurer to have more extra capacity to underwrite other risks. For the reinsurer, it has the advantage to have a continuous premium income, with an expected profit in the long term.

A **proportional** contract divides the premiums and the losses by a certain share. Hence, the primary insurer can transfer more risk to the reinsurer. There are two types of proportional reinsurance. The **Quota Share** splits the premiums and loss payments with a beforehand fixed ratio between cedent and reinsurer. For the **Surplus** contract, the share is calculated by the sum insured divided by the priority level. This ratio is applied to total premiums and determines the allocation between the primary insurer and the reinsurer.

For **non-proportional** contracts, a priority level is set beforehand and the reinsurer covers the amount above until a certain limit. The sum of the priority and the limit is called plafond. It is used to reinsure against large losses and high risks. First, the **Stop Loss** contract aggregates all claims during one period and the loss amount exceeding the priority is compensated by the reinsurer. The **Excess of Loss** contract is a comprehensive feature for the primary insurer to cover against large losses and therefore is used very commonly in private accident (PA) reinsurance. Every individual loss has to exceed the priority level before the reinsurer compensate the primary insurer. For this type of contract, there is also the possibility to agree on a layered structure. In the Figure 2.2 on the next page, an example of a layered Excess of Loss contract can be observed. For instance, the first layer would have a priority of five million and a limit of another five million. Building up on that, the second layer covers the next 10 million and the third layer would cover losses up to 40 million.

To buy different layers instead of one big layer has huge advantages for the primary insurer. It gives an efficient tool to cover specifically against big losses knowing the underlying portfolio. The limit of the lower layers depend on the expected large losses and are bought accordingly. The high layers are bought to protect against very exceptional loss events for a low risk premium. Thereby, the technical provisions can be reduced and the capacity, to take over other risks, increases.

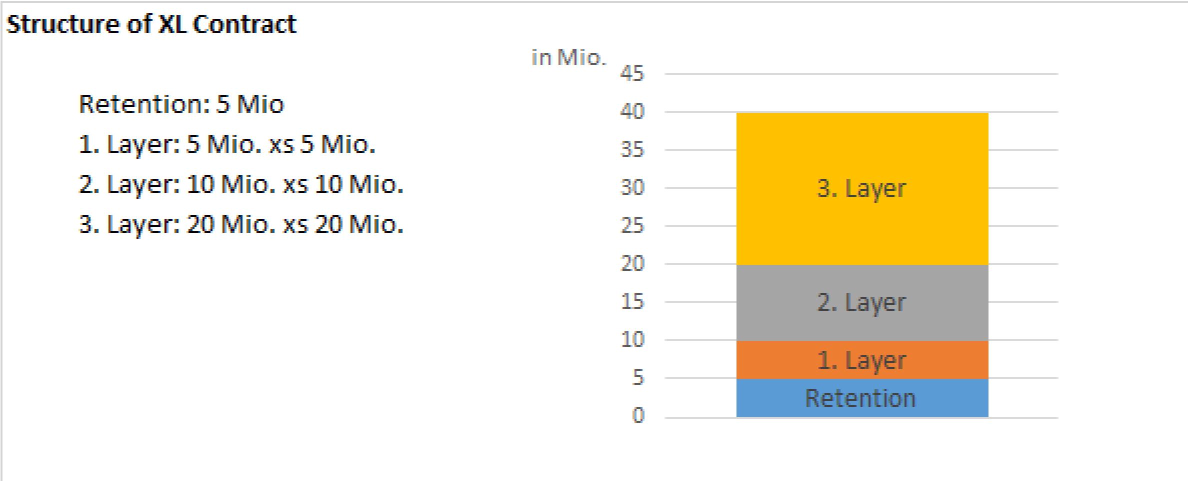

Figure 2.2.: Layering of an Excess of Loss contract

There are also two different forms of Excess of Loss contracts. First of all there is the **Per Risk** contract, which covers against single losses. This means that every loss has to exceed the priority before the reinsurer compensates the loss. A special type of a Per Risk contract is a **Catastrophe Excess of Loss** contract, which cover especially against very large losses and natural catastrophes. For example, the third layer in the Figure 2.2 would be a catastrophe cover. On the other hand there are **Event Excess of Loss** contracts. For example in property insurance, it is agreed on a 72 hours time frame where all losses, which can be led back to one event, are aggregated and compensated if they exceed the priority. Therefore, a higher priority is set for event covers.

The primary insurer tries to optimise their reinsurance structure by weighing up risk cover against premium earnings. Especially for PA insurance business, most of the primary insurer agree on Excess of Loss contracts to reinsure against large losses but not to give away much of the premium. For primary insurer with little PA business, usually a combination of proportional and non-proportional contracts is agreed to get a sufficient reinsurance cover.

Often there are also **additional features** fixed in the contracts. The annual aggregate limit(AAL) is the maximum amount of total losses the reinsurer covers per year. Alternatively it can be agreed that if once the limit is exceeded, the capacity can be reinstated for a certain amount of premium. In addition, it is also important to know that one reinsurance cover is always split among different participants to reduce the risk for one reinsurer.

One of the participants is the leader who negotiates the general conditions and the others have to agree on those conditions.

2.2. Reinsurance Pricing Approaches

As the available data for the reinsurance of the insurers' portfolio is limited, it is more difficult to calculate the adequate risk premium for the covered business. Therefore, two pricing approaches have been established over the years. The Burning Cost method and the Frequency-Severity method both utilise the loss experience to compute the future expected loss. Whereas the latter method includes a Short Tail or Long Tail model. It depends on the line of business and the duration of the development of the losses until their final amount if one or the other is applied. In contrast to observing solely the loss experience, the exposure approach focuses on the underlying portfolio of the cedent.

2.2.1. Experience Pricing

The Experience Pricing is a very widely used approach for reinsurers to calculate the adequate risk premium with limited data. But still some basic information has to be provided by the cedents. This subsection is based on a SCOR internal source.[3]

- **Contract details:** The overall cover in terms of priority, limit and layers have to be known. Moreover, often clauses about indexation[1], exclusions or interest rates are agreed.
- **Cedents underwriting policy:** The Experience Pricing can only be used for portfolios which are consistent over the applied period. Therefore, any changes in the underwriting limits or any tariff changes have to be reported to the reinsurer.
- **Exposure Measure:** To observe the exposure development of the portfolio, usually the loss history or the number of risks is analysed. As, for example, with an increase of the number of risks, more losses can be expected.
- **Loss history:** The loss history is required to build a model to simulate the future losses. For the Short Tail modelling, the ultimate individual losses with at least five years of history has to be known. The Long Tail modelling requires triangulated losses and the claim history has to be more than ten years. Usually the cedent only reports losses above a certain threshold.

[1] will be explained in the following page

The **re-evaluation** of the past premiums and claims is one of the most difficult steps within this pricing approach. The aim is to project the premiums and claims to the pricing year level. The re-evaluation of the premiums should reflect the evolution of the tariffs and developments in the sums insured. For the claims, an increase in the costs because of inflation effects should be taken into account. Often the used indices are derived by the Consumer Price index or Wages & Salaries index. In Germany the GDV provides detailed cost developments for every line of business, which is available to all members.

Further, the distinction between the working and non-working area has to be explained to understand the need of the different pricing approaches. The **working area** indicates the area up to the highest indexed loss that has been occurred. Whereas everything between the maximum loss and the plafond of the contract is called the **non-working area**.

The last requirement for the Experience Pricing approach, is to understand the difference between the **reporting threshold**, the **model threshold** and the **priority level** of the Excess of Loss contract. The losses that the cedents provide have to be always below the priority, because otherwise the pricing does not include all past losses that would have affected the reinsurer. As the reported threshold also has to be re-evaluated, the model threshold has to be above the indexed reported threshold. Otherwise, losses below the reporting threshold could be above the model threshold after re-evaluation.

Burning Cost Method

For reinsurance pricing, the most convenient approach is the Burning Cost method. It is a pure empirical method where it is assumed that the expected future losses equals the past claims. Also, any changes in the portfolio structure are only considered indirectly through the selection of the re-evaluation index. The data requirements are the following. The variable T gives the length of the observed period and N_t represent the number of losses during the year t.

- Premium history P_t with $t \in \{0, ..., T\}$
- Claim history X_{t,i_t} with $t \in \{0, ..., T\}$ and $i_t \in \{0, ..., N_t\}$
- Default index I_t with $t \in \{0, ..., T\}$
- Priority of the contract D
- Limit of the contract L

To re-evaluate the premiums and the claims, the starting amount is multiplied by the rate change of the index' pricing year in relation to the starting year.

Indexed Premium: $P_t^I = P_t \frac{I_T}{I_t}$

Indexed Claim: $X_{t,k_t}^I = X_t \frac{I_T}{I_t}$

The idea of this method is to calculate the occurred historical loss for the reinsurance cover. Therefore, only the loss amounts above the priority up to the contract limit are considered. The sum of the reinsurance losses in relation to the accumulated premiums represents the Burning Costs (BC).

$$BC = \frac{\sum_{t=1}^{T} \sum_{k_t=1}^{N_t} max\{0; X_{t,k_t}^I - D - max\{0; (X_{t,k_t}^I - L)\}\}}{\sum_{t=1}^{T} P_t^I}$$

The application of this approach is very limited as it assumes that future claims will be identical to the claim history. Furthermore, there is no risk premium for the non-working area computed. Therefore, the more advanced Frequency-Severity approach was developed.[6, p. 236ff]

Frequency-Severity Model

The basic idea of this model is to assume that the claims are distributed with a compound distribution with independent and identical random variables. Therefore, the frequency and the severity have to be modelled separately and from the conclusion of the law of total expectation the expected loss of the contract can be computed. First, the following assumptions are made.

The compound distribution S with the random variable N, which represents the number of losses and the random variable X_i, which displays the loss amount.

$$S = \sum_{i=1}^{N} X_i \text{ with } N \text{ and } X_i \ \forall \text{ i are i.i.d random variables}$$

These two frequency distributions are widely used in reinsurance

$$N \sim Poi(\lambda), \ \ N \sim Beta(\alpha, \beta) \text{ where } E[N] = \sum_{k=1}^{\infty} k \cdot P(N = k)$$

For the severity distribution the following three are very common[17]. The variable x_0 represents the model threshold.

$$X \sim Par(\alpha, x_0), \quad X \sim LogGamma(\alpha, \gamma, x_0) \text{ or } X \sim GenPar(\alpha, \beta, x_0)$$
$$\text{where } E[X] = \int_0^\infty (1 - F(X))dx$$

From the law of total expectation, it follows the equation below.

$$E[S] = E[N] \cdot E[X]$$

First the **Short Tail model** is observed. It is used for lines of business where the ultimate loss is known within five years. For example, loss amounts for fire, burglary or machinery breakdown insurance are determined rapidly. The data requirements are identical to the Burning Cost method. Furthermore, the modelling threshold has to be selected. It depicts the level where the frequency and severity distribution starts. As mentioned above, it is necessary that the modelling threshold has to be above the indexed reported threshold and below the priority so as not to miss any loss information.

To determine the frequency distribution, the number of indexed losses which are above the modelling threshold are counted. In the following diagram several fictive indexed losses of a cedent from 2014 to 2019 are shown. The modelling threshold is set at 600.000.

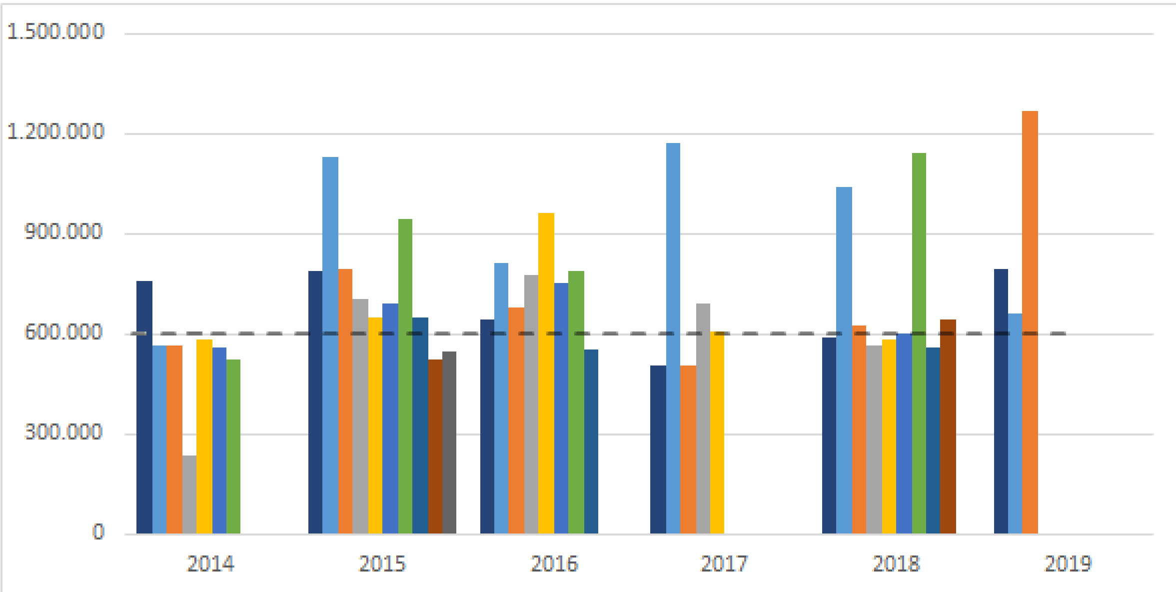

Figure 2.3.: Loss History from 2014 to 2019

Additionally, the number of losses is re-evaluated with the premium development. With an increasing premium it can be assumed that the exposure to the reinsurer also rises. The premium coefficient is calculated by taking the premium of 2020 in relation to the previous years. The As-If number of losses are received by multiplying the number of losses per year with the premium coefficient.

Year	2014	2015	2016	2017	2018	2019	2020
Indexed Premium	90.005.296	92.311.229	94.679.376	97.953.648	98.933.786	98.459.867	99.000.000
Premium Coeff.	1,10	1,07	1,05	1,01	1,00	1,01	
# of Losses	1	8	7	3	5	3	
As-If # of Losses	1,10	8,58	7,32	3,03	5,00	3,02	

Figure 2.4.: Calculation of Frequency

If the Poisson distribution is chosen then the weighted average over the past six years is the parameter λ. Alternatively the parameter can be computed by the linear regression.

For the severity, first the empirical cumulative distribution function (CDF) has to be determined. Therefore, the same occurrence probability for every loss is assumed. Then, the Maximum-Likelihood method is used to fit possible distributions to the sample. The following graph shows the result.

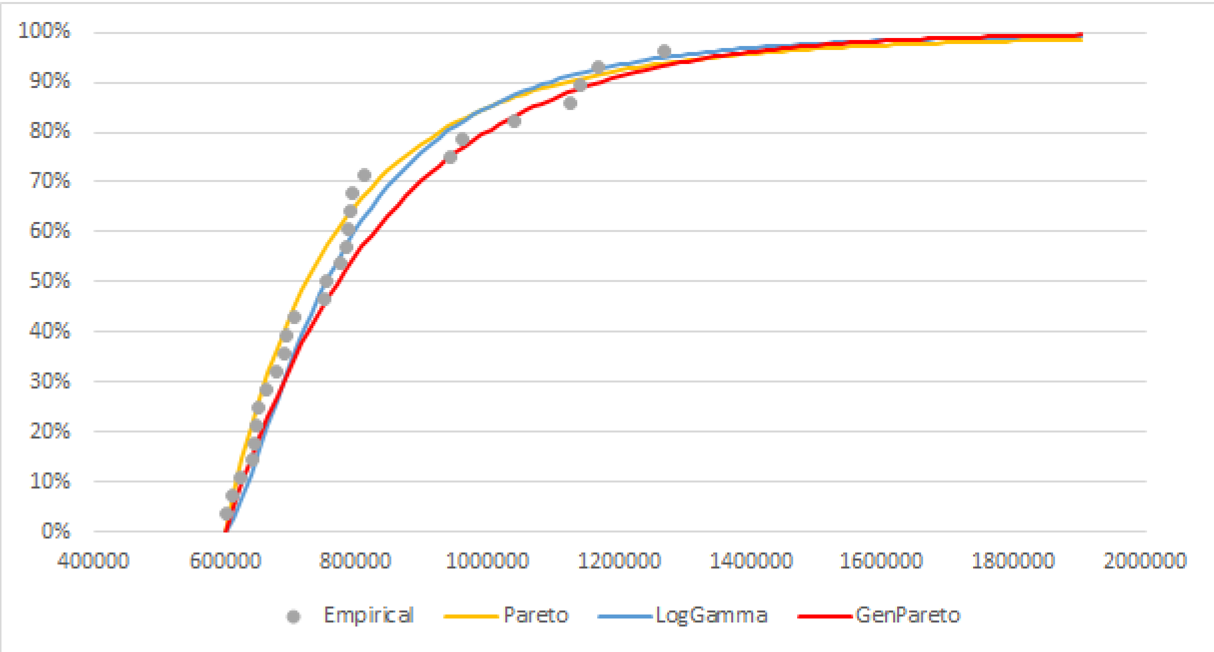

Figure 2.5.: Fitting Severity Distribution

With the help of statistical tests like the Chi-Square test or the Anderson-Darling test, the distribution with the best fitting badness can be chosen. In this example the LogGamma distribution would be used. As result the expected loss can be computed by

$$E[S] = E[N] \cdot E[X] = \lambda \cdot \left(1 - \frac{1}{\gamma}\right)^{-\alpha}$$

The Figure 2.5 also shows the advantage of the Frequency-Severity model over the Burning Cost method. As the highest indexed loss is at around 1,2 million and the limit of the contract is at 2 million, still losses in the non-working area can be simulated by the severity distribution. Thus, the reinsurer can calculate a risk premium for the non-working area.

The **Long Tail model** is used for business lines where the ultimate loss is settled after several years. The motor or general liability insurance are a perfect example for long tail business. As the aim is to understand the development of the loss amounts, the cedent has to provide more detailed loss information. Particularly, for every individual loss, the development of the paid amount and the outstanding amount is required. Equivalent to the two previous models, the premium and losses have to be re-evaluated with an appropriate index. In contrast to the short tail business lines, often a **stabilization clause** is agreed on. Due to the re-evaluation of past losses, the historical individual claim amounts increase substantially. Therefore, the insurer and reinsurer agree on a stabilization clause which limits the loss increase and splits the inflationary effect between the two parties.

For the analysis of the loss development only the claims which affect the reinsurer are considered. Therefore, the priority is subtracted and the maximum loss amount is the reinsurance limit of the contract. As no single loss development model for reinsurer is available so far, the amounts are aggregated per year and the Chain-Ladder Method is applied to the claim triangle to calculate the **loss development factor (LDF)**. The ultimate claim amount per year can be computed by using the Chain-Ladder or Cape Code method.

The following table shows the development of the aggregated losses for the reinsurer since 2014. The green numbers result from forecasting the loss amount including the LDF's. In the last column, the ultimate claim amount for each year is displayed.

Severity	Dev 12m	Dev 24m	Dev 36m	Dev 48m	Dev 60m	Dev 72m	CL Ultimate
2014	1.052.983	745.352	810.431	345.537	365.093	384.568	384.568
2015	156.533	1.512.258	2.945.464	3.158.347	2.411.562	2.540.195	2.540.195
2016	522.566	2.477.060	2.831.966	1.956.876	1.550.728	1.633.444	1.633.444
2017	628.326	2.072.499	1.301.263	1.078.633	854.763	900.357	900.357
2018	797.606	2.925.925	3.390.981	2.810.827	2.227.441	2.346.254	2.346.254
2019	932.035	2.872.560	3.329.135	2.759.562	2.186.816	2.303.462	2.303.462
LDF	3,08	1,16	0,83	0,79	1,05	1,00	(Tail Factor)

Table 2.1.: Loss Amount Development

For the frequency, the procedure is similar. The number of losses that exceed the priority are aggregated and by using the LDF's the ultimate number of losses are calculated for each year.

Frequency	Dev 12m	Dev 24m	Dev 36m	Dev 48m	Dev 60m	Dev 72m	CL Ultimate
2014	3	4	5	3	3	3	3,00
2015	1	7	9	10	10	10,00	10,00
2016	3	8	8	8	8,00	8,00	8,00
2017	1	9	6	5,73	5,73	5,73	5,73
2018	4	9	9,00	8,59	8,59	8,59	8,59
2019	2	6,17	6,17	5,89	5,89	5,89	5,89
LDF	3,08	1,00	0,96	1,00	1,00	1,00	(Tail Factor)

Table 2.2.: Development Number of Losses

The result table 2.3 shows all necessary data to calculate the frequency and severity distribution. The negative reserve indicate a lower ultimate loss in comparison to the latest incurred amount. The As-If Frequency is the number of losses re-evaluated by using the increasing premium income. It is assumed that the frequency of losses is distributed with $Poi(\lambda)$. Therefore, similar to the Short Tail model, λ is the weighted average of the As-If Frequency.

Year	Premium	Incurred	Reserve	Ult. Loss	Loss per Prem.	# of Losses	As-If Freq.	Claim Cost
2014	90.005.296	384.568	0	384.568	0,43%	3	3,3	128.189
2015	92.311.229	2.411.562	128.633	2.540.195	2,75%	10	10,72	248.932
2016	94.679.376	1.956.876	-323.432	1.633.444	1,73%	8	8,37	205.626
2017	97.953.648	1.301.263	-400.906	900.357	0,92%	5,73	5,72	101.767
2018	98.933.786	2.925.925	-579.671	2.346.254	2,37%	8,59	8,66	291.131
2019	98.459.866	932.035	1.371.426	2.303.462	2,34%	5,89	6,82	281.271

Table 2.3.: Result Table of the Long Tail Model

It is more difficult to calculate the severity distribution. The first step is to predetermine the type of distribution, it is suggested to choose one with few shape parameters e.g. the Pareto distribution. To determine the parameter α of the Pareto distribution, the Loss per Premium amount for the working area is set equal to the first moment of the piecewise Pareto distribution. [8, p. 5]

$$E[X] = \left(\frac{D}{\alpha - 1}\right)\left(1 - \left(1 + \frac{L}{D}\right)\right)^{1-\alpha}$$

Solving the above equation for α gives the parameter of the severity distribution. As the frequency and the severity distribution are determined, by using the law of total expectation, the risk premium for this contract can be computed.

2.2.2. Exposure Pricing

An alternative rating is the Exposure Pricing. The idea is to calculate the risk premium by observing the portfolio of the cedent and evaluating the underlying risk. This approach is more adequate than the Experience Pricing if only a few large losses occurred or the underwriting policy of the primary insurer has changed significantly in the past. More often a combination of both approaches is applied. Usually an experience model is used for the working area and merged with an exposure model for the non-working area. For this subsection an internal source of SCOR is used.[14]

For the exposure modelling, the data requirements are higher than for the Experience Pricing. Usually the primary insurer does not deliver individual data for every insured person but rather aggregated **risk profiles**. The number of risks, the total sum insured and the premium are given per band. In the following table a risk profile is displayed. This is the minimum data needed for applying the exposure rating. The average sum insured is just the total sum insured divided by the number of risks.

Min	Max	# of Risks	Total SI	Ø SI	Premium
0	500.000	10.000	2.550.000.000	255.000	3.000.000
500.001	1.000.000	5.000	3.930.000.000	786.000	4.000.000
1.000.001	1.500.000	3.000	3.300.000.000	1.100.000	1.200.000
1.500.001	2.000.000	20	36.000.000	1.800.000	17.000
2.000.001	2.500.000	5	10.500.000	2.100.000	3.000

Table 2.4.: Example of a Risk Profile

Another important information is the ground up **loss ratio** for every individual cedent. The loss ratio is the average amount of the premium that is expected to be used to cover the claims. Even though there is a significant discrepancy of the loss ratios among the different clients, only a few primary insurer provide this information. Hence, a market loss ratio is used.

For the evaluation of the risk profile an **exposure curve (EC)** is used. There are two classes of exposure curves. The class of the Increased Limit Factor (ILF) are commonly used for the liability business. Especially, the Riebesell Factors are a very widely used exposure curve. It gives the percentage of risk premium increase if the sum insured doubles. For insurance business with a fixed sum insured, e.g. personal accident or property insurance the **SwissRe curves** and the **MBBEFD distribution** are applied. They just display the share of risk premium the primary insurer retains for a certain deductible. In the following both exposure curves are presented.

SwissRe Curves

The SwissRe curves are also known as Gasser Curves as they were developed by Peter Gasser within a fire insurance study for Switzerland. There are six different shapes which are widely used by European Reinsurer. They are dependent on the parameter $Y_i \quad i \in \{0, 1, 2, 3, 4, 5\}$. The interpretation is straight forward, for $Y = 5$ the primary insurer would keep around 80% of the premium for a deductible of 25% of the maximum sum insured. The other 20% of the premium would go to the reinsurer for covering losses above the deductible. In the following figure 2.6, the SwissRe curves are displayed.[13, p. 130]

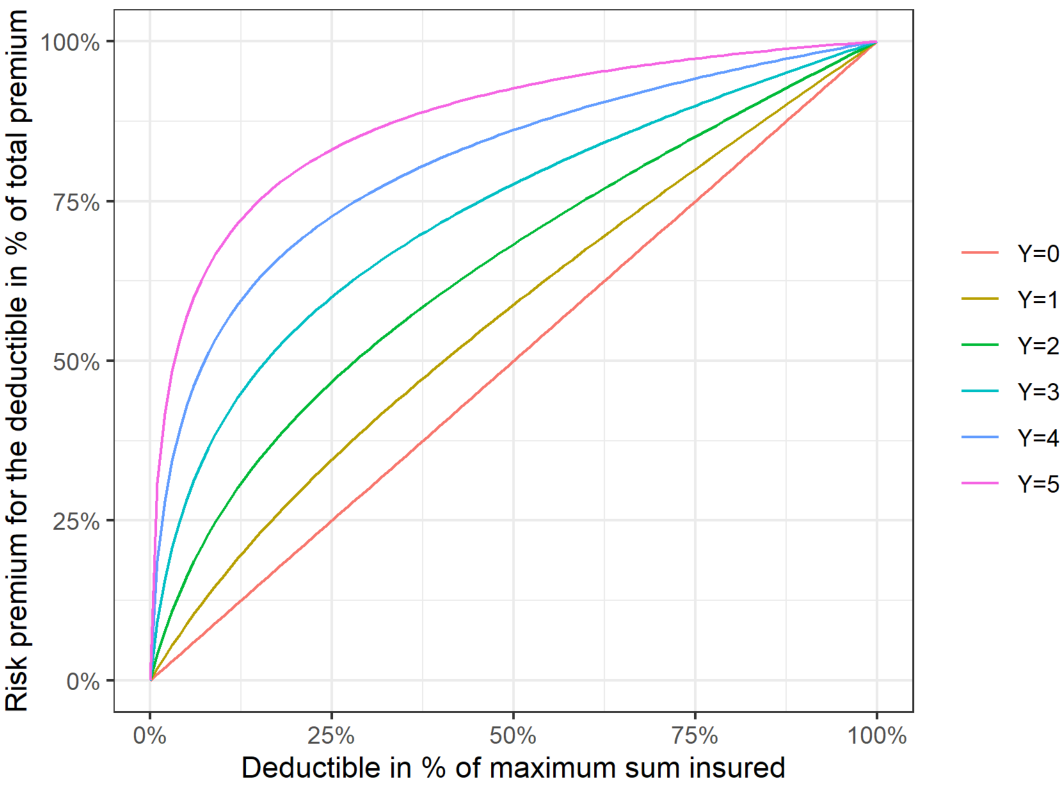

Figure 2.6.: SwissRe Exposure Curves

MBBEFD Exposure Curve

This two-parameter exposure curve is based on the three distributions Maxwell-Boltzmann, Bose-Einstein and Fermi-Dirac (MBBEFD), which are usually used in statistical mechanics. Nevertheless, Stefan Barnegger showed in a paper that the MBBEFD distribution class can also be used for modelling losses. Moreover, he could link the distribution to the SwissRe curves through parametrization.

The exposure curve is defined as

$$G(x)_{\{a,b\}} = \frac{\ln(a+b^x) - \ln(a+1)}{\ln(a+b) - \ln(a+1)}$$

with the regarding distribution function.

$$F(x) = \begin{cases} 1 & x = 1 \\ 1 - \frac{(a+1)b^x}{a+b} & 0 \leq x < 1 \end{cases}$$

The parameters a and b have to be selected so that $G(x)_{\{a,b\}}$ is a real, increasing and concave function on $[0, 1]$. Hence the curve parameter g is defined as the inverse of the total loss probability p with

$$g = \frac{a+b}{(a+1)b}; \qquad a = \frac{(g-1)b}{1-gb}$$

Therefore, the condition $0 \leq p < 1$ holds only for $g \geq 1$ and for $b \geq 0$ the exposure curve is a real function. As a result the following restrictions hold for the class of the MBBEFD distribution function.[4]

$$G_{b,g}(x) = \begin{cases} x & \text{if } b = 0 \text{ or } g = 1 \\ \frac{ln(1+(g-1)x)}{ln(g)} & \text{if } b = 1 \text{ and } g > 1 \\ \frac{1-b^x}{1-b} & \text{if } b = 0 \text{ or } g = 1 \\ \frac{ln\left(\frac{(g-1)b+(1-gb)b^x}{1-b}\right)}{ln(gb)} & \text{if } b > 0 \text{ and } b \neq 1 \text{ and } bg \neq 1 \text{ and } g > 1 \end{cases}$$

To get the one-parameter MBBEFD exposure curve, the two parameters b and g have to be approximated for every SwissRe curve. Then an exponential function is fitted to these resulting curves. The one-parameter MBBEFD exposure curve is then defined as

$$G_Y(x) = G_{b_Y, g_Y}(x)$$

with

$$b(Y) = e^{3{,}1-0{,}15(1+Y)Y}; \qquad g(Y) = e^{0{,}78+0{,}12Y)Y}$$

If the risk profile, the loss ratio and the parameter of the exposure curve are determined, it is straightforward to execute the exposure approach. Therefore, the numbers of the above risk profile of table 2.4 are recalled. Furthermore, a loss ratio of 50% is assumed. The reinsurance contract is an Excess of Loss contract with priority of 500.000 and a limit of 1.500.000.

Excess of Loss: 1.500.000 xs 500.000								
			Layer in % of Ø SI		Empirical Curve			Risk premium
Premium	Ø SI	Loss Ratio	D	D+L	G(D)	G(D+L)	Difference	per Layer
3.000.000	255.000	50%	196%	784%	100%	100%	0,00%	0
4.000.000	786.000	50%	64%	254%	93%	100%	6,87%	137.489
1.200.000	1.100.000	50%	45%	182%	88%	100%	12,27%	73.637
17.000	1.800.000	50%	28%	111%	79%	100%	20,88%	1.775
3.000	2.100.000	50%	24%	95%	76%	99%	22,92%	344

Table 2.5.: Result Table Exposure Approach

The risk premium has to be computed for every band separately. For example, the calculation for the fifth band are as below.

$$
\begin{aligned}
D &= \frac{\text{Priority}}{\varnothing\ \text{SI}} = \frac{500.000}{2.100.000} = 24\% \\
D + L &= \frac{\text{Priority} + \text{Limit}}{\varnothing\ \text{SI}} = \frac{500.000 + 1.5000.000}{2.100.000} = 95\%
\end{aligned}
$$

These ratios are then evaluated with the selected exposure curve. For this example a SwissRe curve with parameter $Y = 4$ is chosen. Evaluating the fifth band, the orange part of the SwissRe curve shows the burden of the risks.

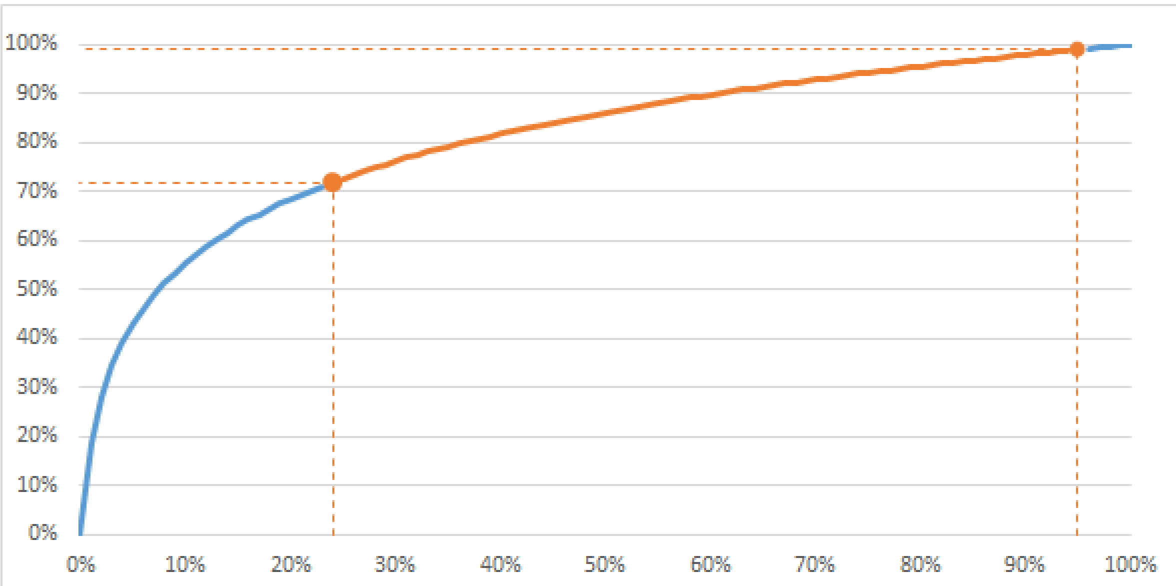

Figure 2.7.: Evaluation of Fifth Band

Finally, the risk premium per band can be calculated.

$$\begin{aligned}\text{Risk premium} &= \text{Premium} \cdot \text{Loss Ratio} \cdot \text{Difference} \\ &= 3.000 \cdot 50\% \cdot 22,92\%\end{aligned}$$

As it is possible that the average sum insured is below the priority, no risk premium would be calculated for that band. Hence, often it is assumed that the sums insured are Beta distributed within the band. Therefore, one would use the average sum insured as expected value and a default assumption for the standard deviation of the Beta distribution. The density outside of the band has to be zero.

Following the above, it needs to be explained how to get from the exposure approach to a Frequency-Severity Model. It is useful because in the fifth chapter both approaches are going to be compared to identify the difference in the result. First it has to be recalled that the expected loss(EL) to the layer $L\ xs\ P$ is calculated by

$$EL[L\,xs\,P] = freq_P \cdot ES[L\,xs\,P] \tag{2.1}$$

To calculate the frequency it is necessary to choose a layer where all losses are total losses. Therefore, a very small layer is chosen or mathematically speaking the layer of the amount ϵ tends towards zero. After rearranging equation 2.1 it follows.

$$freq_P = \lim_{\epsilon \searrow 0} \frac{EL[\epsilon\,xs\,P]}{ES[\epsilon\,xs\,P]} = \lim_{\epsilon \searrow 0} \frac{EL[\epsilon\,xs\,P]}{\epsilon} = \lim_{\epsilon \searrow 0} \frac{G(\frac{P+\epsilon}{SI}) - G(\frac{P}{SI})}{\epsilon} \cdot RP = \frac{RP}{SI} G'(\frac{P}{SI})$$

If the frequency at the priority level and the above severity distribution $F(x)$ are known, then the excess frequency at any point $x > P$ can be computed.

$$freq_x = freq_P \cdot (1 - F(x))$$

By rearranging the equation one receives the severity distribution.

$$F(x) = 1 - \frac{freq_x}{freq_P}$$

3. Market Study of the German Private Accident Insurance

This chapter should give an introduction to the German private accident insurance. First of all, the variety of the product features is going to be explained. Furthermore, the development of the premiums and the profitability is going to be analysed. To get an overview of the loss development, a closer look at the trends of the claims severity and frequency is taken. Also, a comparison between the primary insurance and the reinsurance market situation is drawn. Most of the analysed data is published by the GDV and BaFin, which is accessible to every member.[1]

3.1. The Private Accident Insurance

In the German market, there are 127 primary insurers which provide a PA insurance[1]. Hence, a variety of product features is offered. Also, it is differentiated between individual and group policies, where the latter mostly covers whole families within one policy. Most of the private accident insurance contracts are based on the terms and conditions (AUB) published by the GDV. The AUB determines the scope of the insurance and defines the accident scenario for which the insurer is liable.[9]

In the following, the main benefits, the additional features and special contract types are explained. There are three main benefits which are included in most of the PA contracts.

Disability Benefit

This benefit is the basic idea of the private accident insurance. It should support the insured person in case of an accident against financial issues, whether for the costs of the healing process or the discontinuation of the salary payments. The beneficiary receives a lump sum payment dependent on the basic sum insured, the progression model and the degree of disability. The **basic sum insured** is fixed in the contract. On the market it can be observed that the agreed sums depend mainly on the chosen progression model. The **progression model** is very crucial for the benefit payments. In the figure 3.1, five different shapes are displayed. The x-axis represents the disability degree and the y-axis the payment in percent of the basic sum insured. A progression model of 0% is a linear payment dependent on the disability degree. For example, a disability degree of 50% would lead to a payment of 50% of the basic sum insured. But during the last years, many primary insurer offer progression models up to 1000%, which means that in the case of a 100% disability degree the benefit payment would be ten times the basic sum insured. By analysing the market environment it can be concluded, that for low progression models, a basic sum insured up to one million Euro is offered. For higher models the primary insurers provide a maximum basic sum insured of up to €500.000.[16]

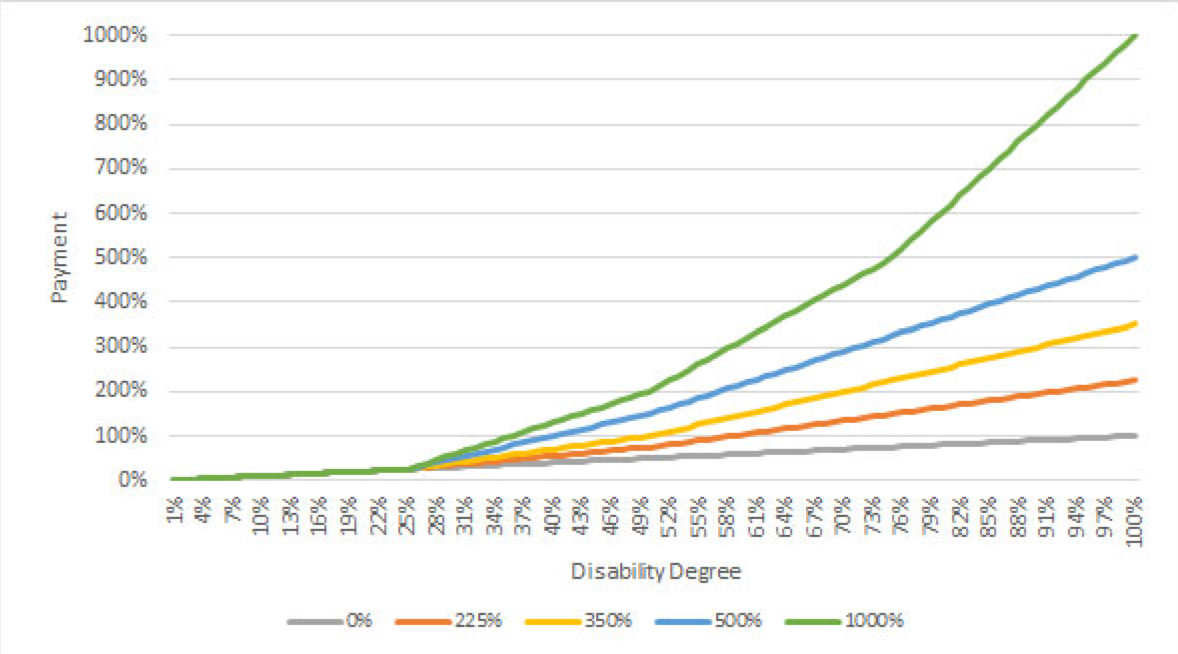

Figure 3.1.: PA Progression Models

The **degree of disability** is determined by the schedule of compensation. The AUB suggests a possible disability degree distribution for different types of injuries. For example, the loss of a whole arm would result in a degree of disability of 70%. Also, there are schedules of compensation for special occupational groups which are more adequate. But in case a disablement is determined the vast majority lie between 1% and 20%, although the average disability degree increased over the last years.[9, p. 6]

Pension Annuity

More than twenty years ago, a pension annuity was included within the private accident insurance. It is a monthly predetermined payment to the insured person. Market observations show that the annuity is usually between €500 and €3000 monthly. In most of the contracts it is stated that, the pension annuity is paid if the diagnosed disability degree is at least 50%.

Death Benefit

The death benefit covers in case of a deadly accident. The range of sums insured is huge and can reach up to one million Euro. However, the market study shows that an average sum insured of €50.000 is common.

Additional Benefits

Within the last several years the primary insurers try to make the PA insurance more attractive. Therefore, they offer a variety of additional features and widen their scope of cover. By default, most contracts include cosmetic operations after an accident or cover salvage costs. For an additional premium, the insured person can agree on a daily hospital allowance to cover a discontinuation of the salary payments in the short term.

Special Private Accident Contracts

To attract specific age groups, primary insurers invented contracts for children and seniors. The **senior accident insurance** is defined by a wide catalogue of services of assistance. These services can include emergency calls, financial support or shopping aids. The **child invalidity supplementary insurance** is an additional insurance that is bought in connection with a basic child PA insurance. The insurer pays a pension annuity in case the child is diagnosed with a degree of disability of 50% or more, as a result of an accident or illness. Furthermore, several years ago a contract with a **guaranteed premium refund (UPR)** was introduced. UPR policies combine the classical PA insurance with a saving part. As soon as the contract expires, the policyholder is going to get the full premium with a surplus participation refunded.

3.2. Market Environment

This section should give an overview of the general market environment from the perspective of the primary insurer and the reinsurer. The main indicators such as profitability, number of insured persons and premium development are observed. The analysis is based on BaFin data of insurance[1] and reinsurance[2] companies.

3.2.1. Business Line Profitability

The personal accident insurance is one of the lines of business with the highest consistent profitability regarding P&C. The figure 3.2 represent the underwriting (UW) result for primary insurer in comparison to reinsurer from 2010 to 2018. In 2018, the insurers could achieve a net UW result of 18,6% which is an absolute profit of around €1,061 billons. The average UW result from 2010 to 2018 is 17,5%. In comparison, the reinsurer were less profitable with an average UW result of 15,1% over the same period. Especially, in 2015 a huge discrepancy can be observed. The data show, that the gross loss ratio was similar to the other years but the net claim expenses were around 9% lower for primary insurer. From that, it can be concluded that more large losses occured the hence more claims were ceded to the reinsurer.

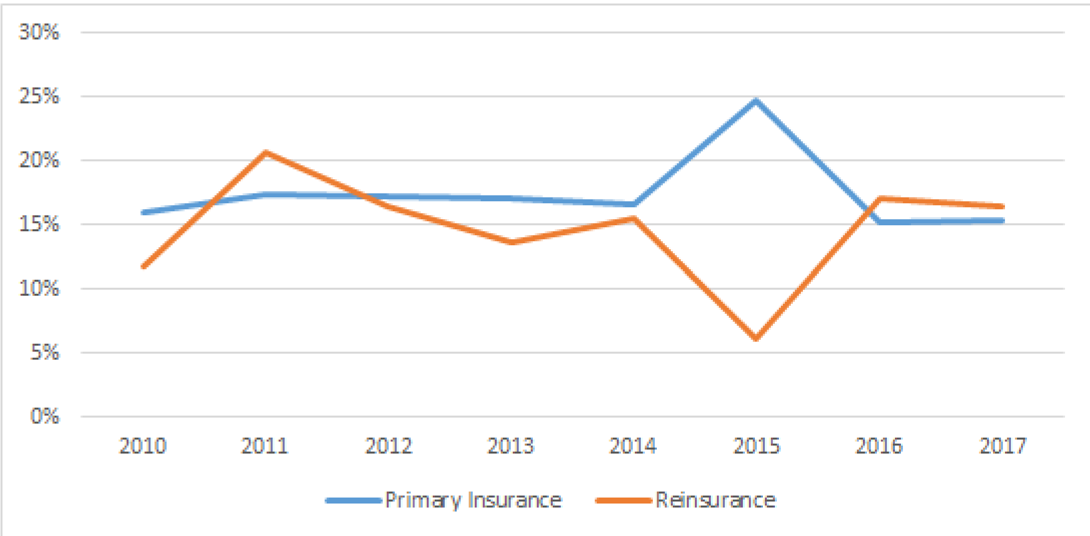

Figure 3.2.: Profitability Comparison Primary Insurer vs. Reinsuer

3.2.2. Number of Insured Persons

It is important to differentiate between the number of contracts and the number of insured persons, because a group policy count as one contract. Hence, it is more useful to compare the absolute number of persons which are covered by a personal accident insurance. The figure 3.4 shows the development of the number of insured persons over the last decade and additionally the share of UPR policies on the right-hand y-axis. Since 2009, the number of risks decreased steadily from 73 million to 70 million, with an exception in 2013. Also, the UPR policies dropped by 22% since 2009 and only 2,3 million contracts remain. One reason is the low-interest environment which makes this product less attractive.

The development for the whole line of business is mainly driven by the demographic change in Germany. The numbers indicate that there is a significant shift within the composition of the insurance portfolios. The insurance density of people above 65 increased, while in the lower age group the number of contracts decrease every year. One reason is that young people tend to sign rather occupational disability insurance than PA insurance, as the insurance density for people below 35 is higher for occupational disability insurance(see A). Analysing the internal SCOR data, an average age of 49 years can be observed, with an increase of six years over the last decade. It is expected, that this trend will go on, as the market penetration for people from the age of 65 is still low and a lot of insurers provide special products for this age group.

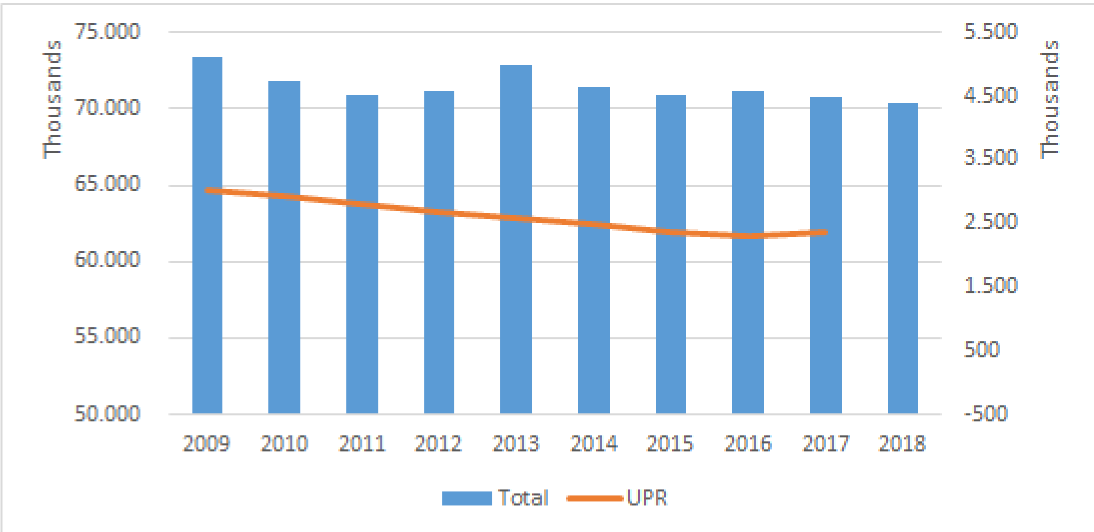

Figure 3.3.: PA Number of Insured Persons[2]

[2]the number for the UPR policies in 2018 is not available

3.2.3. Premium Development

With an overall gross premium of €6,656 billions in 2018, the personal accident insurance makes up 8,6% of the German P&C market. With a decline of 0,2%, the premium income decreases marginally in comparison to last year and on average a stable yearly increase of 0,3% over the last ten years can be seen. For 2019, a slight increase of around 1,5% is forecasted. This implies, although the number of contracts fell, including UPR contracts, it has no major impact on the premium. This is due to dynamic premium adjustments of a certain percentage, fixed in the PA contracts.

Analysing the reinsurance data is more difficult. On the one hand, the BaFin reinsurance data does not include all reinsurance companies that are acting on the German market. On the other hand, larger insurance groups use internal cessions to keep the premium income for this very profitable line of business. Nevertheless, some conclusions can be drawn with reservation. The gross premium income in 2018 was €1,417 billions, which represent around 20% of the total PA premium and increased by almost 5% in comparison to 2017. Also, a slight yearly growth of 1,3% since 2009 can be observed.

The figure 3.4 shows the gross premium development since 2009 for primary insurer and reinsurer. The y-axis is given in million Euro.

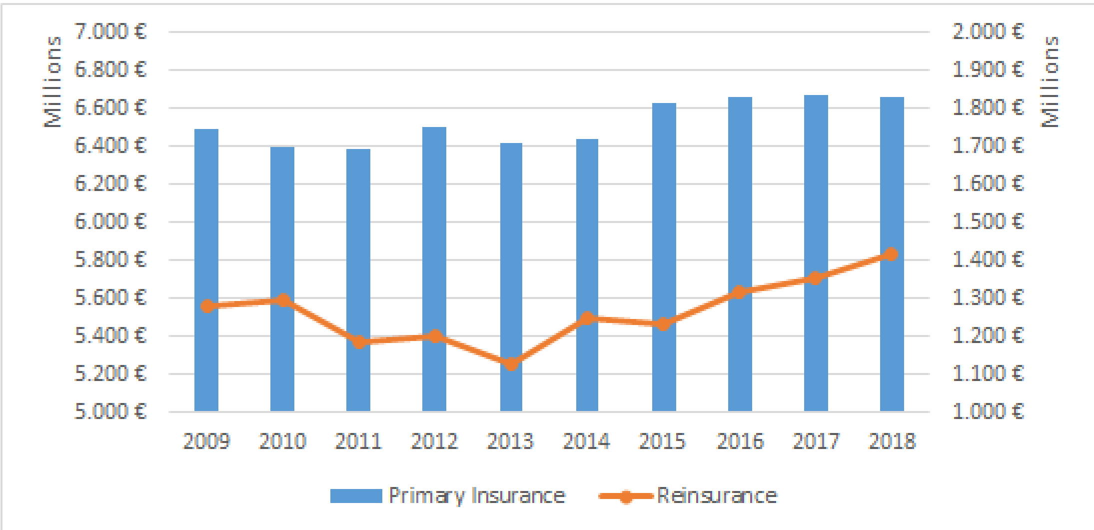

Figure 3.4.: Gross Premium Comparison Primary Insurer vs. Reinsuer

3.2.4. Major Primary Insurance Companies

The figure 3.5 shows the complete list of all acting companies in the private accident market with an earned gross premium income of more than €100 millions. Furthermore, some key figures such as reinsurance cession and number of contracts are given.

Insurer	Earned Gross Premium (mio EUR)	Reinsurance Cession (mio EUR)	No of Contracts
ALLIANZ VERS.	1.295,2	311,0	3.936,0
ERGO VERSICHERUNG	618,3	1,3	1.815,0
R+V ALLGEMEINE VERS.	404,8	19,9	1.400,0
DEBEKA ALLGEMEINE	346,5	0,0	1.965,0
SIGNAL IDUNA ALLG.	294,6	17,0	1.097,0
AACHENMüNCHENER VERS.	263,5	54,7	2.315,0
AXA VERS.	237,5	1,2	815,0
GENERALI VERSICHERUNG	199,9	60,8	1.868,0
LVM SACH	196,0	17,2	961,0
HDI GLOBAL SE	143,4	38,6	98,0
WÜRTT. VERS.	139,5	21,1	733,0
GOTHAER ALLGEMEINE AG	138,2	1,0	700,0
BAYER.VERS.VERB.AG	130,5	23,6	1.038,0
DEVK ALLG. VERS.	115,4	25,9	938,0
STUTTGARTER VERS.	105,8	2,5	535,0
CONTINENTALE SACHVERS.	104,0	2,5	585,0
BASLER SACH AG	103,1	4,5	433,0
Remaining Market Participants	1.774,1	305,1	49.125,0
Total	6.610,3	907,9	70.357,0

Figure 3.5.: PA Insurer with at least 100 Millions Premium Income

Only six companies hold over 50% of the German market for personal accident insurance. Although the leader, Allianz Insurance, dominates the market with a share of nearly 20%, their share has decreased by 5% within the last ten years. A huge discrepancy can be seen in the number of contracts per insurer and their premium income. While the remaining market participants with a premium income less than €100 millions hold 60% of the contracts, their earned premium amount is only 24% of the market premium. Observing the reinsurance cession of the individual primary insurer, it can be confirmed that the big insurer as Allianz, General or HDI cede in proportion the most risks.

3.3. Trends of the Claim Development

Especially for the pricing of reinsurance contracts, it is essential to observe the trends in the claim development. Based on the findings, the index for the re-evaluation of the premium and losses is chosen. As the GDV provides explicit data of the claim frequency and severity developments of the primary insurance PA market, a profound analysis can be conducted.[12][11]

3.3.1. Analysis of the Combined Ratio Development

The combined ratio (CR) is the share of the total premium, which is required to cover the costs of the insurance company. It consists of the loss ratio (LR), which is the share to cover the claim expenses, and the cost ratio, that includes all operating and acquisition costs of the insurance company. A CR below 100% implies an operating profit.

A combined ratio of 77,4% in 2018 is very satisfactory for the primary insurer and indicates a profitable year. Observing the last ten years, the LR increased by two percentage points from 58,2% in 2009 to 60,6% in 2018. For the year 2019, the GDV predicts a LR of 61%. So far, the stable profitability of the PA business results from a slight decrease of the cost ratio in the last five years. But in an exceptional year like 2017, where the operational costs are higher than on average, the primary insurer will make less profit.[11, p.6]

The figure 3.6 below, shows the combined ratio in percent split into the loss ratio and the cost ratio.

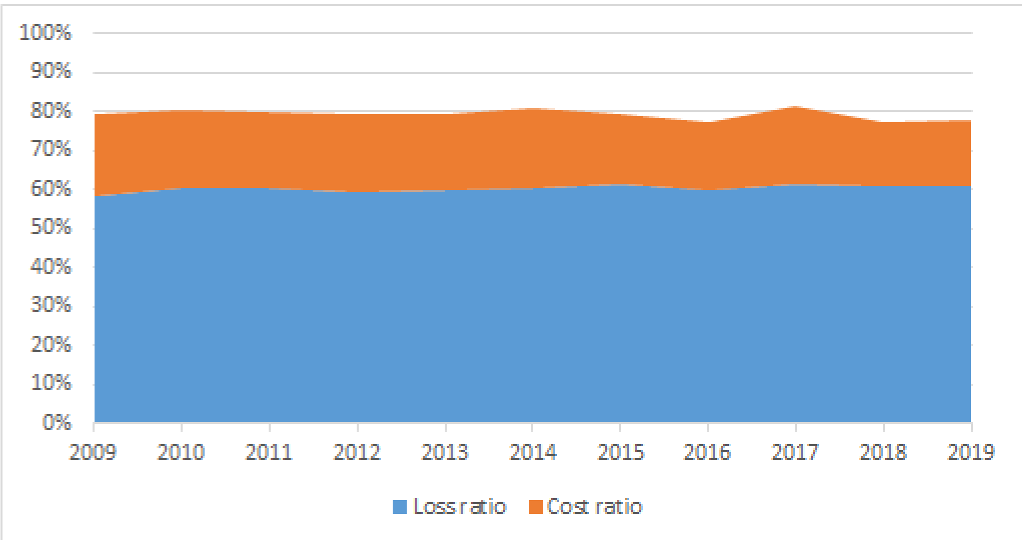

Figure 3.6.: Loss Ratio and Cost Ratio

To analyse the steady increase of the loss ratio, a closer look of the claim frequency and severity development is taken in the figure 3.7. The severity indicates the average claim amount and the frequency, the loss probability. In 2018, 3,2% of the contracts produces claims with an average amount of €4204. Whereas the severity increases steadily in average by 2% every year, the frequency development has two significant leaps in 2010 and 2016.

There are several reasons why the primary insurers face higher claim expenses. On the one hand, insurers are trying to keep the sales up by widening the scope of cover, offering higher basic sums insured and introducing new progression models. But more importantly, the average age of the insurer portfolios' increases tremendously. The GDV number shows the claim cost rises with progressive age and especially around 55 a tipping point is reached, in which the contracts are not profitable any more.[10, p.17]

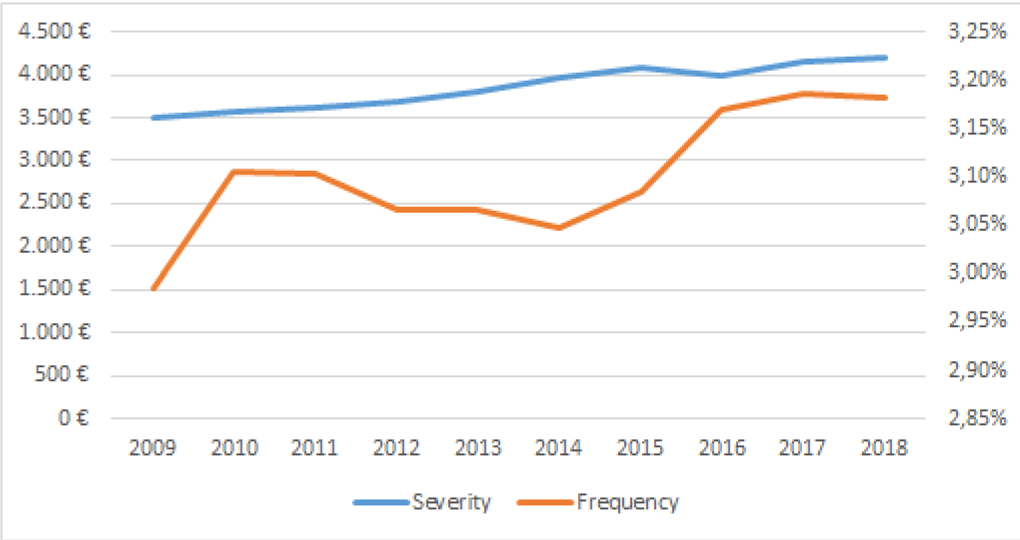

Figure 3.7.: Primary Insurance Frequency and Severity

The figure 3.8 emphasises the observations from above. It shows the development of the average premium and the loss per exposure from 2009 to 2019. The average premium increased by €24 since 2009, which corresponds to a yearly growth of around 1,4%. However the loss per exposure went from €104 to €138 in 2019, which implies an increase of 2,8% per year. As the loss per exposure increases twice as fast as the average premium, the ratio changes significantly. In 2019, the primary insurer used almost 53% of the premium income to cover their claim costs, whereas in 2009 it was only roughly 46%.

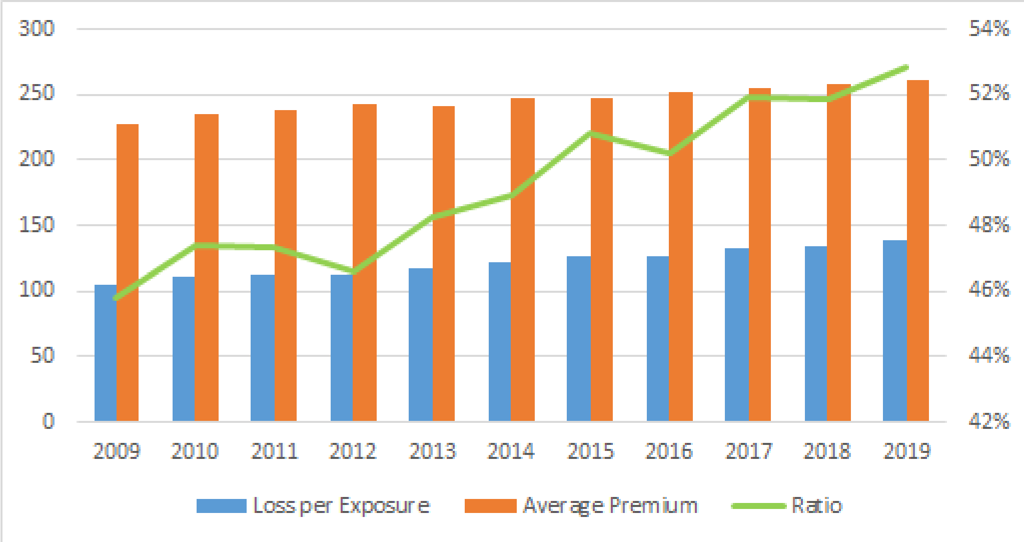

Figure 3.8.: Comparison Average Premium vs. Loss per Exposure

3.3.2. Exclusions in the Private Accident Insurance

The primary insurer try to protect themselves against cumul events by excluding general accident scenarios. The AUB provide a list of the most common accident scenarios, which most of the insurers include in their PA contracts. In the following, a closer look is being taken on the two most essential exclusions.[9, p. 10ff]

As the market environment gets more competitive, the primary insurers are softening their exclusion for **infectious diseases** in recent years. While the GDV recommends including only hydrophobia and tetanus in the PA contracts, many companies are extending their cover to any kind of infectious diseases.

By including the **passive war risk** in the contracts means that the insured person is fully covered in countries of war or even civil war, if the insured is not actively participating in this war. But the GDV recommends excluding accidents due to the use of atomic, biologic and poisoning weapons.

4. Large Loss Comparison of SCOR Clients

To get an overview of the claim development of the private accidents insurance of the SCOR clients, an experience large loss model is build. First, a Short Tail model is presented to analyse the claim frequency and severity development of the cedents. In the course of that, the issues of pricing private accident insurance contracts, solely with an experience approach, are going to be shown. Furthermore, the differences between pricing with a Long Tail and Short Tail model are going to be worked out. All the analysis are based on data provided by the cedents. Due to reasons of data security, only aggregated loss information can be presented.

4.1. Data Basis for the Experience Modelling

SCOR provides reinsurance for 18 primary insurance companies in the German PA market. Adapted to every cedent, an individual reinsurance solution is elaborated. In general, the smaller primary insurer ask for a combination of Quota Share and Excess of Loss reinsurance. But most of the primary insurers agree only on an Excess of Loss per Risk or Event reinsurance contract to keep the biggest part of the premium.

For the modelling, the insurers are split into three groups primarily according to their premium volume. But also it can be examined, that the insurers within those groups, offer similar benefit amounts and hence have a similar loss history. Analysing the scope of the reinsurance covers within the different groups, the tendency can be observed, that the plafond increases with the premium volume. At the same time, smaller primary insurer agree on a lower priority level in general. The loss history reaches back to almost thirty years of experience for some insurer. Yet, the complete loss data for every insurer only dates back to 2005. For the Short Tail model it can be drawn on more than two thousand individual ultimate losses, which represents a decent loss data basis to build up an accurate experience model. The table 4.1, displays the data basis for the Short Tail model.

Size of Insurer	Premium in Mio. €	# of Insurer	# of Losses	Plafond in Mio. €
Small	< 40	8	1288	2 - 4
Medium	40 - 100	4	435	3,5 - 5
Big	>100	3	524	4 - 10

Table 4.1.: Data Basis for the Large Loss Short Tail Model

As more extensive data is necessary for the Long Tail model, less primary insurers provide the triangulated loss history. Therefore, the small and medium size insurers are combined to get a sufficient loss data foundation. The table 4.2 below, gives an overview of the data for the Long Tail experience model.

Size of Insurer	Premium in Mio.	# of Insurer	# of Losses
Small+ Medium	<100	7	1215
Big	>100	2	465

Table 4.2.: Data Basis for the Large Loss Long Tail Model

4.2. Results of the Short Tail Modelling

4.2.1. Examination of the Claim Trends

Based on the losses the SCOR clients had to face, an experience Short Tail model is going to be build. The claim frequency and severity is going to be analysed at different model thresholds, to examine trends more accurately. Additionally, the cedents provide their losses at different reporting thresholds, which has to be considered in the modelling. For the indexation of the premiums, the average premium growth on the market is taken. On the claim side, the losses are adapted by the market increase of the average loss per exposure. Both information are provided by the GDV.[12, p. 71]

Short Tail Model xs € 500.000

The number of indexed losses above the model threshold depend on the reporting threshold and the chosen index. A model threshold of € 500.000 is chosen as all small and medium size insurers can be considered in the comparison. As the big insurers' provide their loss history at a higher threshold, they can not be taken into account for this model. In the table 4.1 below, the available loss data is displayed.

Size of Insurer	# of losses > Treshold
Small	102
Medium	163

Table 4.3.: Number of indexed Losses above € 500.000

The model frequency represent the number of losses per one million premium. Analysing the frequency development of the small and medium size cedents, two conclusions can be drawn. First it is observable, that the trends are contrary. The small insurer face an increase of the claim frequency and the medium size insurer vice versa. Second, the volatility of the losses is stabilizing at almost 0,06 losses per one million premium within the last seven years. Whereas between 2007 and 2011 the frequency was varying from 0,02 to 0,1 losses.

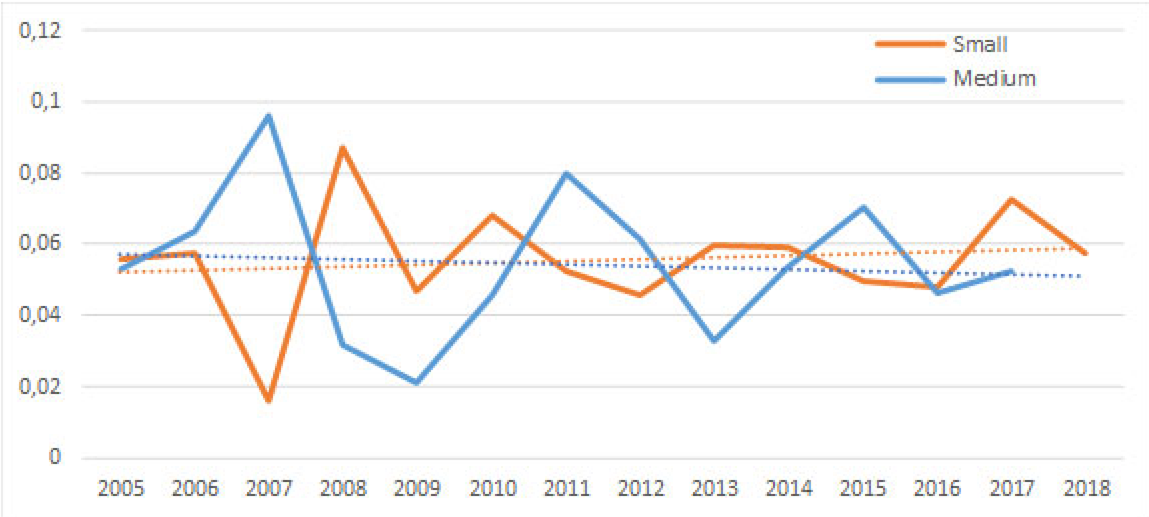

Figure 4.1.: Model xs € 500.000 Development of Frequency

The figure 4.2, shows the severity development of the small and medium size insurers. The severity represents the average claim amount in thousand Euro. For both groups of primary insurers, no clear trend is observable over the whole period and a rather stable development can be seen. Still, the small insurers show a positive trend within the last four years.

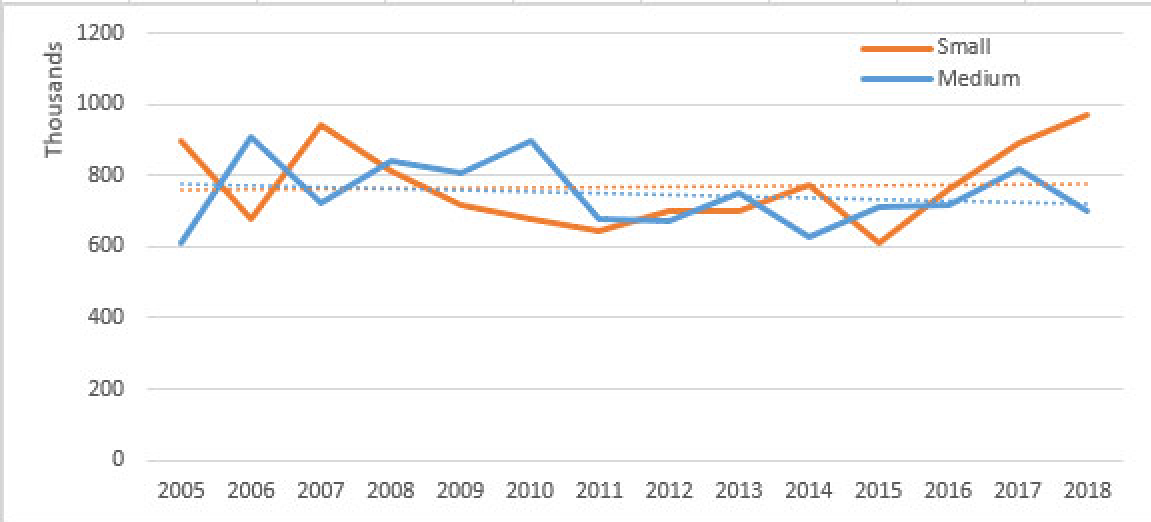

Figure 4.2.: Model xs €500.000 Development of Severity

Short Tail Model xs € 1.000.000

As less claims occur above €1.000.000, it is more difficult to build a model with a higher threshold. Therefore, the small and medium size insurer are combined to obtain a sufficient data basis. For the big insurer one cedent could not be considered, because he provides its loss history above the model threshold. In the table 4.4 the number of losses above the threshold for both groups of primary insurer are shown.

Size of Insurer	# of losses > Treshold
Small + Medium	38
Big	31

Table 4.4.: Number of indexed Losses above €1.000.000

Again, two different trends in the frequency of the claims can be observed. On the one hand, the small and medium insurers reveal a constant development, with a high volatility of the claim occurrences. On the other hand, the big insurers have a decreasing trend of the number of losses above €1.000.000. Also, in comparison to the above model, the frequency lays at a significant lower level.

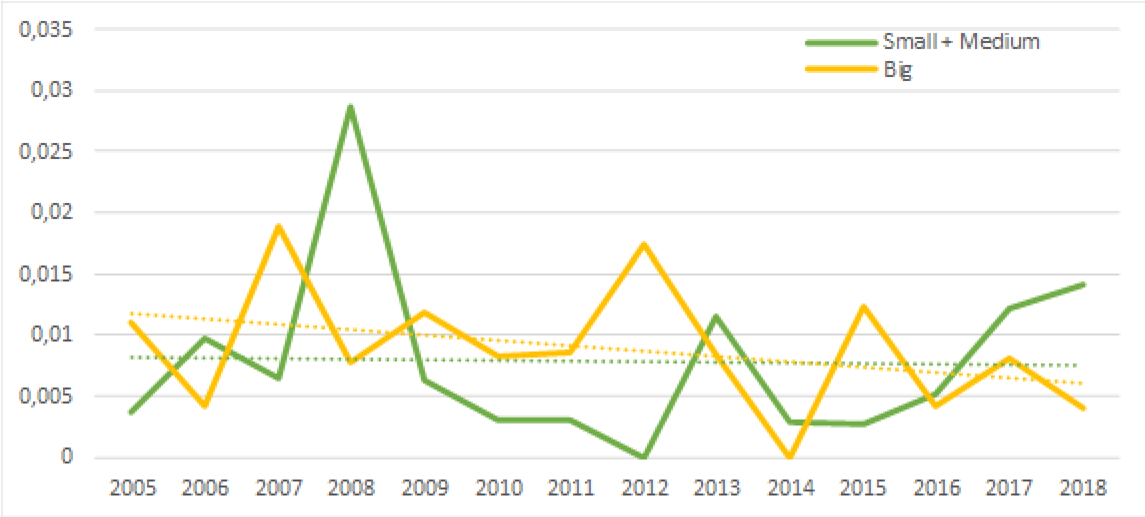

Figure 4.3.: Model xs €1.000.000 Development of Frequency

For the claim severity, both groups develop constantly since 2005. Even if the severity for the big insurers is on a higher level, the average claim costs are not much above the model threshold. Hence, one can conclude that most of the claims are only slightly above one million Euro.

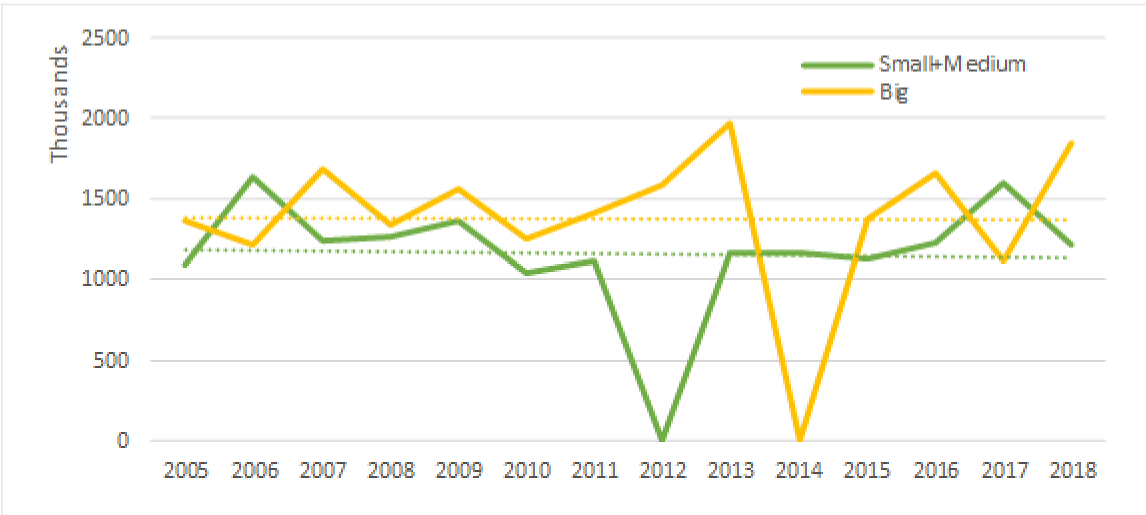

Figure 4.4.: Model xs €1.000.000 Development of Severity

4.2.2. Model Severity Distribution Fit

As for the Short Tail experience model, the severity distribution has to be computed, the distribution fit is crucial for the model accuracy. The figure 4.5 shows the model fit for a threshold at €300.000 and €500.000 for the group of small cedents. Analysing possible distribution functions for the private accident insurance business, it can be suggested to use the LogGamma distribution. In the figure below, every square represent one claim. It can be concluded that the chosen distribution would simulate future losses accurately based on the loss history.

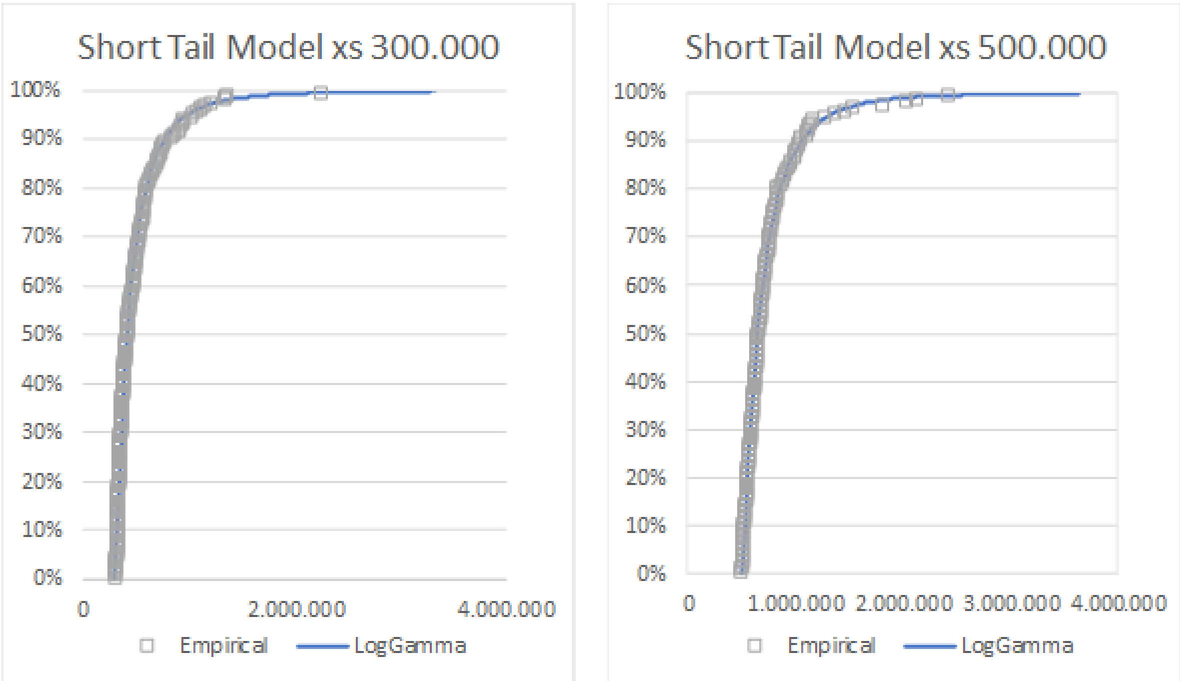

Figure 4.5.: Experience Short Tail Distribution Fit

A very different setting can be found, if the model threshold is higher and less loss history is available. The figure 4.6 below, shows the distribution fit for big insurers with a threshold of one million. It reveals that, the experience approach is inaccurate in fitting a distribution to high losses for the private accident insurance. As the private accident insurance is a sum insurance, the loss amounts are similar and therefore line up vertically. Hence, no distribution function can be fitted to the loss observations. As a result, a development of an exposure-based approach is useful to avoid the difficulty to model high losses.

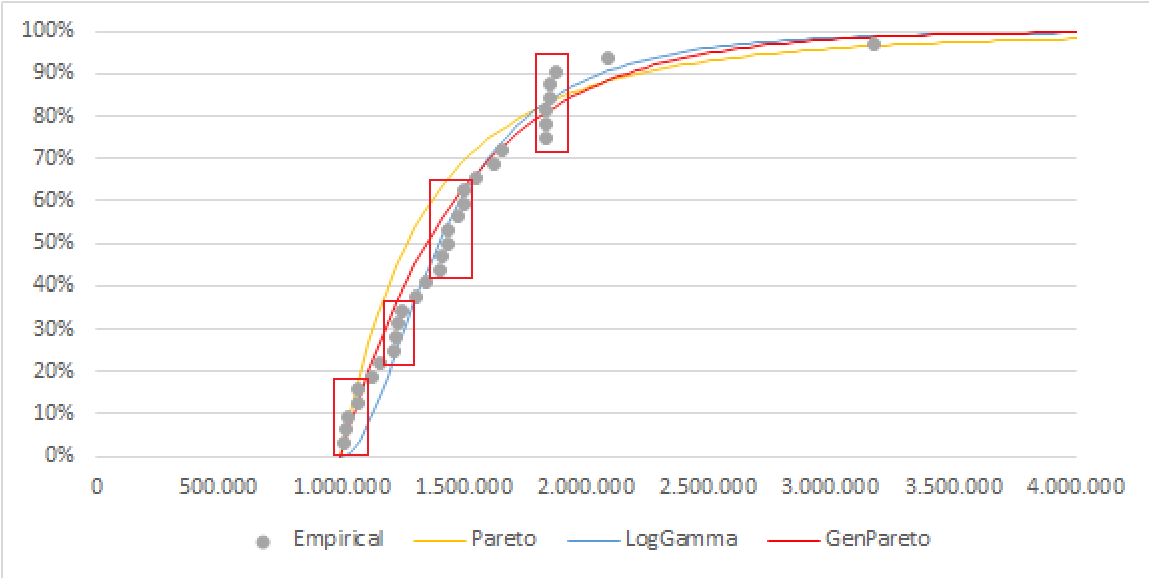

Figure 4.6.: Distribution Fit of the Short Tail Model of Big Insurer

4.3. Results of the Long Tail Modelling

As there is less loss data available for the Long Tail modelling, the group of small and medium size insurers are combined. The claim development of the big insurers can not be analysed, as only two insurers make their data at very different thresholds available. For the claim re-evaluation, the same indices as in the Short Tail model are used. For the observation, again the period from 2005 to 2018 is examined. In the table 4.5, the number of considered losses is displayed.

Size of Insurer	# of losses > Treshold
Small + Medium	193

Table 4.5.: Number of indexed Losses above the Long Tail Model Threshold

Analysing the LDFs resulting from the triangulated losses, the latest three years are remarkable. For the private accident insurance, the deadline for determining the final degree of disability is set at the end of the of the third year after the accident [9, p. 5]. The impact can be observed in the table 4.6, where the LDFs in the first three years are clearly above one. This indicates that the insurance benefits increase within the first three years. After this period, the LDFs are moving around one. It is because, only in exceptional situations, for example individual court decisions would adapt the final degree of disability.

Dev.	1. Year	2. Year	3. Year	4. Year	5. Year	6. Year	7. Year	8. Year	9. Year	Tail Factor
Severity	1,73	1,27	1,14	1,01	1,02	0,98	0,99	1,03	1,04	1,00
Frequency	1,75	1,20	1,12	0,99	0,99	0,97	0,99	1,02	1,01	1,00

Table 4.6.: Loss Development Factor for the Long Tail Model

For the comparison of the claim frequency and severity, the same primary insurers for the Short Tail and Long Tail model are chosen. In the figure 4.7, the impact of LDFs on the frequency development of the Long Tail model can be observed. The frequency of the Short Tail Model is significantly lower in the last two years in comparison to the Long Tail model. Hence, in the Short Tail model the latest two to three years should be re-evaluated differently in order not to neglect the frequency growth within the first three years.

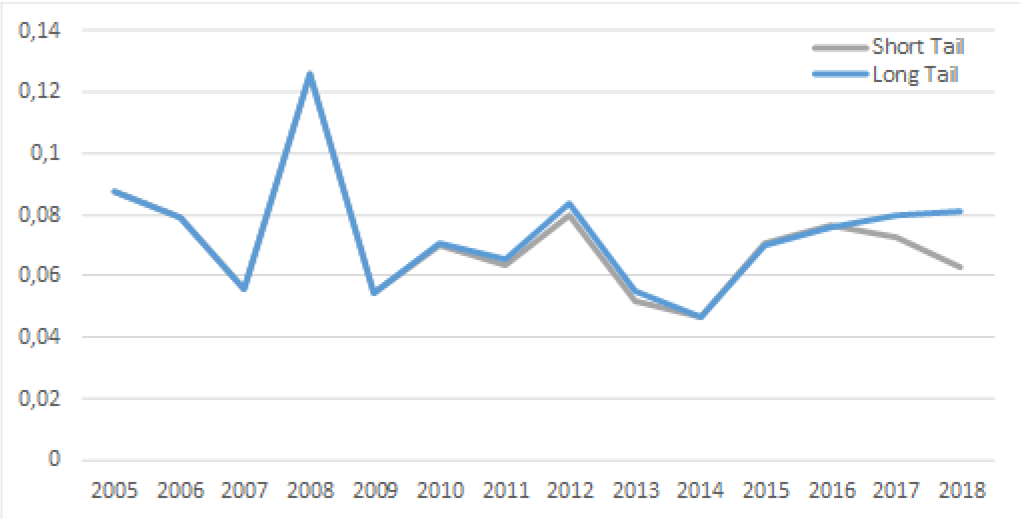

Figure 4.7.: Comparison of the Frequency

The severity is measured as the total claim amount in percentage of the total premium. The ultimate claim amount also depends on the reserving method that is used. For this example, the Chain-Ladder method is applied. The effects of the positive claim development in the first years can also be seen, whereas there is also some minor deviation for earlier years.

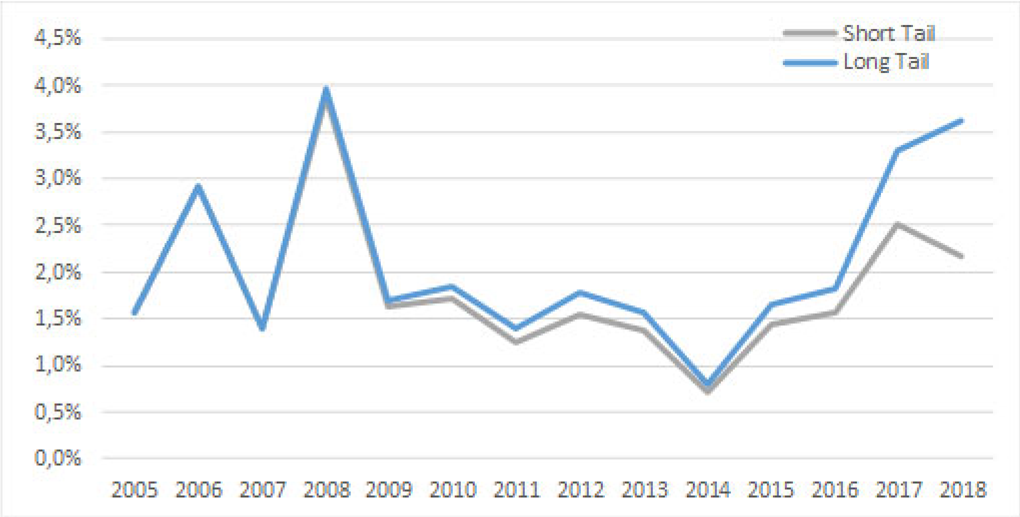

Figure 4.8.: Comparison of the Severity

5. The Exposure-Based Pricing Approach

This chapter presents the developed exposure-based approach for the private accident insurance. The idea of the method is going to be explained in detail and applied on the available cedent data. As it is going to be shown in the first section, that the risk profiles are very inconsistent across the different insurers, the model is adapted to make it applicable to every cedent. Also, the findings from chapter two regarding higher benefit models are going to be included in the calculations.

5.1. Data Basis for the Exposure Modelling

For the exposure approach usually the risk profiles are sufficient. But to compute the parameter of the exposure curve for the private accident line of business, it is necessary to evaluate the whole portfolios of the individual primary insurers. Hence, five insurers provide their portfolio data with a different degree of information. For the model, the age, the pension annuity, the basic sum insured and the progression model per insured person are necessary. The age and the pension annuity are used to calculate the present value of the pension according to the mortality tables provided by the German Society of Actuaries (DAV)[5]. Unfortunately, one cedent only provides the data per contract type and can therefore not be considered. Nevertheless, in total there is information about 786.000 insured persons available. For the insurer 1, every risk within a group contract will be treated as a different insured person. The second insurer provides only the maximum payment for the disability benefit. Additionally, he makes data available about the distribution of the progression model within the portfolio. With that information, the progression model is simulated for every insured person and the basic sum insured can be calculated. The table 5.2 gives an overview of the data basis for the development of the exposure approach.

Insurance	Age	Pension	Disability SI	Progression	# of insured	Group Contracts
Insurer 1	✔	✔	✔	✔	294.057	✔
Insurer 2	✔	✔			281.456	
Insurer 3	✔	✔	✔	✔	175.687	
Insurer 4	✔	✔	✔	✔	36.091	
Insurer 5	Data not per insured person given					

Table 5.1.: Data Basis with individual Portfolio Information

To test the computed exposure curve parameter, the risk profiles are used to apply the exposure approach. As the risk profiles are a more common information that is shared between primary insurer and reinsurer, there is data for 13 cedents available. But most of the risk profiles contain incomplete information. Necessary are the number of risks, the total basic sum insured, which consists of the present value of the pension and additionally the maximum disability benefit, and the premium. From the individual data, the risk profiles can be easily created. Other cedents often provide separate tables for the pensions and the disability benefit, which can not be used. Furthermore, many primary insurer split the risk profiles per progression model, which can be combined into one profile. The table 5.2 below shows the data foundation regarding the risk profiles.

Insurance	# of risks	Total SI	Premium	Note
Insurer 1	✔	✔	✔	
Insurer 2	✔	✔	✔	
Insurer 3	✔	✔		
Insurer 4	✔	✔	✔	
Insurer 5	Data not per insured person given			
Insurer 6	✔	✔	✔	
Insurer 7	✔	✔	✔	Only DB per Progression model
Insurer 8	✔	✔		
Insurer 9	✔	✔		
Insurer 10	✔	✔	✔	Only xs Threshold
Insurer 11	✔	✔	✔	Only DB per Progression model
Insurer 12	✔			Only DB per Progression model
Insurer 13	✔	✔	✔	All information separetly

Table 5.2.: Overview of Cedents Risk Profile Information[3]

For the model, the **disability degree distribution** is essential. Later it will be shown that the exposure curve parameter is very dependent on this distribution. The figure 5.1 shows the observations of a SCOR internal private accident insurance loss study. The

[3]DB is short for disability benefit

height of the bars represents the probability of the diagnosed disability degree. As for data safety reasons, the y axis can not be shown. But nevertheless remarkable conclusions can be drawn. The disability degree up to 25% make up already 92% of the cases. It can be concluded, that for most of the benefit payments the progression model does not apply. Furthermore, in only 2% of the cases the disability degree is at least 50% and hence the primary insurer has to pay the pension annuities. Due to the determination of the disability degree according to the schedule of compensation, the disability degree peaks at specific numbers[9, p. 6]. Especially, the probability of a disability degree of maximum 100% is disproportionately high, as a combination of injuries lead to a high disability degree.

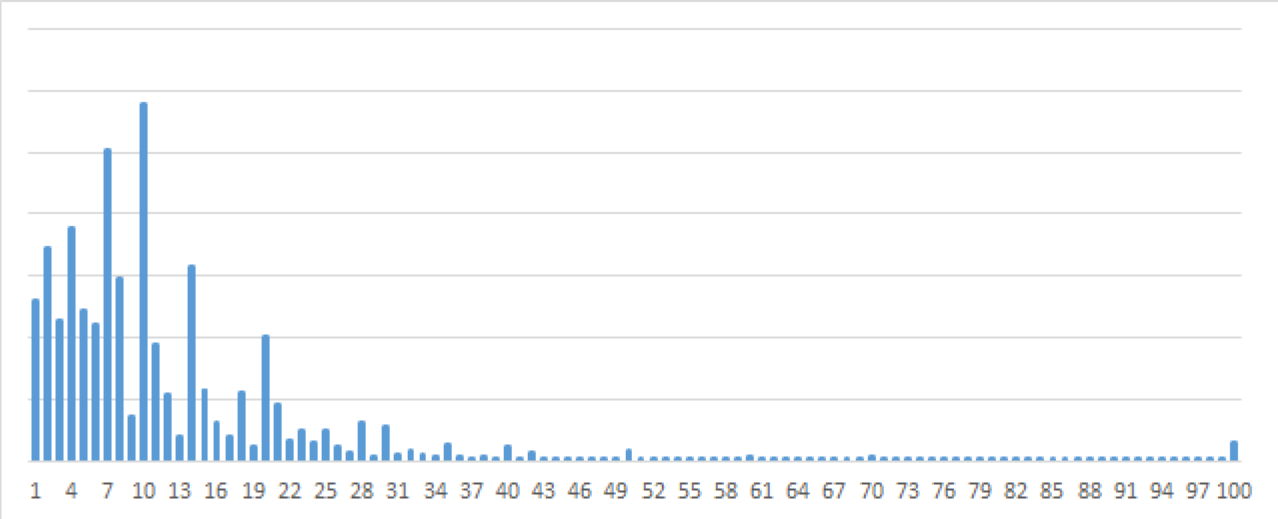

Figure 5.1.: Disability Degree Distribution

5.2. Illustration of the Implementation of the Approach

This sections presents the idea of the exposure-based pricing approach developed for the PA insurance. As for some cedents, only the risk profiles are available, the aim is to find a general exposure curve parameter which is applicable to every primary insurer. Therefore, the individual data is taken to compute the parameter and further used as a test set to verify the results. Additionally, for the cedents which provide individual data, the model is adapted to compute the severity and frequency distribution for the reinsurer.

The basic idea is to calculate the expected loss above a series of thresholds for the whole portfolio of the primary insurer and conclude from that the premium share between the primary insurer and the reinsurer. The expected loss is calculated by multiplying the benefit amounts with the occurrence probability of corresponding disability degree. The considered compensations of the reinsurer consists of the present value of the pension annuity (PVP) and the disability benefit, which results from the basic sum insured and the progression model. Hence, there are three possibilities in which the reinsurer has to compensate the primary insurer.

1. The present value of the pension annuity is above the threshold
2. The disability benefit is above the threshold at a disability degree below 50%
3. The sum of the present value of the pension annuity and the disability benefit is above the threshold at a minimum disability degree of 50%

5.2.1. Application of the Approach on an Exemplary Portfolio

For the explanation of the approach, the method is applied on a portfolio with only four insured persons. For simplicity reasons, only the progression model 500% is considered. In the table 5.3, the benefits for all the insured persons are displayed.

Insured	PV Pension Annuity	Disability SI	Progression
Person 1	0	40.000	500%
Person 2	90.000	0	0%
Person 3	80.000	10.000	500%
Person 4	25.000	35.000	500%

Table 5.3.: Portfolio for Model Explanation

Some assumptions are made to calculate the expected loss for this portfolio. The variable i indicates the degree of disability.

- The possibility of a disability degree of 0% is excluded.
- The disability degree is distributed as in 5.11 and defined as $Inval(i)$ for $i \in \{1, ..., 100\}$.
- The progression model 500% processes as in 3.1 and is defined as $Prog(i)$ for $i \in \{1, ..., 100\}$.
- The maximum benefit is set at 2.000.000.

First of all, the total expected loss is computed, which is equivalent to a threshold of 0. The PVP is paid in case of the disability degree is at least 50% and the disability benefit is paid from the first degree of disability. The sum of the expected loss per person is the total expected loss for the portfolio.

Total Expected Loss:

$$\text{Per Person:} \quad PVP \cdot \sum_{i=50}^{100} Inval(i) + SI \cdot \sum_{i=1}^{100} Prog(i) \cdot Inval(i) \tag{5.1}$$

$$\text{Total Expected Loss:} \quad 7.753 + 1.959 + 3.680 + 7220 = 20.612 \tag{5.2}$$

For the expected loss above a threshold, individual calculations for every insured person have to be carried out. The outcome is highly dependent on the composition of the disability benefit and the present value of the pension annuity. Computing the expected loss above 50.000, the **insured person 1** agreed only on a disability benefit of 40.000 with a progression model of 500%. As the disability benefit exceeds the threshold at a disability degree of 12%, the sum insured is multiplied with the sum of the multiplication of the progression and the disability degree, starting at $i = 12$. **The insured person 2** benefits only from the pension annuity. As the PVP is above the threshold, the sum starts at the payout disability degree of 50%. The last two policyholder in this exemplary portfolio, include a pension annuity and a disability benefit in their contract. Therefore, the expected loss is computed as the sum of the individual benefits. It has to be evaluated if the PVP, disability benefit or the sum of both is above the threshold. For the **insured person 3** the maximum disability benefit would not be above the threshold, but the sum of the benefits exceed 50.000. Hence, the expected loss is calculate by starting the sum at $i = 50$. The benefit composition of the **insured person 4** is more complicated. On the one hand, the disability benefit exceeds the threshold at a disability degree of 49%. On the other hand, the PVP itself is below 50.000, but the sum of both benefits is above the threshold. As a result the sums start at different disability degrees.

Expected Loss above Threshold 50.000

$$\text{Person 1:} \quad SI \cdot \sum_{i=42}^{100} Prog(i) \cdot Inval(i) = 3.135$$

$$\text{Person 2:} \quad PVP \cdot \sum_{i=50}^{100} Inval(i) = 1.959$$

$$\text{Person 3:} \quad PVP \cdot \sum_{i=50}^{100} Inval(i) + SI \cdot \sum_{i=50}^{100} Prog(i) \cdot Inval(i) = 2433$$

$$\text{Person 4:} \quad PVP \cdot \sum_{i=50}^{100} Inval(i) + SI \cdot \sum_{i=49}^{100} Prog(i) \cdot Inval(i) = 3.013$$

In total: 10.540

The idea for the computation of the expected loss for a threshold of 100.000 is the same as above. In the case of the **insured person 3** and **insured person 4**, it has to be considered that the sum of the benefits exceed the threshold at a higher disability degree as where the PVP payment would start.

Expected Loss above Threshold 100.000

Person 1:	$SI \cdot \sum_{i=65}^{100} Prog(i) \cdot Inval(i) = 2.103$
Person 2:	Present value of pension annuity not above the threshold
Person 3:	$PVP \cdot \sum_{i=58}^{100} Inval(i) + SI \cdot \sum_{i=58}^{100} Prog(i) \cdot Inval(i) = 1.799$
Person 4:	$PVP \cdot \sum_{i=60}^{100} Inval(i) + SI \cdot \sum_{i=60}^{100} Prog(i) \cdot Inval(i) = 2.392$
In total:	**6.294**

For a threshold of 150.000, it can already be noticed that the disability degree has to be very high so that the benefits exceed the threshold, even the benefit payments of two contracts are not necessary to be considered for the expected loss.

Expected Loss above Threshold 150.000

Person 1:	$SI \cdot \sum_{i=83}^{100} Prog(i) \cdot Inval(i) = 1.508$
Person 2:	Present value of pension annuity not above the threshold
Person 3:	Maximum benefit not above the threshold
Person 4:	$PVP \cdot \sum_{i=80}^{100} Inval(i) + SI \cdot \sum_{i=80}^{100} Prog(i) \cdot Inval(i) = 1.704$
In total:	**3.212**

The last step of computing the empirical exposure curve is to get the burden for the primary insurer at the series of thresholds. The total expected loss with no deductible is equivalent of receiving the total premium. The ratio of the expected loss at a threshold and the total expected loss gives the exposure for the primary insurer at the specific threshold. With the formula 5.3 below, the CDF of the empirical exposure curve can be computed.

Empirical Exposure Curve

$$\text{Empirical EC} = 1 - \frac{EL(\text{threshold})}{EL(\text{total})} \tag{5.3}$$

The table 5.4 shows that already with a deductible of 50.000, the primary insurer would keep almost 50% of the premium. In the next section, it is going to be shown that this form is very typical for the PA line of business. As the maximum benefit is 200.000, the threshold at that point adopts the value one.

Threshold	Empirical EC
50.000	0,48
100.000	0,69
150.000	0,85
200.000	1

Table 5.4.: CDF of the empirical exposure curve

The figure below displays the empirical exposure curve.

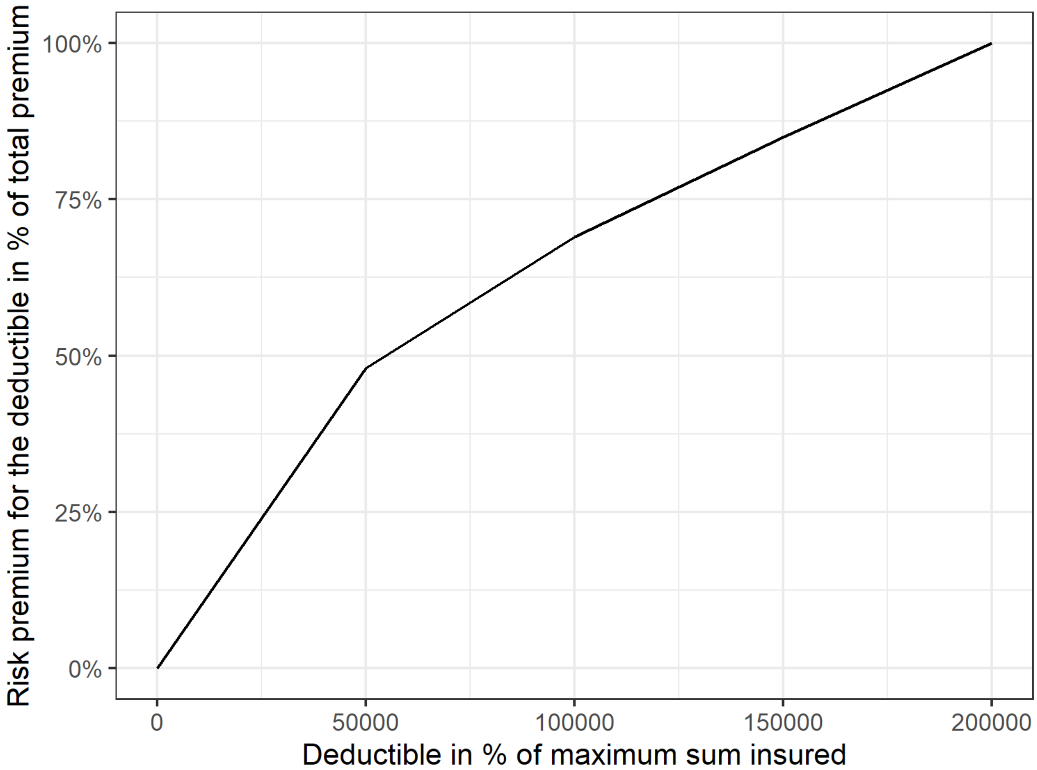

Figure 5.2.: Empirical Exposure Curve

5.2.2. Implementation in R

The code for the model is implemented in R. In the following, the pseudo code for the calculation of the exposure curve and the adapted approach are presented. The **Algorithm**

1 is divided in three steps. In the **first step**, the expected value for each individual at several thresholds is going to be calculated. Therefore, two for-loops are installed. One counts the threshold in 100 equidistant steps until the maximum benefit and the other runs through all the insured persons of the cedents portfolio. The main part of the code are the rows from 10 to 16. Here, the expected loss of the insured person at the specific threshold is calculated. The if-else-clause determines which case of the three possibilities that have been shown in subsection 5.2 has to be applied. The "if" statement deals with the possibilities of a PVP above the threshold or a disability benefit above the threshold from a disability degree below 50%. In the "else" statement, the case that the sum of the disability benefit and the PVP exceeds the threshold from a disability of 50% is covered. The **second step** is to compute the empirical CDF. In **step 3** the MBBEFD distribution and the SwissRe exposure curves are fitted to the empirical CDF. For the fitting of the MBBEFD distribution a R package[7] is used, which applies the Maximum-Likelihood method to get the parameter b and g. As it is not possible to estimate the parameter of the SwissRe exposure curves directly, the least square method is used to get the optimal parameter Y.

Algorithm 1 Calculating the Exposure Curve Parameter

```
Require:
 1: Maximum benefit payment MaxPay
 2: Progression model Prog, disability benefit DB and present value pension annuity
    PVP per insured person
 3: Disability degree distribution
Ensure: Parameter for exposure curve
 4: Step 1: Calculate expected value for every individual
 5:
 6: for Threshold = 0 in 100 Steps until MaxPay do
 7:    for all Insured Persons do
 8:       if Maximum Benefit > Threshold then
 9:
10:          if PVP > Threshold or DB for Disability Degree below 50 > Threshold then
11:             Expected value of PVP with disability degree starting at 50% +
12:             Expected value of DB with disability degree so that DB is above threshold
13:          else
14:             Expected value of [PVP + DB] with disability degree starting above 50%
15:          end if
16:
17:       end if
18:    end for
19:    Total = sum of expected value of all insured person per threshold
20: end for
```

```
Step 2: Compute the empirical CDF

for i = 0 to 100 do
    CDF = 1 - Total[i]/Total[0]
end for

Step 3: Fit MBBEFD distribution and SwissRe exposure curve

Fit MBBEFD distribution with MLE
Fit SwissRe with least square method

print  Parameter for MBBEFD and SwissRe
```

With the individual data per insured person, the risk premium split for the primary insurer and the reinsurer can also be computed directly. Therefore, the code is being adapted to the **Algorithm 2**. In contrast to the original code, only the priority level k has to be predetermined. Moreover, all data requirements are the same as for the Algorithm 1. As the expected loss is evaluated above a certain priority, only one for-loop is necessary to run through all insured persons. The main adjustment can be found within the rows 7 and 12 on the next page. As the reinsurer only compensates the loss excess the priority, the amount k is subtracted (row 8 and 11) by the benefits before calculating the expected loss. The risk premium is just the sum of the excess expected loss from every individual insured person. Furthermore, optionally the severity and the frequency distribution at the priority level can be computed, in order to compare the result with the experience model. The detailed explanation can be found in subsection 2.2.2.

Algorithm 2 Calculating the Excess of Loss Risk Premium

Require:
Priority level k
Progression model $Prog$, disability benefit DB and present value pension annuity PVP per insured person
Disability degree distribution
Ensure: Risk Premium
Calculate Risk Premium at k

for all Insured Persons **do**

  if Maximum Benefit > Threshold **then**

    if PVP > Threshold **or** DB for Disability Degree below 50 > Threshold **then**
      Expected value of $[PVP + DB - k]$ with disability degree starting at 50%
      or with disability degree so that DB is above threshold
    else
      Expected value of $[PVP + DB - k]$ with disability degree starting above 50%
    end if

  end if

end for

print $RiskPremium$ = sum of risk premium for all insured persons at k

Optional: Calculate Severity Distribution and Excess Frequency for model comparison

Procedure as explained in subsection 2.2.2

Frequency at Priority Level P
$freq_P = RP(P) - RP(P + \epsilon)$

Severity Distribution:
$F(x) = 1 - \frac{freq_x}{freq_P}$

5.3. Modelling Results Summary

The results of evaluating the cedents data with the developed exposure model are shown in this section. The first part presents the exposure curve fit for the MBBEDF distribution and the Swiss Re curves. Furthermore, the characteristic of the empirical curve shape is explained by means of the influence of the disability and pension benefit. The second subsection compares the result of the risk premium calculations with an experience pricing and works out the advantages of applying the exposure model for the non-working area.

5.3.1. Exposure Curve Fit

In total, there is detailed portfolio information for four individual insurer available. Observing the benefits, it can be concluded that the insurers offer similar disability basic sums insured, progression models and pension annuities. Hence, a homogeneous portfolio, including all around 786.000 insured persons, can be set up and evaluated by the exposure model. The maximum total benefit is fixed at two million, as only several contracts can exceed this limit. The figure 5.2 below, shows the fit of the MBBEFD distribution and the Swiss Re exposure curve to the empirical exposure curve resulting from the exposure model. The x-axis shows the deductible of the primary insurer in percent of the maximum sum insured of two million. On the other hand, the y-axis presents the risk premium the primary insurer would keep for the deductible in percent of the total premium.

For this model it is assumed that the premium is equal to the expected loss, because any external or internal expenses are not taken into account.

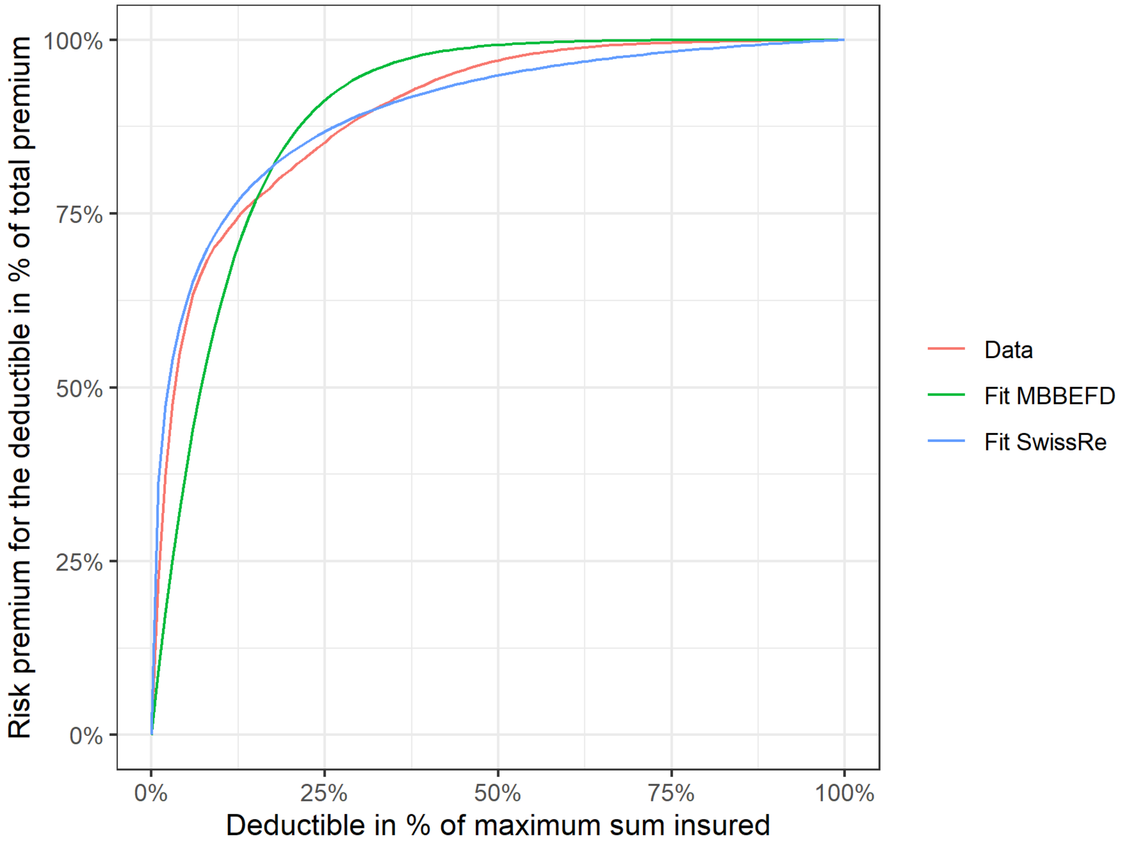

Figure 5.3.: Exposure Model Fit

The most obvious finding is that the empirical exposure curve increases very steep in the beginning and flattens already at around 70%. Thus, already a low deductible means that the primary insurer can keep most of the premium. Furthermore, a deductible of 70% would lead to a retention of 99% of the premium. As a result, one can conclude that the expectation of high benefit payments above €1,4 million have a very little probability. Analysing the fit of the **MBBEFD distribution**, it can be observed that the curve form is even stepper and does not fit the empirical exposure curve. Especially, in the area of high benefits, above one million, almost no risk premium would be allocated. For the fit of the **Swiss Re** exposure curve, a better result can be achieved. The shape of both curves are very similar, but the Swiss Re exposure curve flatten slower and therefore allocates more risk premium to high benefit payments.

To understand the characteristic of the shape of the empirical exposure curve better, the share of the disability benefit and the pension benefit is evaluated separately. The maximum benefit is still set at two million, which can have the effect that the exposure curve flattens earlier as the benefits observed separately are lower. Nevertheless, as it can be seen in figure 5.4, it is very significant that the exposure to the benefits is very different.

In general, the risk amount allocated to the primary insurer is lower for the pension benefit as the curve is always below the empirical exposure curve of the disability benefit. Also, it is clear that the total empirical exposure curve has to lie in between the two curve, as it consists of both benefits. Dependent of the proportion of the disability benefit to the pension benefit, the total empirical exposure curve fits closer to one or the other curve.

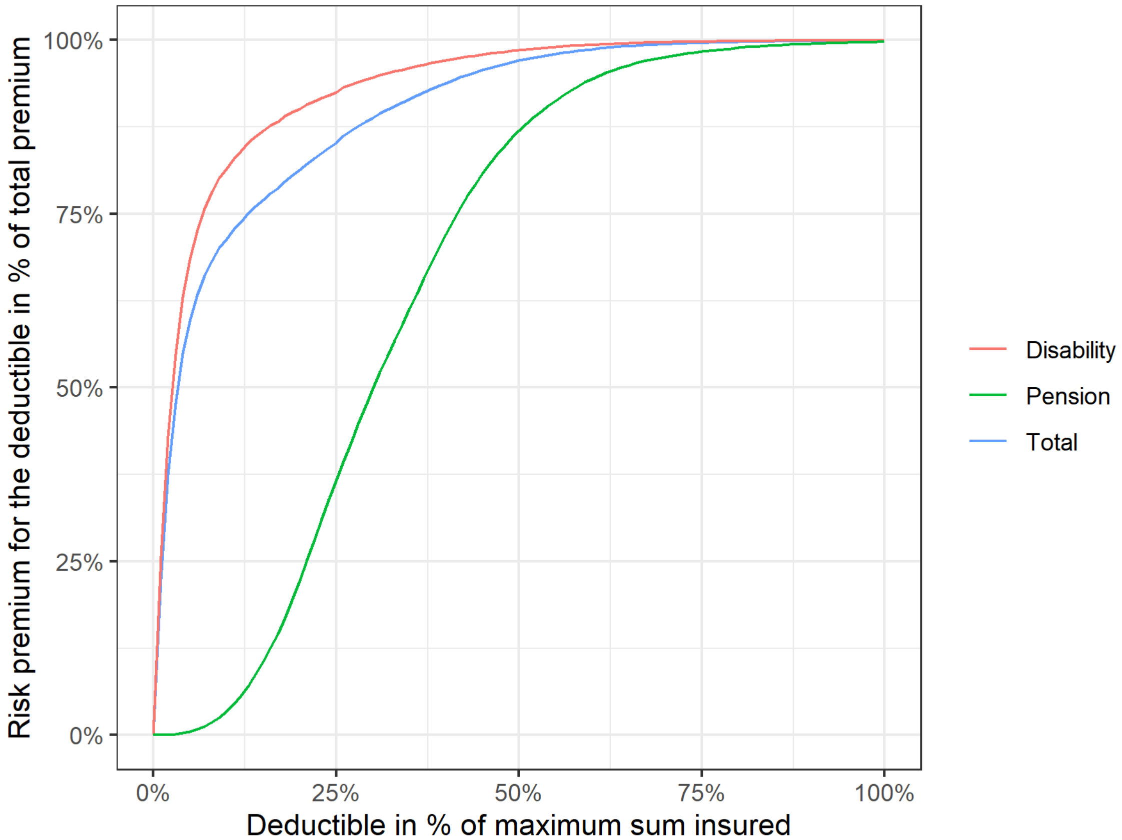

Figure 5.4.: Exposure by Type of Benefit

First, it has to be pointed out that the pension benefit is paid from a disability degree of 50% and the whole benefit is compensated at once by the reinsurer. Therefore, the shape of the exposure curve is very smooth and even reminds of the normal distribution. To understand the shape of the curve, it has to put in context with the distribution of the present value of the pension benefit, which is presented in the histogram in the figure 5.5. Here, the x-axis shows the sum insured and the y-axis the total count. The flat increase in the beginning can be explained because there are only a few insured persons with a low pension benefit. It results from the fact that even if the pension annuity is low, the PVP is dependent on the age and can reach therefore a higher amount. Furthermore, the steep increase between 10% and 70% is a consequence of the distribution of the pension benefits. Most of the PVPs are located in the interval between €70.000 and €700.000. The curve flatten slower as there are still several benefits up to two million.

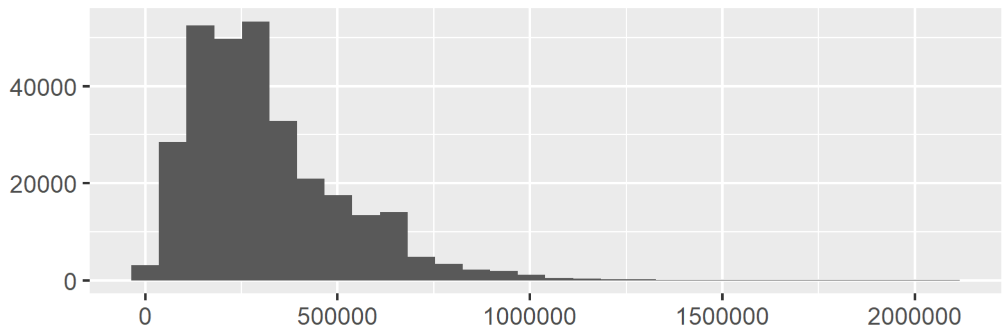

Figure 5.5.: Distribution of Present Value of the Pension Annuity Benefit for all Cedents

The exposure to the disability benefit shows a completely different shape. Considering the aspects that the compensation increases with the disability degree and the distribution of the disability degree, the shape of the exposure curve can be explained. High compensations are reached with a very low probability. Therefore, the empirical exposure curve rises very steep in the beginning and hence a high amount of the premium can be kept by the primary insurer with a low deductible. Observing the distribution of the disability benefit, it can be noticed that most of the sums insured are between €400.000 and €800.000. Therefore, proportionally less risk premium is allocated to the primary insurer for a deductible between 10% and 60%. The maximum disability benefits are higher than the pension benefits in general and can reach up to €6.000.000. Nevertheless, the low probability of a full compensation lead to a flattening of the exposure curve. As a result a deductible of 60% make up already 99% of the risk premium.

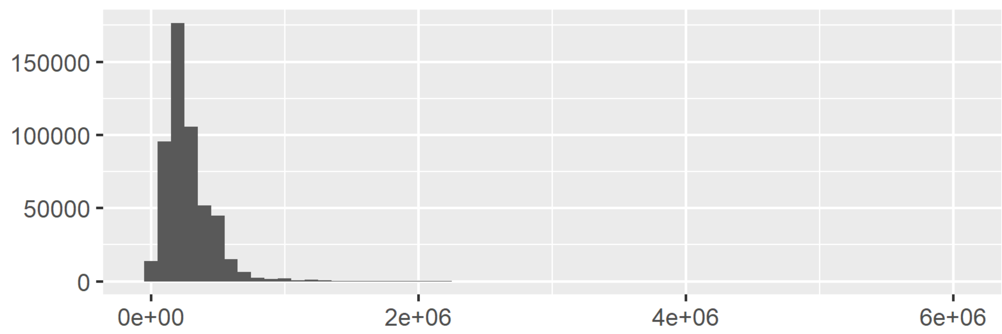

Figure 5.6.: Distribution of Disability Benefit for all Cedents

An overview of the results of evaluating the insurers individually is provided in table 5.5. As the MBBEDF distribution did not provide a convenient fit to the empirical exposure curve, it will not be considered as possible exposure curve. For the insurer 2 and 3, a maximum sum insured of €2,8 million is chosen because these are the only portfolios where several contracts exceed two million. Additionally to the Swiss Re parameter, the proportion of the disability benefit to the pension benefit is presented. The Swiss Re parameter is ranging from 4,3 to 5,7 which is notable higher than for other lines of business. Indicating that in general, the primary insurer need a lower deductible to keep most of the premium. Observing the proportion of disability benefit to pension benefit, the tendency can be noted that a higher share of pension benefits lead to a lower Swiss Re parameter. The reason is that the expected compensation of pension benefit is higher.

	SwissRe	Proportion Disability	Proportion PVP
All Insurer	5,26	65,7%	34,3%
Insurer 1	5,71	68,2%	31,8%
Insurer 2	4,26	59,8%	40,2%
Insurer 3	5,56	74,7%	25,3%
Insurer 4	4,54	55,1%	44,9%

Table 5.5.: Summary Evaluation of the individual Insurer

To show an example where the fit is more problematic, the results of the insurer 4 are presented in the figure 5.7. The Swiss Re exposure fit would allocate the risk premium differently and especially would shift more risk from the middle part to a higher deductible. The shape of empirical exposure curve is not as smooth as it could be observed before. Particularly in the beginning, after a steep increase the curve has a convex form between a deductible of 10% and 40%. Moreover, the exposure curve flattens already at around 60%.

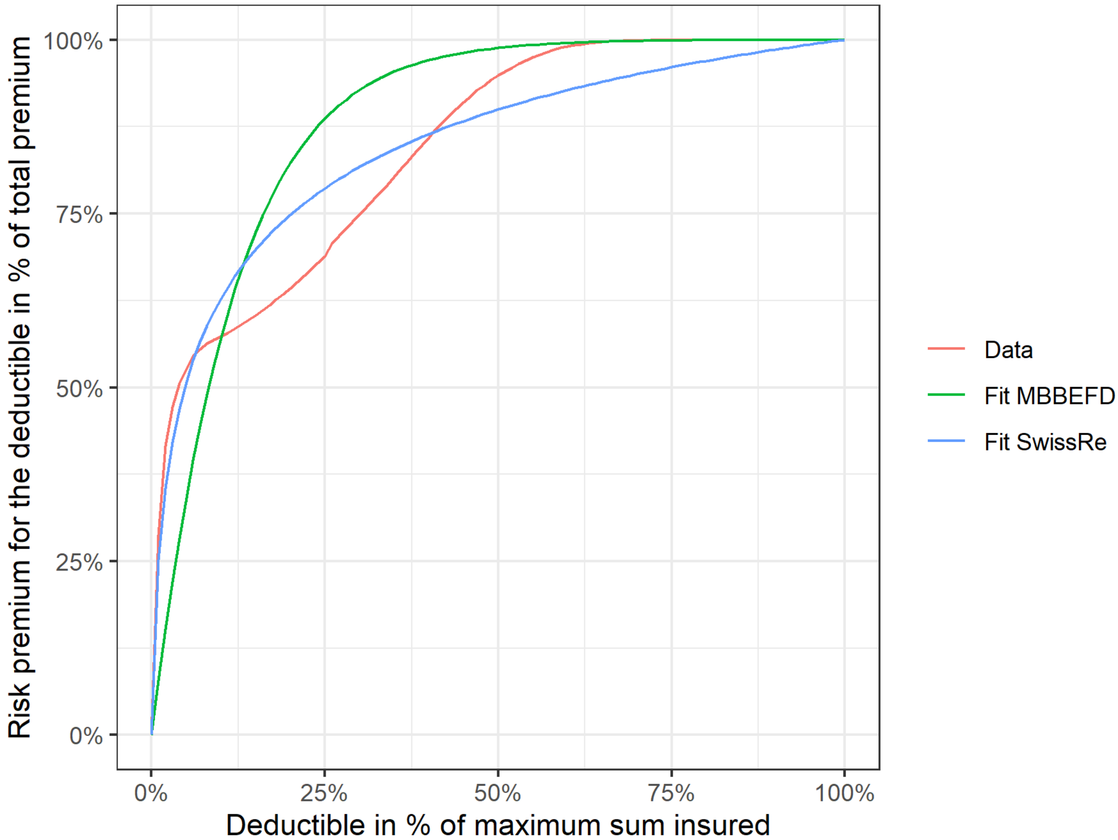

Figure 5.7.: Exposure Curve Fit Insurer 4

Evaluating the exposure of the share of the disability benefit and the pension benefit separately, it can be observed that the empirical exposure curve is driven by two effects. First of all, the exposure curve of the disability benefit increases linear between a deductible of 10% and 40%, which can be led back to the distribution of the disability benefit. In the figure 5.9, it can be seen that the disability benefit peaks between € 500.000 and € 600.000, which is much higher than in the other portfolios. The convex part is a result of the high proportion of the pension benefit in comparison to the total benefit.

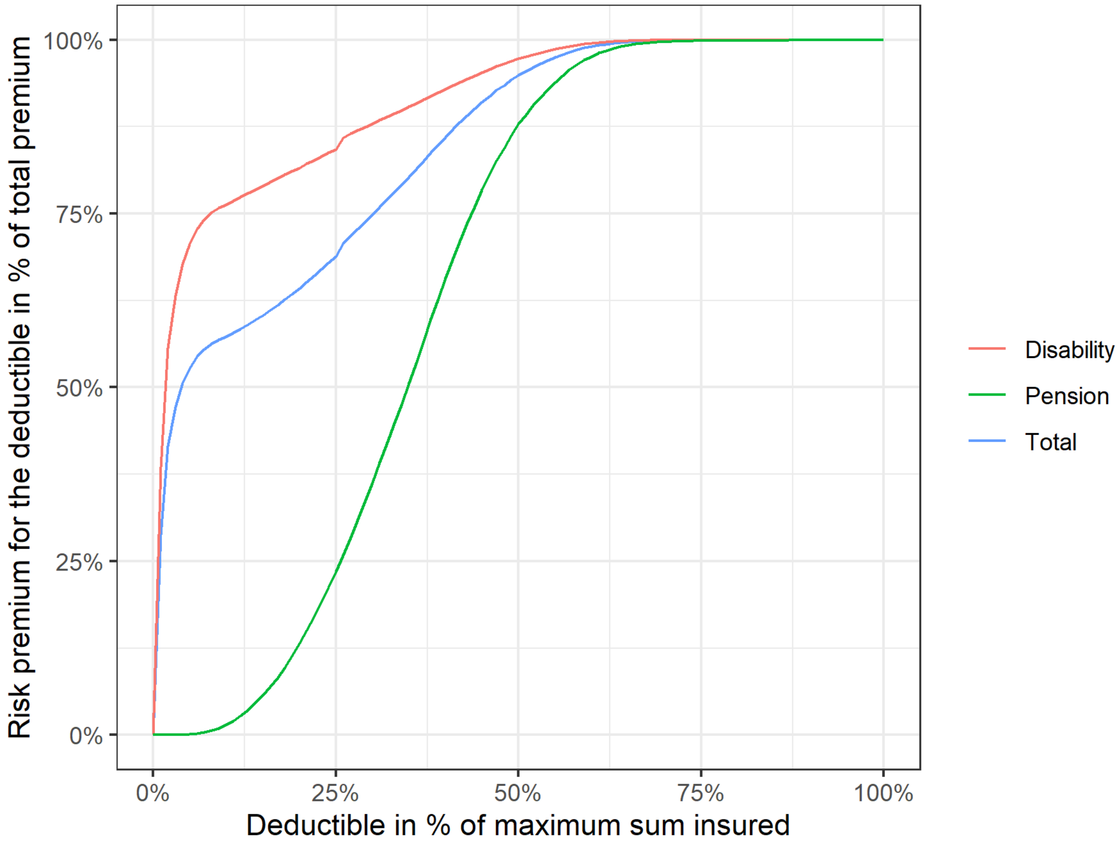

Figure 5.8.: Exposure by Type of Benefit Insurer 4

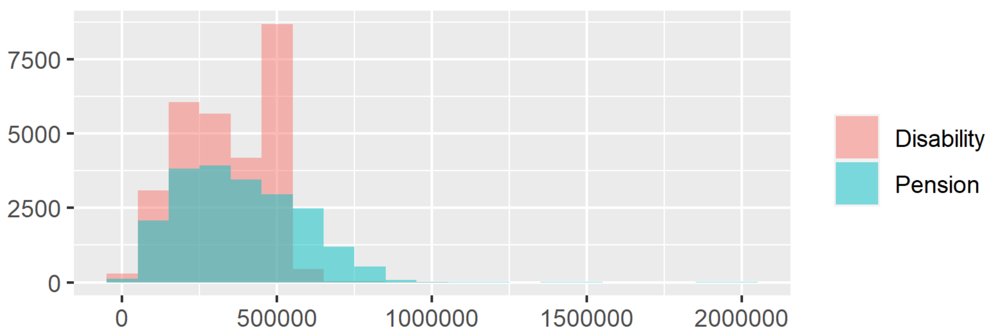

Figure 5.9.: Distribution of Disability Benefit and PVP for insurer 4

5.3.2. Comparison Exposure vs. Experience Modelling

This last sections compares the pricing result of the exposure model, with the experience model. In particular, the excess frequency functions are set against each other. For the exposure model the empirical frequency is obtained by computing the risk premium at several threshold as explained in the Algorithm 2. The excess frequency from the experience model is derived by the frequency and the severity distribution that have been chosen. Moreover, a blended excess frequency function is presented to show how both models can be combined to get a more adequate pricing model. Therefore, a merging function is initialised to blend the two functions between the transitions points. More precisely, the experience frequency $f_1(x)$ is taken until the first transition point θ_1, further the blended function until the second transitions point θ_2 and afterwards the excess frequency $f_2(x)$ of the exposure model is applied. Dependent on the transition points, the exposure or the experience frequency is weighted heavier. As the combined function is decreasing, it has to hold that $f_1(\theta_1) > f_2(\theta_2)$. The function to merge both curves is defined as following.

$$f_{\text{comb}}(x) = \begin{cases} f_1(x) & x < \theta_1 \\ f_1(\theta_1) + [f_1(x) - f_1(\theta_1)] \cdot \frac{f_1(\theta_1) - f_2(\theta_2)}{f_1(\theta_1) - f_1(\theta_2)} & \theta_1 < x < \theta_2 \\ f_2(x) & x > \theta_2 \end{cases}$$

For the comparison, the results of the insurer 1 and 4 are used. Both primary insurer offer contracts with benefits up to two million. Therefore, the modelling is applied on an Excess of Loss contract with a threshold of € 400.000 and a limit of € 1,6 million. But the experience pricing of the portfolios is very different to each other as the loss history varies significantly.

The **insurer 1** has a much more bigger PA portfolio containing 294.000 risks. Hence, the primary insurer had to deal with a higher amount of losses and can provide a more extensive loss history. In total 2.500 losses are reported, where 107 are above the model threshold of € 400.000. The index to re-evaluate the premium and the losses are derived by the developments presented in chapter 3. Hence, the highest re-evaluated losses are around € 1,6 million. As the loss information is comprehensive, an adequate experience pricing can be set up. For the frequency, the Poisson distribution is chosen with a parameter $\lambda = 3,4$. Fitting the severity distribution to the empirical losses, the Log Gamma distribution with parameters $\alpha = 3,46$ and $\gamma = 1,46$ achieves the best model badness. To get a smooth transition from the experience to the exposure excess frequency, the transition points are set at € 1,2 million and € 1,8 million. In the figure 5.10 below, all three excess frequency functions can be seen.

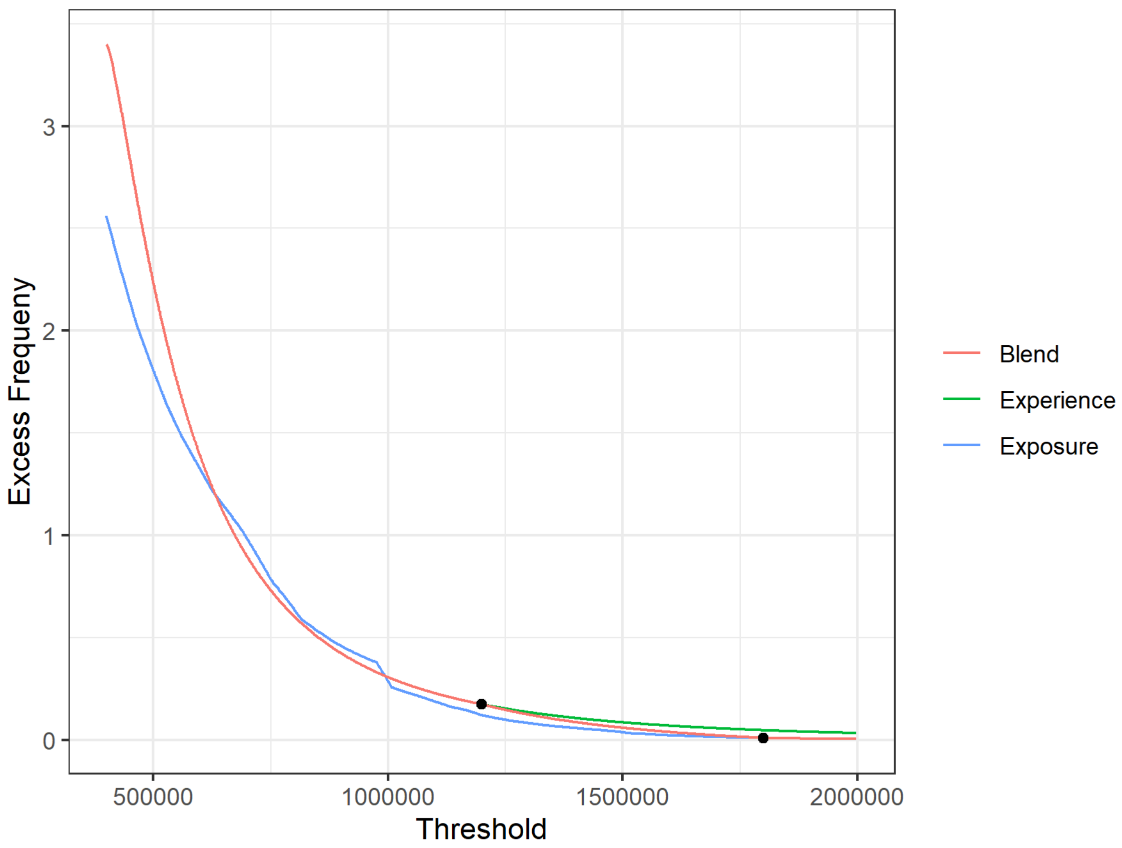

Figure 5.10.: Comparison Excess of Loss Frequency Insurer 1

It can be observed that the experience and the exposure functions intersect several times, but in general are close to each other. The table 5.6 gives an overview on how the experience and the exposure model would allocate the risk premium in comparison to the blended model. Observing the risk premium for the whole contract, it is remarkable that the experience model would charge a higher risk premium than the exposure model. As the first transition point is set at €1,2 million, the experience function is weighted higher and therefore the blended risk premium is closer to the experience risk premium. One reason for the wide spread of the risk premium is the risk premium allocation for the non-working area. The exposure model would charge only one fourth of the experience model, as the underlying portfolio is taken into account and only few contracts can reach high losses above €1,6 million.

Priority	Limit	Experience	Exposure	Blended
400.000	1.600.000	846.677	727.481	825.774
1.600.000	400.000	20.015	4.901	6.485

Table 5.6.: Risk Premium Distribution per Model for Insurer 1

The next evaluation provides a good example why an additional exposure model leads to a more adequate pricing of a PA portfolio. Therefore, the **insurer 4** is going to be analysed. The portfolio contains around 36.000 risks and hence only 224 losses are reported. The model threshold is set at €200.000 and 59 re-evaluated loses exceed that amount. As the highest loss is at €1,2 million, it is more difficult to build an accurate experience pricing model. Nevertheless, a Poisson distributioin with parameter $\lambda = 2,19$ is obtained and for the severity, again the Log Gamma distribution with $\alpha = 3,19$ and $\gamma = 2,57$ provides the best model badness. For the transition points, €800.000 and €1,4 million are selected. The results of the excess frequency are presented in the figure 5.11.

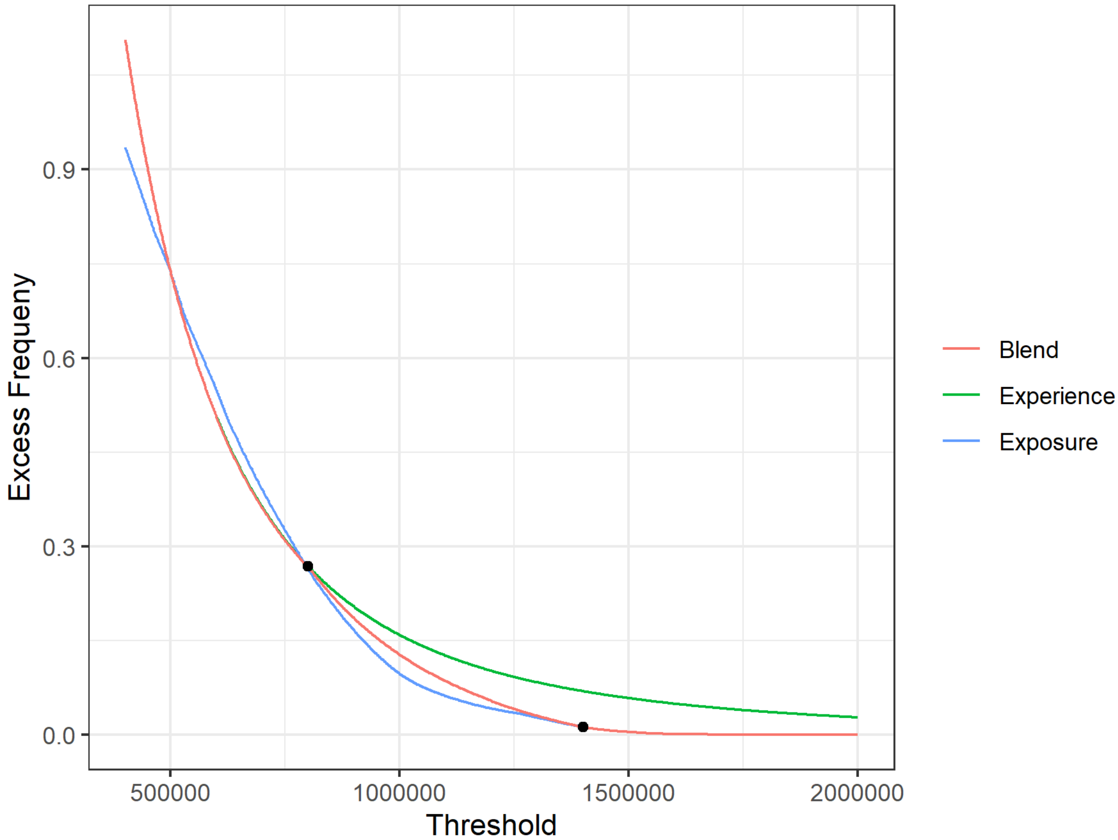

Figure 5.11.: Comparison Excess of Loss Frequency Insurer 4

The summary of the risk premium allocation in the table 5.7, shows the advantages for blending the experience and the exposure model. The excess frequency of the experience model is always above the exposure model from the first transition point. As there is no loss experience above €1,2 million, it is very insecure to model the non-working area up to €2 million. Examining the risk premium for the whole contract, there is a difference of around almost €60.000 between both models. This can be mostly led back to the risk allocation for the non-working area. As it can be observed in the second row of the table, that the charged risk premium for the area of losses between € 1,2 million and € 2 million, is almost € 40.000 lower for the exposure model. As a result, blending both excess frequency functions brings in the advantages of both models.

Priority	Limit	Experience	Exposure	Blended
400.000	1.600.000	337.230	280.228	289.724
1.200.000	800.000	43.773	6.884	7.782

Table 5.7.: Risk Premium Distribution per Model for Insurer 4

6. Conclusion

The aim of this thesis was to develop an exposure-based pricing model for personal accident reinsurance contracts. Examining the data of the primary insurers provided by SCOR, the portfolio information was mostly given as risk profiles. But for four primary insurers detailed information for every insured person was available and therefore an accurate model to compute the risk premium could be set up. Furthermore, with the individual portfolio information, parameters for the Swiss Re exposure curve could be estimated to price reinsurance contracts where only the risk profiles are available.

Another objective of the thesis was to examine the German personal accident insurance market to identify trends of the loss development. It could be figured out that the combined ratio is constant over the last year but the loss ratio was increasing. It is a result of an increasing claim frequency and severity due to higher benefits and a widening of the scope of the cover by the primary insurer. Additionally, it could be discovered that the insurers face a significant ageing of their portfolios, which lead to higher and more probable compensations.

To compare the market study with the SCOR's clients, a large loss experience model was set up. Therefore, the cedents got split up into three groups according to their premium income and the amount of benefits. The results did not coincided exactly with the market observations. For the small and middle size insurer it could be seen that the frequency and severity have a stable development since 2005. For the big insurer even a decreasing frequency trend could be observed. Hence, the increasing claim expenses observed on the market are not driven by the large losses. Furthermore, a comparison of the Long Tail and Short Tail experience pricing have led to the conclusion that the claims severity and frequency develops within the first three years significantly. Therefore, the re-evaluation of the last three years has to be adapted in the Short Tail model. In the last part of this chapter, the problems of modelling personal accident reinsurance with an experience approach were examined. Especially, for cedents with little loss history, it could be shown that the losses line up vertically as contracts with the same sum insured are offered. Hence, it is not accurate to fit a distribution to the empirical losses.

For developing the exposure model it could be drawn on the information of 786.000 insured person of four different insurers. Furthermore, a SCOR internal study provided the disability degree distribution, which is fundamental for the developed exposure approach. The idea was to calculate the expected loss for every individual insured person and evaluate the total expected loss for the primary insurer at several thresholds. As a result, an empirical exposure curve solely based on the primary insurers portfolio was received. Next, the MMBEFD distribution and the Swiss Re exposure curve were fit to the empirical data. It could be concluded that the Swiss Re exposure curve provides the best fit to the empirical exposure curve. Evaluating all insurers, the Swiss Re parameter ranges from 4,2 to 5,7. Thereby, it could be discovered that the parameter is dependent on the proportion of disability benefits to pension benefits. As conclusion, the proportion can be used as controlling parameter and a higher proportion of pension benefits indicates a lower Swiss Re parameter. For the portfolios with the individual data, another approach was developed to calculate the risk premium and the excess frequency above a threshold. Thus, a direct comparison between the experience pricing and the exposure based pricing could be set up. As a finding it could be observed that, especially for the non-working area the exposure approach provides a more adequate pricing. Thus, this additional pricing approach can be used in connection with the experience approach to receive a more accurate pricing for personal accident reinsurance contracts.

To conclude, the findings from the chapter 3 are brought into context with the results of the exposure modelling. Similar to the market development, the analysed primary insurer have a stable or slight decreasing premium income. Hence, the exposure to the reinsurer is shrinking and less large losses are expected in the future. Moreover, it was mentioned that the primary insurer offer higher disability benefits with progression models up to 1000%. Observing the portfolios, it can be concluded that the progression model 1000% makes up only 1,7% of all the progression models and therefore has no huge impact on the exposure yet. Also, there are only 583 insured person with a maximum benefit above two million. A more significant influence on the exposure pricing has the distribution of the disability degree. So far, only a market observation can be used as no individual distribution per primary insurer is available. Another chance to improve the exposure approach is to receive the accident probability for personal accident insured person per primary insurer. But these improvements can only be included if the primary insurer provides the relevant data.

A. Comparison of Occupational Disability and PA Insurance

The figure A.1 shows a comparison of the market penetration of the occupational pension and the private accident insurance in Germany. As the occupational pension is a rather new product, less older people signed this kind of contract. Nevertheless, the occupational pension is gaining market share, especially within younger age groups.

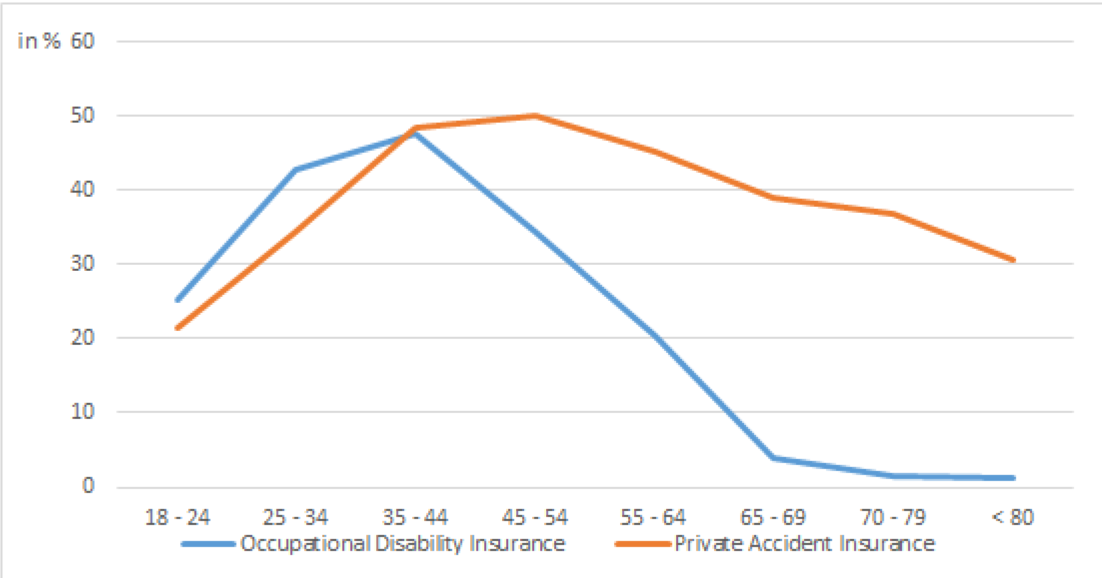

Figure A.1.: Market Penetration Comparison of Occupational Pension and PA

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Statutory Declarations

I declare that I have authored this thesis independently, that I have not used other than the declared sources, and that I have explicitly marked all material which has been quoted either literally or by content from the used sources. I furthermore declare that this thesis has not been submitted to any other board of examiners yet.

Frankfurt, 31.08.2020

(Signature)