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# ALM Model: The contribution of Fuzzy Logic to behaviour modelling

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## *Abstract*

*The reforms ushered in by Solvency 2 introduce a new method for the valuation of insurance company balance sheets. The principle of "fair value" is now used to measure assets and liabilities: asset values are calculated at market value, while for liabilities the Best Estimate method is used. In life insurance and, more specifically, in the life insurance savings segment, it is necessary to model all possible interactions between the asset portfolio and the portfolio of liabilities. ALM (Asset Liability Management) models seek to forecast all of these interactions by integrating – over and above the usual financial and technical assumptions – assumptions pertaining to the behaviour of both policyholders and management. These behavioural laws, such as the laws governing policy surrender, for policyholders, and the policy governing credited interest rates, for management, can lead to problems in terms of their calibration, their modelling and, more broadly, their justification.*

*Given the decisive impact of behavioural laws when it comes to establishing Best Estimate values for liabilities, the objective of our study is to propose an alternative to the modelling of behavioural laws that is traditionally implemented within ALM models. The focus of our interest is a theory founded on human logic that has been widely tried and tested in other sectors of activity, such as manufacturing and heavy industry: fuzzy logic.*

*In the course of our study, we use fuzzy logic to model the behaviour of policyholders when it comes to economic lapses and that of insurers when it comes to the target rate credited to policyholders. The model we have built allows us to end up with a quantified decision while also simulating a human thinking process where the criteria are expressed in linguistic ways. Our results are encouraging. We demonstrate that the fuzzy approach allows us both to justify and generalize the ACPR's economic surrenders function and that a fuzzy modelling of the policy governing credited interest rates allies optimization and an accurate representation of management's policy.*



[1] The expected or mean value (probability weighted average) of the present value of future cash flows to settle contract obligations, projected over the contract's runoff period, taking into account the time value of money, and using a term structure of interest rates (i.e., a yield curve).
[2] Autorité de Contrôle Prudentiel et de Résolution. The ACPR is responsible for supervising the banking and insurance sectors in France.

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# 1 Traditional behavioural modelling methods in an ALM model

In order to carry out this study, we had to implement an ALM model that was able to determine the Best Estimate values for a fictitious life insurance company and, more specifically, for a simple life insurance savings vehicle.

## 1.1 Asset-liability interactions

Within the context of our life insurance company, the asset-liability interactions arise from the options and guarantees that accompany the savings vehicle and that are directly linked to the insurer's assets and the way the insurer manages them.

For example, the profit participation clause is a guarantee that gives the policyholder access to a minimum percentage of the interest earned by the insurer on its invested assets. The amount that will be made available to policyholders under this clause will thus depend on the structure of the insurer's assets (composition, maturity, risks, etc.) and its evolution over time. The valuation of the insurer's obligation to its policyholders cannot be dissociated from the insurer's future decisions concerning the management of its assets.

A Best Estimate valuation of liabilities must necessarily include the fair value values of the options and guarantees in play. To this end, it is preferable to use simulation techniques via an ALM model.

Below is a simplified example of the valuation of the minimum guaranteed rate (MGR) derived using a deterministic or stochastic approach :

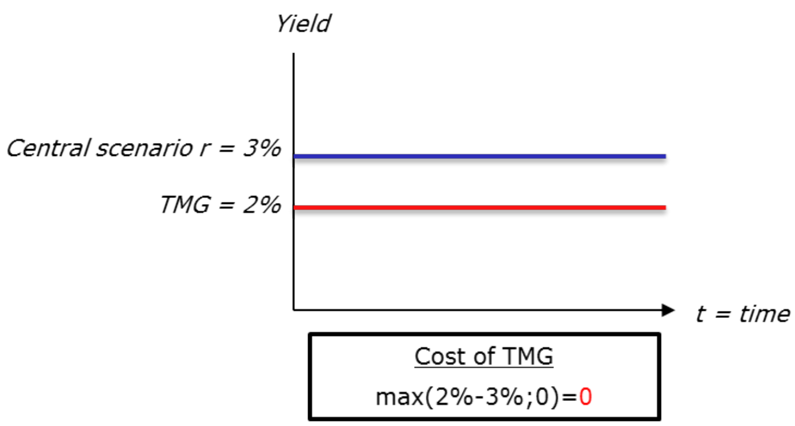



**FIGURE 1 –** Deterministic approach

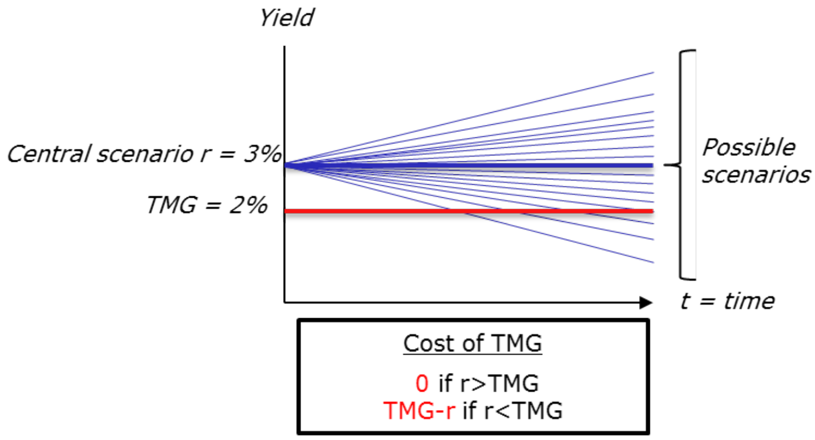



**FIGURE 2 –** Stochastic approach

Based on an average situation, a deterministic scenario does not correctly assess the cost of an MGR guarantee, since in this unique scenario the insurer is able to honour its obligation via the return on its assets. Conversely, using a stochastic approach [1], we are able to measure the cost of the guarantee at its fair value [2].

## 1.2 ALM model

The use of an Asset-Liability-type forecasting model is essential for capturing all of the asset-liability interactions. The simulation technique used in the ALM tool produced in the course of our study is the Monte-Carlo method, which is based on the Law of Large Numbers. As a reminder, this method involves running a large number of scenarios, generated independently, in order to obtain an approximation that is close to the true Best Estimate value.

1. All scenarios, determined at random, around an average.
2. For a sufficient number of scenarios

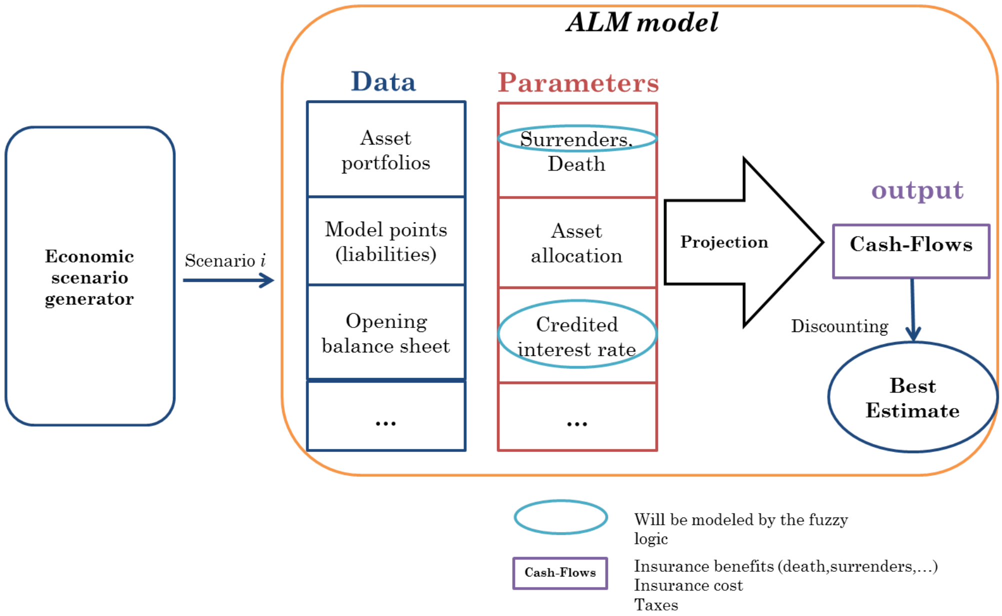



**FIGURE 3 –** ALM Model

The ALM tool works in the following manner. To start, the risk neutral economic scenario generator generates 1000 economic scenarios that it transmits to the heart of the ALM tool, whose role is to project, for each scenario, the company's activity over the forecasting horizon[3]. To do so, the model needs data pertaining both to the company's assets (structure, allocation, etc.) and its liabilities. The policyholders are grouped into homogenous categories, usually called the model point. In addition, the model requires a certain number of assumptions, some of which pertain to the behaviour of policyholders (laws of surrender, mortality, etc.) and others related to the decision-making on the part of the insurance company's management (asset allocation, credited interest rate policy, etc.). After making business projections, the model produces the insurance company's cash flows, which, after they have been discounted at the risk-free rate[4], give us the Best Estimate for the scenario that has been generated. Using the law of large numbers, the company's Best Estimate is the average of the scenarios generated.

$$\text{Best estimate} = E^{Q\otimes P}\left(\sum_{n=1}^{30} \delta_n \times C_n\right) \approx \frac{1}{1000}\sum_{i=1}^{1000}\sum_{n=1}^{30} \delta_n^i \times C_n^i$$

Behavioural modelling occupies a central place in ALM models. However, despite having a non-negligible impact on Best Estimate valuation, choosing the behavioural models and justifying these choices poses a number of problems. This is the case in particular for the two behavioural laws introduced in the following two subsections.

3. 30 years per assumption.
4. As a reminder, the Best Estimate value is determined in a risk neutral universe.

## 1.3 Economic lapses

### 1.3.1 Lapses' option

Lapse is an option embedded in the policy that allows the policyholder to recover, when he or she wishes, all or a portion of his or her savings. There are two kinds of surrenders :

- **Structural lapses**

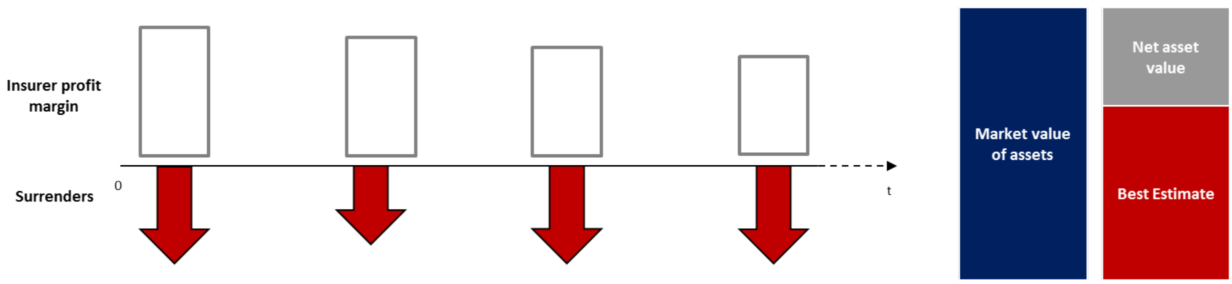
Structural surrenders are surrenders that are explained by the structure of the insurer's liabilities (policy clauses, age of policyholders, policy terms).

- **Economic lapses**

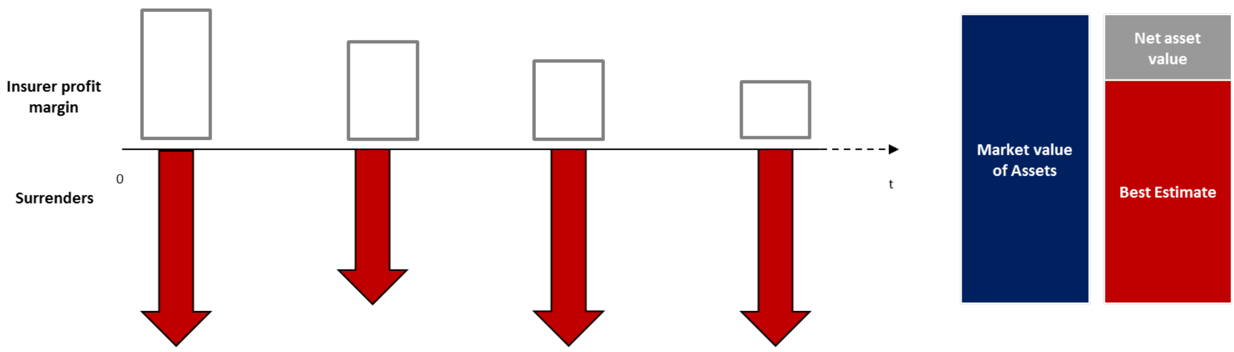
Economic or cyclical lapses are surrenders that are linked to the economic situation, and more specifically to how the insurance company is doing in this situation.

### 1.3.2 The challenge and importance of estimating surrenders under Solvency 2

In savings, the insurer's margin over the term of the policy is related to its ability to hold onto the policyholder's invested savings over the long term. In fact, the larger this amount the bigger the base of investment income to be shared between the insurer and the policyholder.

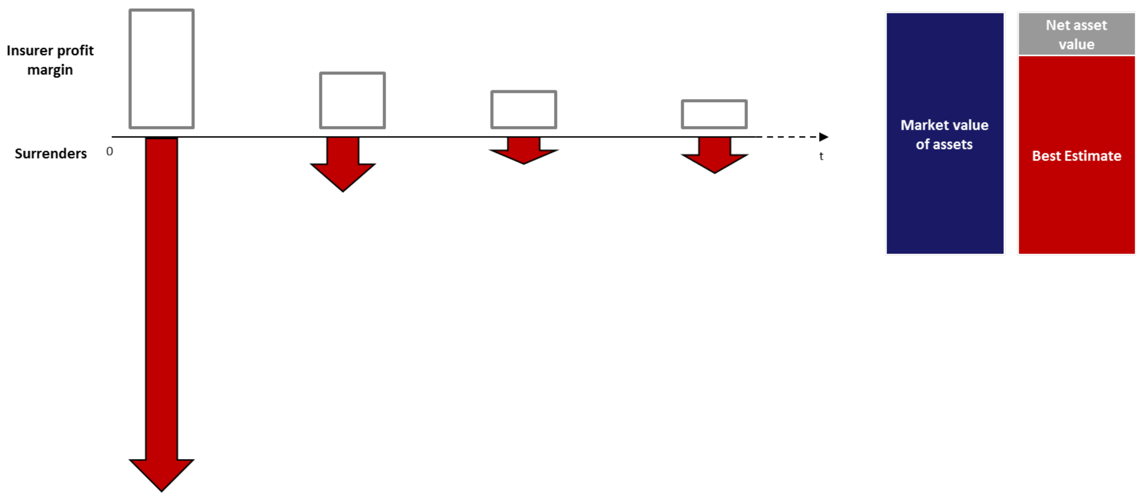



When the invested assets decrease substantially over the term of the policy, for example via an increase in the pace of surrenders, the insurer's investment margin naturally decreases as well and, as a result, its Net Asset Value diminishes while the Best Estimate value of its liabilities increases.



The distribution of the initial wealth between the insurer (Net Asset Value) and the policyholder (Best Estimate) depends on the insurer's ability to generate investment income from the premiums paid in by its policyholders. Consequently, the surrender risk is an important one for the life insurer because a direct increase in outflows through surrenders will have a significant impact on its Net Asset Value.

Indeed, the surrender risk is highlighted in the Solvency 2 regulatory framework for the calculation of economic capital. The massive surrender[5] shock penalizes insurers severely by reducing their invested assets at the start of the forecast and, by the same token, having a considerable impact on its future margins.




The risk of massive surrenders is thus one of the most important that life insurers face, especially in terms of the regulatory capital to immobilize. It is important for insurers to really understand surrender behaviour patterns and model them correctly.

### 1.3.3 Modelling economic lapses

When modelling structural lapses, insurance companies can generally count on having enough data about the past to define a surrender law. Conversely, estimating economic surrenders is a far more complex task.

Economic surrenders are related to market conditions. Modelling these surrenders means taking into account policyholders who, in reaction to the economic situation, will cash out their policy if they think they can get a better return by investing in a competing product. Modelling these economic surrenders poses a problem for insurance companies because insurers do not have historical data[6] or concrete elements that would allow them to identify a law.

5. Mass surrender of 40% of the policies.
6. For a long time, insurance companies generally followed the market in terms of interest credited to policyholders,

In $2010$, the ACPR published additional national guidelines for QIS $5$, a law of economic surrenders based on the gap between the credited interest rate offered by the insurance company and the rate offered by the competition.

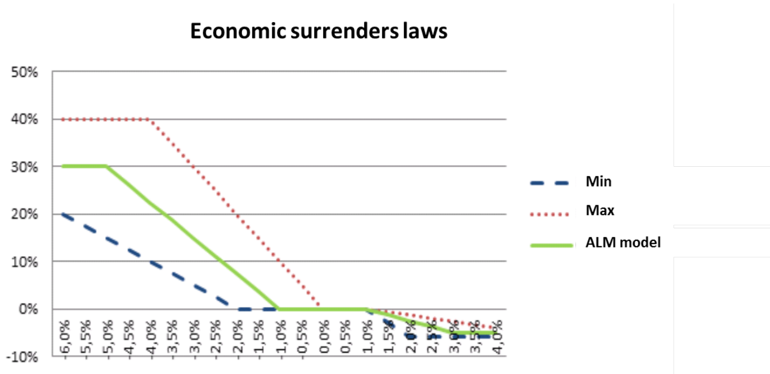



**FIGURE 4 –** Economic surrenders laws

$$
RC = \begin{cases} RC_{max} & \text{si} \quad TS - TA < \alpha \\ RC_{max} \times \frac{TS-TA-\beta}{\alpha-\beta} & \text{si} \quad \alpha \leq TS - TA < \beta \\ 0 & \text{si} \quad \beta \leq TS - TA < \gamma \\ RC_{min} \times \frac{TS-TA-\gamma}{\delta-\gamma} & \text{si} \quad \gamma \leq TS - TA < \delta \\ RC_{min} & \text{si} \quad TS - TA \geq \delta \end{cases}
$$

On the basis of $6$ parameters, the ACPR proposes two economic surrender laws, one of them corresponding to a minimum floor and the other to a maximum floor for surrenders. It is recommended that insurers calibrate their law in the interval between this minimum and maximum threshold.

The surrender law that the ACPR proposes represents a first approach to modelling economic surrenders, but it has the following limitations :

- A lack of elements that could be used to calibrate the surrender function and, more broadly, its justification. Insurers don't know how and on what to calibrate this ACPR function. There are no concrete elements or data that would allow them to justify one calibration over another.
- A single criterion that triggers surrenders : the gap in rates of return. The economic surrender function proposed by the ACPR thus assumes that policyholders are rational.
- A single surrender law for everyone : all policyholders, regardless of their age, the value of their investment, etc., react in a similar way to this gap in credited interest and adopt the same surrender behaviour. But even though this gap is indeed the main driver of economic surrenders, it is not absurd to think that other factors may motivate policyholders to cash out their policy or not.

not factoring in economic surrenders at all. While today some insurers do take economic surrenders into account, they remain difficult to capture, in particular when it comes to measuring the impact of adverse deviations of these surrenders under extreme case scenarios.

We have identified other factors that help to explain the economic surrender dynamic. More specifically, these factors are viewed as elements that can influence the sensitivity of a policyholder to the observed gap in credited interest rates between his or her insurer and the competition.

| **Factors** | **Impact on sensitivity to spread in interest rates** |
|---|---|
| **Age of the policyholder** | Generally, young policyholders have a lower amount invested, do not respond to market fluctuations, and are more interested in the long-term return on the investment. Consequently, they pay less attention to the amount in savings and are less sensitive to gaps in interest rates. |
| **Seniority of contract** | If seniority of the contract is small, policyholders are less inclined to surrender than policyholders who have held their policy for a long time, as the latter are eligible for the tax advantages associated with life insurance. |
| **Policyholder inertia** | Policyholders will not necessarily surrender their policy if their insurer has one bad year but may do so if the returns offered by the latter are consistently lower than the competition over many years. |
| **Acquisition fees Or Surrender penalty** | Some policies contain clauses that call for penalties for surrender ; in some cases the acquisition fees on premium payments can make it unprofitable to cash out a savings policy in order to sign up with the competition. |

In the course of our study, we used the following as factors that trigger economic surrenders : gap in credited interest rates, the age of the policyholder, and the number of years the policy has been in force.

## 1.4 The credited interest rate policy

### 1.4.1 Why credit interest to policyholders ?

It is in a life insurance company's interest to offer an interest rate that satisfies policyholders in order to avoid having to face economic surrenders. The policy on crediting interest to policyholders, which is decided on by company management, must be re-transcribed into the ALM model in the same way other relevant company policies are - asset allocation, for example, and the policy on harvesting unrealized capital gains. This is what is generally referred to as the modelling of the insurer's behaviour or the modelling of management actions.

### 1.4.2 Modelling management actions

Generally speaking, in ALM models these actions are represented in the form of either a static function or an optimization problem under restricted circumstances.

For example, standard, static-format modelling of the credited interest rate target may be expressed in the following manner :

$$Taux_{Servi_t} = 100\% \times Taux_{Concurrent_t}$$

Thus, for the period where the insurer decides to adjust the value of the savings of its policyholders [7], the tool will seek to adjust at a minimum the interest rate or return offered by the competition. We should signal, however, that whether this objective is achievable will depend above all on the company's financial resources (investment gains, etc.) and the insurers obligations to the policyholder (minimum guaranteed rate, etc.).

Like the modelling of economic surrenders, though, the traditional modelling of management actions often shows limitations.

It is often observed that the management of an insurance company is not involved in this modelling process and that the task is most often left to actuaries and developers. Be that as it may, management remains responsible for its expression inside the company, whether that takes the form of written policies or a forecasting model.

The more heavy and complex the modelling, the more removed management will be. In addition, this modelling responds to optimality criteria that are not necessarily verified in reality, particularly in a stressed universe. We need to remember that a decision made by a human being is not always the most optimal and has a subjective component.

Moreover, if a simplified, static approach is adopted, the modelling strays considerably far from reality and can above all lead to inconsistencies in the models that will impact the Best Estimate valuation.

This first section has demonstrated the limits and problems that arise in connection with the modelling of behavioural laws in an ALM model. Our study consists of proposing an alternative to the standard modelling of behavioural laws presented above, based on fuzzy logic. More specifically, we apply the theory of fuzzy logic to the modelling of policyholder behaviour (the laws of economic surrender) and a management action (the credited interest rate policy).

## 2 Fuzzy logic, alternative modelling of the behaviour of human beings

The human being is confronted with complex problems that he or she must resolve with the help of approximate data. The theory of fuzzy sets allows us to define a robust conceptual framework for these methods of reasoning. Rather than modelling the behaviour of the human being using precise, numerical values, fuzzy logic allows us to describe approximate variables in a qualitative way. The aim of our study is to roll out a new way of modelling human behaviours like surrenders,

7. Generally at the end of the year or the end of the quarter.

but also management decision-making, via fuzzy logic. The ALM model that we previously built can be used to test and compare the theory with traditional methods.

## 2.1 Why fuzzy logic ?

Most of the problems with which human beings are confronted can be modelled mathematically, but is this truly a realistic way to represent reality ? Real world problems sometimes require imprecise and uncertain information. For example, let's take the behaviour adopted by an individual as she approaches a three-coloured traffic light. The human being will not say to herself, "if the light is red, if I am less than $50$ meters from the light, and if my speed is $46.52$km/h, then I will hit the brakes." Rather, she says, "if the light is red, if I am close to the light, and if my speed is average, then I will slowly apply my brakes." The fact of assessing the data in a way that is approximate and imprecise and not in a way that is strictly quantitative is fuzzy reasoning. Thus, the human brain works on fuzzy logic.

This fuzzy approach was first developed in $1965$ by Lotfi A. Zadeh, a professor at the University of California, Berkeley, from his theory of fuzzy sets generalizing the mathematical theory of conventional sets. Lotfi A. Zadeh factored in the imprecisions and uncertainties of human reasoning by allowing an element to belong to a set not with absolute certainty but rather with a certain degree of membership.

Numerous applications of fuzzy logic were developed in consumer appliances and electronics, where imprecision in the data renders automation using the conventional methods impossible. It was in Japan that fuzzy logic really took off in a big way. Starting in $1980$, washing machines with no dials and cameras with auto-focus using the fuzzy approach began to appear. Having proven its worth in other areas, such as finance and medical diagnosis, fuzzy logic is now a reality that we have decided to apply to human behaviour in insurance, with a view to building the best possible models for such behaviour in light of the uncertainties they represent.

## 2.2 Elements of the theory of fuzzy logic

As with any mathematical concept, it is important to understand the theory and the principal ideas behind fuzzy logic in order to grasp how it works.

Before providing a theoretical definition of fuzzy sets, it may be useful to consider an example. Let's take the size of an individual. When can we begin to say that an individual is tall ? The answer to this question is difficult because defining a minimum height requirement for membership in this category is a subjective decision. Indeed, the minimum height might be 1m71, 1m75 or even 1m80. So the set of tall individuals is fuzzy.

Fuzzy logic rests on the theory of fuzzy sets, which is a generalization of the theory of conventional sets (Boolean theory). A fuzzy set is a set to which a thing can belong to a certain degree.

A fuzzy set $A$ in the universe of discourse $X$ is characterized by a membership function $\mu_A(x)$ that at each point $x$ in $X$ associates a real number in the interval $[0, 1]$. $\mu_A(x)$ represents the degree of membership $x$ in $A$.

A fuzzy set is totally characterized by its membership function. If $\mu_A(x) = 40\%$ then $x$ belongs to $A$ awith a $40\%$ degree of membership. With membership functions, we can thus belong to a set only $40\%$ (not totally), whereas in conventional set theory, there is no happy medium to be had : we either belong to the set or we dont belong to the set.

- **Fuzzy theory :**

$$\forall x \in X; \mu_A(x) \in [0, 1]$$

- **Conventional theory :**

$$\forall x \in X; \mu_A(x) = 1 six \in A; \mu_A(x) = 0 sinon$$

As in the conventional theory, operators are used to establish logical links between fuzzy sets and to handle them easily. We call these operators the fuzzy operators. So we can define the operators "AND", "OR" and "NO" in order to calculate the degrees of membership and create new fuzzy sets. In fact, we can belong to an "AND" set, to an "OR" set or "NOT" belong to either.

The correspondences between the conventional operators and the fuzzy Zadeh operators are set forth in the table below.

| | **Zadeh operators** | **Probabilistic operators** |
|---|---|---|
| **Intersection : AND** | $\mu_{A \cap B}(x) = Min(\mu_A(x), \mu_B(x))$ | $\mu_{A \cap B}(x) = \mu_A(x) \times \mu_B(x)$ |
| **Union : OR** | $\mu_{A \cup B}(x) = Max(\mu_A(x), \mu_B(x))$ | $\mu_{A \cup B}(x) = \mu_A(x) + \mu_B(x) - \mu_A(x) \times \mu_B(x)$ |
| **Negation : NO** | $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ | $\mu_{\bar{A}}(x) = 1 - \mu_A(x)$ |

The theoretical elements we have just mentioned comprise the fundamental bases for understanding fuzzy logic reasoning as well as the pathway for achieving real results via the fuzzy universe. Now we are ready to study fuzzy reasoning.

## 2.3 Fuzzy logic reasoning applied to economic surrenders

Using real data on human beings and their environment as its starting point, fuzzy logic works in the fuzzy universe to model as best it can the behaviour of these humans when faced with certain problems. But it has to return to the real world in order to offer a precise conclusion with respect to these problems. This robust methodology can be broken down into three steps : **fuzzification, fuzzy inference and defuzzification**.

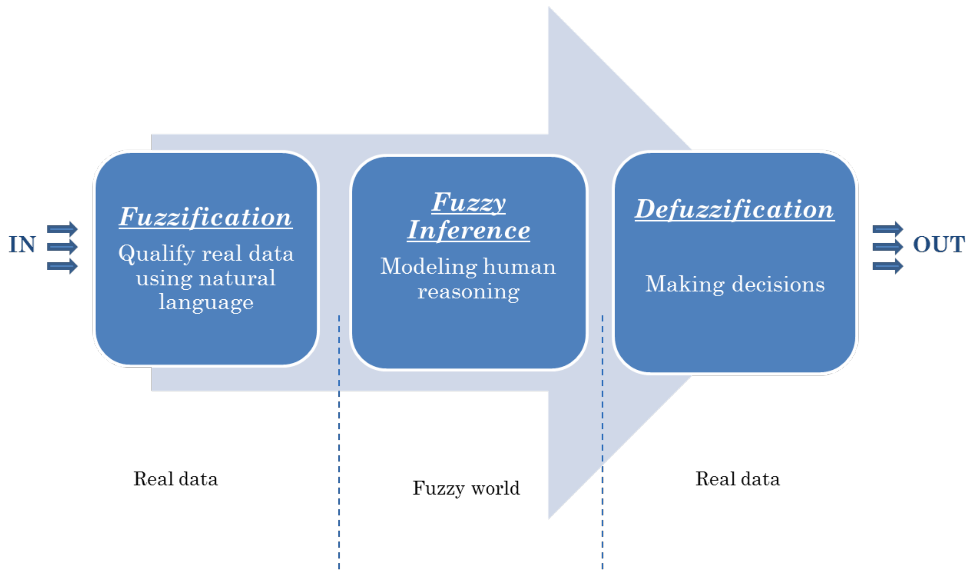



**FIGURE 5 –** Fuzzy logic reasoning

### 2.3.1 Fuzzification

**Fuzzification** is the step that takes us from the real world to the fuzzy world, *i.e.*, it allows us to qualify real data using natural language. The goal is to quantify the fuzzy : to transform numerical values into fuzzy data. To do this, we have to determine the model's fuzzy input variables (variables that will enable us to make choices regarding the conclusion of the problem) and the model's output variable (percentage of economic surrenders). Each variable must be associated with fuzzy sets and their related membership functions.

In the case of economic surrenders, we characterize the surrender decision-making through three fuzzy variables and the following fuzzy sets :

- **The spread between credited rates** (interest credited under the policy - rate offered by the competition), which allows us to assess changes in the behaviour of policyholders in the face of market conditions. This gap can thus be strongly negative, slightly negative, close to zero, slightly positive, or strongly positive.
- **How long ago the policy was purchased** (policy's seniority), which can influence the surrender decision via the tax reductions that it suggests. This can be defined as low, average, or high.
- **The age of the policyholder**, which guides surrenders depending on how he or she plans to use the accumulated savings (fructification or transmission). When we talk about the age of

policyholders, it seems natural to use the categories “adult” and “retired”. But we wanted to draw a distinction within the “adult” category, creating a separate category for young adults. This distinction reflects the fact that adults can adopt rather unique behaviour with respect to surrenders, because they are more proactive than young adults, who are less interested in their savings. In addition, because they generally don’t have the same level of savings as older adults, they adopt a different behaviour towards surrenders.

The output variable is the surrender rate. It is viewed as very negative, negative, close to zero, positive, or very positive.

| Variables | Input | | | Output |
|---|---|---|---|---|
| | *Spread between credit rates* | *Policy's seniority* | *Age of the policyholder* | *Surrender rate* |
| Fuzzy sets | Strongly negative<br>Slightly negative<br>Close to 0<br>Slightly positive<br>Strongly positiv | Low<br>Average<br>High | Young adult<br>Adult<br>Retired | Very negative<br>Negative<br>Close to 0<br>Positive<br>Very positive |

**FIGURE 6 –** Fuzzy variables

The membership functions for each fuzzy set defined above appear on the graph below.

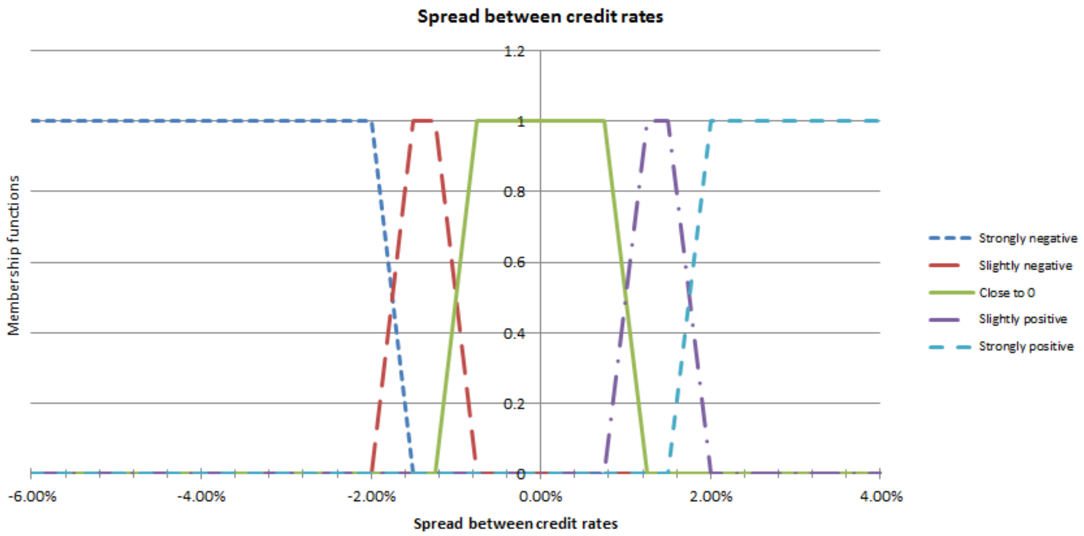



**FIGURE 7 –** Membership functions for spread of credit rates

To define the other membership functions for the fuzzy sets associated with the gap in the credited rate variable, we asked ourselves the following question : what is the tipping point, *i.e.*, what gap in the rate between the insurer’s credited interest rate and that of the competition triggers the policyholder to consider the possibility of surrendering the policy ? In answering this question, we can determine if this return gap is “slightly negative”, “close to zero”, or “slightly positive”.

Individuals begin to get interested in another policy if it offers them a rate that is at least $1\%$ more than what their own insurer guarantees. In this case, they begin to see surrender as a possibility. Before $1\%$, they will be more perplexed and will not find it necessary to surrender their policy (possible gains are not enough and the acquisition fees associated with the new policy have to be taken into account). To highlight this change in attitude that occurs around $1\%$, we decided that if the gap were $-1\%$, it would be considered to be $50\%$ close to zero and $50\%$ slightly negative [8]. But to ensure that the passage from the close to zero to the slightly negative set is gradual, the membership function for the slightly negative gap in rates of return begins with a gap of $-0.75\%$.

Expert opinions, common sense, and solid arguments are the essential factors that have allowed us to determine the relative membership functions of each remaining fuzzy set [9].

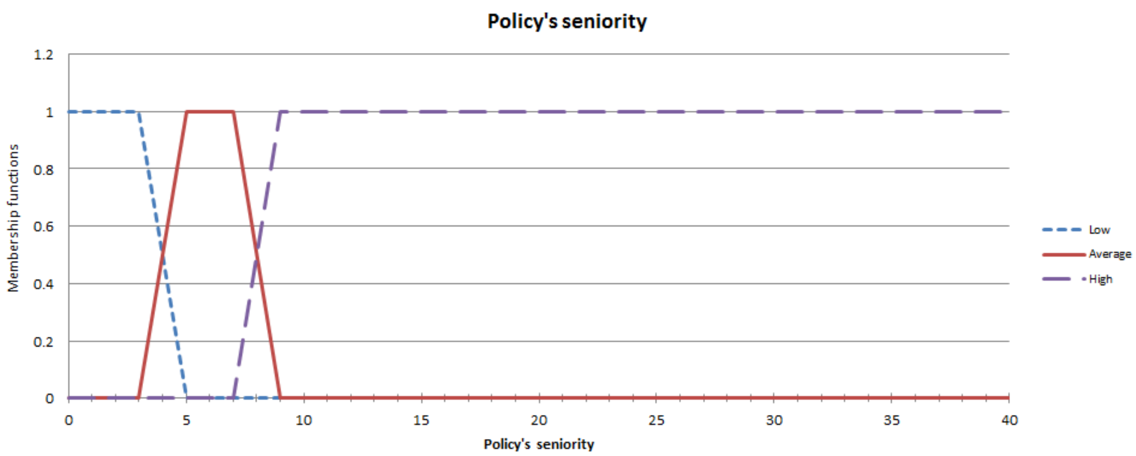



**FIGURE 8 –** Membership functions for policy's seniority

8. Remember that the gap in rates is defined in the following manner : insurers interest credited competitor rate.
9. For more details on the rationale behind each of the membership functions, see the published memo on **http ://www.ressources-actuarielles.net.**

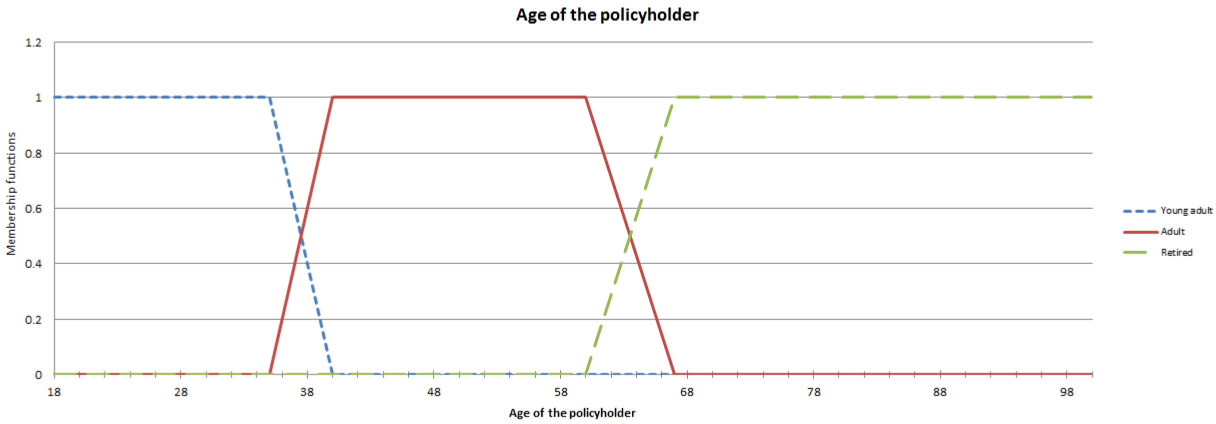



**FIGURE 9 –** Membership functions for age of the policyholder

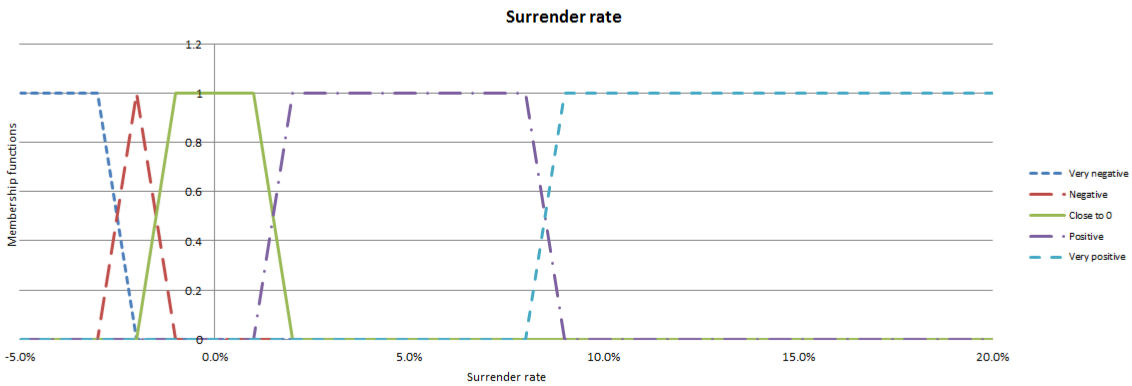



**FIGURE 10 –** Membership functions for surrender rate

Once this step is completed, we are in the fuzzy universe.

### 2.3.2 Fuzzy inference

As with any other kind of thought or reflection, fuzzy logic works using decision rules that are formulated as implications : there is a conclusion associated with each proposed problem. These fuzzy rules are stated in natural language : **IF... AND... THEN...** They are grouped in what we call the **decision matrix**. The proposed rule is represented by a gap in the rate of return, the age of the policyholder, and the number of years the policy has been in force (our input variables). For example, a rule might be : **IF** the rate gap is strongly negative **AND** the policyholder is an adult **AND** the policy has been in force for many years **THEN** surrenders are very positive.

Calibrating the decision matrix is based on a robust process of justifications and expert appraisals.

For example, young adults who have not held their policy for very long are characterized by a low level of invested savings and little knowledge of the life insurance market. Hence, they are not proactive and do not necessarily take an interest in their savings. They prefer to invest in "Livret A" passbook savings accounts and/or PEL home savings plans. In addition, as young adults, they are not thinking about estate planning ; their focus is on prevention (they are not looking to make a profit or capital gains ; their goal is to set money aside). Consequently, they are not very sensitive to the rate spreads that may exist between the interest credited to them as a policyholder and the competitor's rate : the surrender is hence close to zero, regardless of the gap between returns. [10].

Decision matrix of this study is :

| Surrender rate | Policy's seniority & Age of the policyholder | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Low | | | Average | | | High | | |
| Spread of credit rate | Young adult | Adult | Retired | Young adult | Adult | Retired | Young adult | Adult | Retired |
| Strongly negative | Close to 0 | Positive | Positive | Positive | Positive | Positive | Positive | Very positive | Positive |
| Slightly negative | Close to 0 | Close to 0 | Close to 0 | Close to 0 | Positive | Close to 0 | Close to 0 | Positive | Close to 0 |
| Close to 0 | Close to 0 | Close to 0 | Close to 0 | Close to 0 | Close to 0 | Close to 0 | Close to 0 | Close to 0 | Close to 0 |
| Slightly positive | Close to 0 | Close to 0 | Close to 0 | Close to 0 | Negative | Close to 0 | Close to 0 | Negative | Close to 0 |
| Strongly positive | Close to 0 | Negative | Negative | Negative | Negative | Negative | Negative | Very negative | Negative |

**FIGURE 11 –** Decision matrix for surrender rate

Now let's take an individual with the following characteristics : $61$ years old and has held the policy for $3.5$ years. In addition, the gap in rates observed in the market between the competitor's rate and the insurer's rate is $-1.7\%$. Based on the membership functions defined above, this individual is $86\%$ adult and $14\%$ retired, but in terms of the length of time he has held his policy, his situation is $75\%$ low and $25\%$ average and the gap in rates is $40\%$ strongly negative and $60\%$ slightly negative. So this individual belongs to six different fuzzy sets, which triggers eight different decision rules. The conclusion of each decision rule can be read in the decision matrix we built earlier.

| Spread of credit rate | AND | Policy's seniority | AND | Age of the policyholder | THEN | Rachat |
|---|---|---|---|---|---|---|
| Strongly negative | AND | Low | AND | Adult | THEN | Positive |
| Strongly negative | AND | Average | AND | Adult | THEN | Positive |
| Strongly negative | AND | Low | AND | Retired | THEN | Positive |
| Strongly negative | AND | Average | AND | Retired | THEN | Positive |
| Slightly negative | AND | Low | AND | Adult | THEN | Close to 0 |
| Slightly negative | AND | Average | AND | Adult | THEN | Positive |
| Slightly negative | AND | Low | AND | Retired | THEN | Close to 0 |
| Slightly negative | AND | Average | AND | Retired | THEN | Close to 0 |

**FIGURE 12 –** 8 decision rules

The advantage of fuzzy logic lies in its ability to trigger a decision rule even if the proposition is not $100\%$ true. It is important to know the degree of truth of the proposition, called the activation degree. The characteristics of the individual are what enable us to assign him a precise degree of membership for a fuzzy set per variables. The minimum of these membership degrees will thus be

10. For more details on the argument used to build the decision matrix, see the published memo on *http ://www.ressources-actuarielles.net.*

the activation degree for the rule. If the individual is only a $20\%$ member of one of these sets, the proposition cannot be more than $20\%$ true.

Let's now look again at the first possible decision rule for our sample individual : this individual is a $40\%$ member of the "strongly negative gap in rates" set, a $75\%$ member of the "low policy duration" set and an $86\%$ member of the "adult" set. So the activation degree of this rule is $40\% = MIN(40\%, 75\%, 86\%)$.

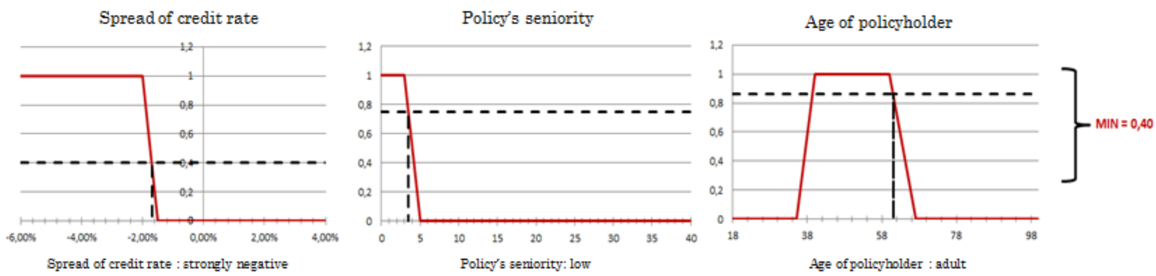



**FIGURE 13 –** Activation of the first decision rule ($R1$)

Then comes the moment to assess the decision made per fuzzy rule. Each rule conclusion is represented by a membership function. Determining this function involves taking the minimum between the activation degree of the rule in question and membership function of the fuzzy set proposed by the rule's conclusion. This method is called Mamdani's **implication**[11].

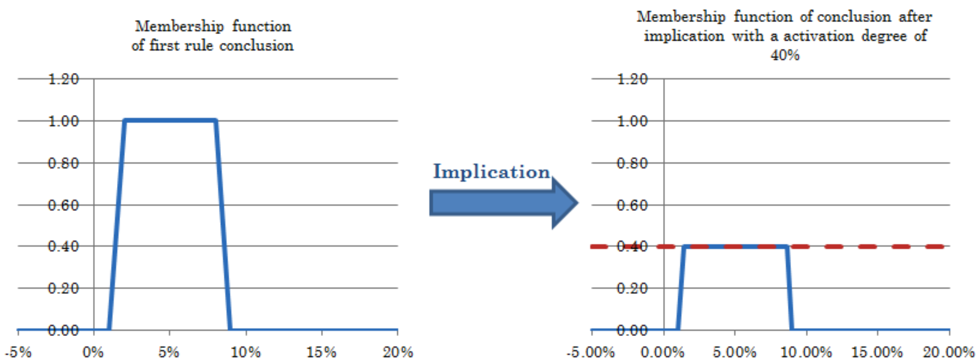



**FIGURE 14 –** Mamdani's implication

The characteristics of an individual can trigger several decision rules (8 in our example). A conclusion has been attributed to each fuzzy rule. We aggregate the set of these conclusions to get to the final conclusion. Since any conclusion can be more or less true, the membership function of

11. Other methods of implication exist in Fuzzy Logic, such as the Larsen implication, which entails taking for the conclusion function a percentage of the membership function of the fuzzy set proposed by the rule's conclusion. This percentage is the rule's activation degree.

the final conclusion is the maximum of the membership functions of each conclusion (an “OR” logic translates as a maximum in fuzzy logic).

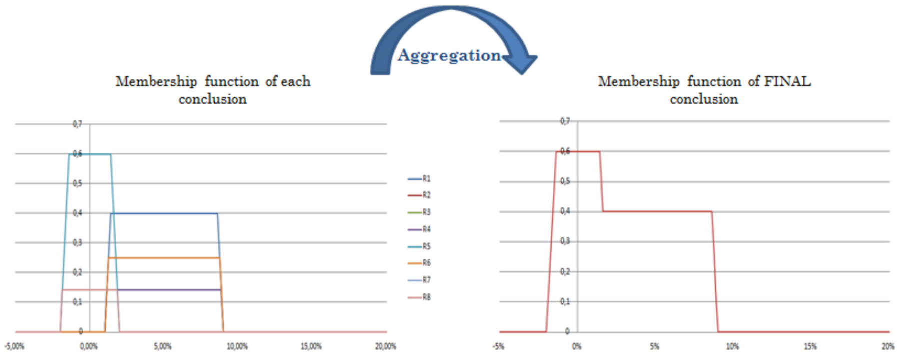



**FIGURE 15 –** Conclusions’ aggregation

### 2.3.3 Defuzzification

This final step allows us to return to the real world and quantify the decision made by the policyholder (*i.e.*, to find a surrender percentage relative to the policyholder).

Different methods are applicable during this step : the **Centre Of Gravity** method (COG), which consists of taking as your value the abscissa of the centre of gravity of the area under the final conclusion membership function, or the **Mean of Maximum method**(MM), which consists of taking as your value the abscissa the mean of the output values having the highest membership degree.

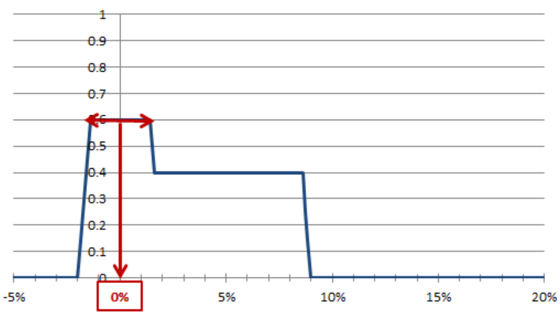



**FIGURE 16 –** Defuzzification with Mean of Maximum method

**The mean of maximum method** engenders a surrender rate of $0\%$ for our sample individual. This method offers the advantage of being simple to implement. In fact, it is sufficient to note the maximum of the membership function, look at the abscissas which have this maximum as their ordinate and take the mean. However, there are some strong disadvantages that offset this advantage. During a study of sensitivities, we noticed that the results obtained using this method are unstable. Indeed, the surrender rate value can vary enormously in the face of very minor input data variations. A surrender rate can, for example, go from $10.5\%$ to $0\%$, simply by adding one year to the length of time the policy has been in force, all other things being equal.

The most disturbing thing about this method is that it does not give the impression of using the entire final conclusion membership function. In choosing to look only at the maxima, we simply skip over a big chunk of the curve. So we can imagine that some of the individuals intentions are not represented. We force the individual to belong to just one surrender group when in fact he has the possibility of belonging to several. In our example, we ultimately force our individual to be a member of the “close to zero” group when in fact he could just as well belong to the “positive” group.

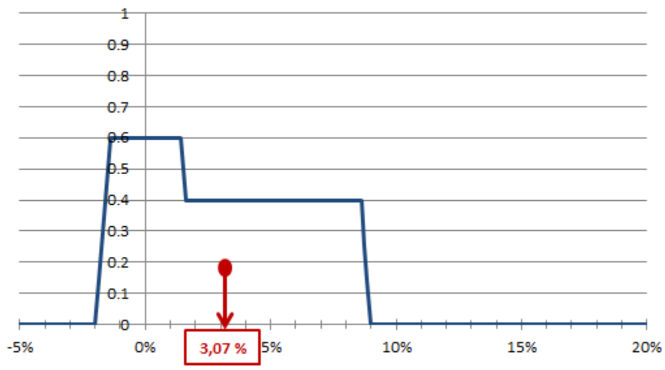



**FIGURE 17 –** Defuzzification with Centre Of Gravity method

**The COG method** generates a surrender rate of $3.07\%$ for our “sample individual” This method, which is the opposite of the previous one, is stable with respect to changes in data input. In addition, it allows us to take into consideration all the output data, as if we assumed that the individual selected via his data related to his age, the age of his policy, and gaps in rates could belong to several groups. In our example, the individual can belong to two groups. Lastly, it is the very essence of the principle of fuzzy logic to not be a member of just one set at a time. Accordingly, this is the method we have chosen for our study.

**Observation :** When we state that an individual adopts a surrender rate of $3.07\%$, this means - roughly speaking - that in a group of people who have the same characteristics as this individual (observed rate gap, age of policyholder and how long he has held the policy), $3.07\%$ of the group’s global mathematical reserves are surrendered.

For an adult whose policy has been in force for a long time, we obtain the following law of surrenders. It should be noted that it respects the ACPR’s recommended minimum and maximum cap.

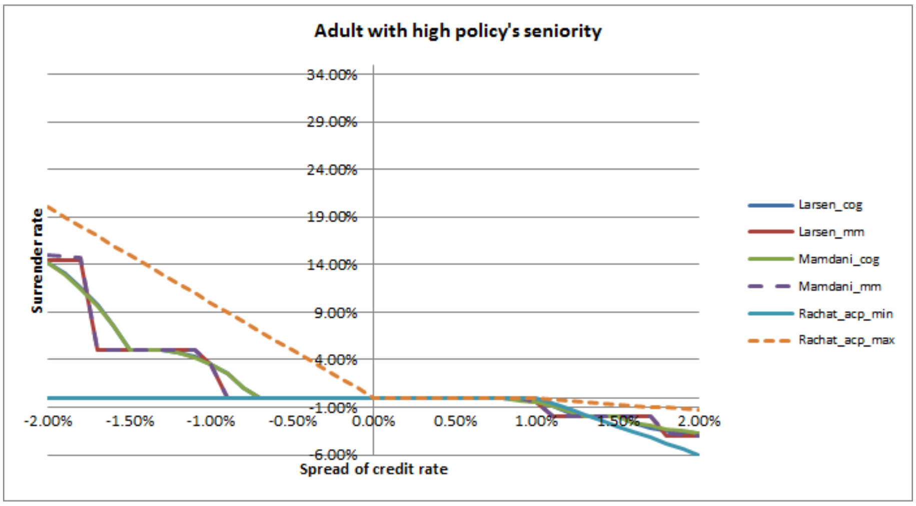



**FIGURE 18 –** Adult whose policy has been in force for a long time

We note that fuzzy logic reasoning is a long and complex process that requires numerous decisions and assumptions that must be vigorously and convincingly justified.

## 2.4 Fuzzy logic reasoning applied to the credited interest rate policy

The fuzzy approach, explained with precision for economic surrenders, is applied in exactly the same way to modelling the credited interest rate target. We have assumed that management determines this target based on the company's solvency and financial position. The indicators used are :

- **The level of the profit-sharing reserves** expressed as a percentage of the mathematical reserves. The profit-sharing reserves can be considered as low, adequate, or high.
- **The return on invested assets** compared with the competition. Interest or investment income will be considered as low, appreciable or high.
- **The Solvency 1 ratio** : Capital over solvency margin required for savings products (since we are working on savings products denominated in euros, the required solvency margin is equal to $4\%$ of mathematical provisions). It should be stressed that this ratio depends on the insurer's appetite for risk and that whether or not the solvency margin is deemed to be "Passable", "Good" or "Very Good" will depend on the insurance company's policy.

In this fuzzy reasoning, our output variable - the credited interest rate - is expressed as a percentage of the competing rate. The credited interest rate policy is the objective or target set by the company in terms of adjusted or reset policyholder mathematical reserves. Based on the health of the insurer's earnings, it can be "Not Very Competitive", "Competitive" or "Very Competitive".

The decision matrix built to resolve the credited interest rate target is the following :

| Credited interest rate policy | Financial products &PPE | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Low | | | Appreciable | | | High | | |
| Solvency 1 ration | Low | Adequate | High | Low | Adequate | High | Low | Adequate | High |
| Passable | Not Very Competitive | Not Very Competitive | Not Very Competitive | Not Very Competitive | Competitive | Competitive | Not Very Competitive | Competitive | Competitive |
| Good | Not Very Competitive | Not Very Competitive | Not Very Competitive | Competitive | Competitive | Competitive | Competitive | Competitive | Very Competitive |
| Very good | Not Very Competitive | Not Very Competitive | Competitive | Competitive | Competitive | Very Competitive | Competitive | Very Competitive | Very Competitive |

**FIGURE 19 –** Decision matrix for credited interest rate policy

For a level of PPE that varies between $0.30\%$ and $3\%$, here is the target credited interest rate used via fuzzy logic based on investment income for a solvency level of $135\%$.

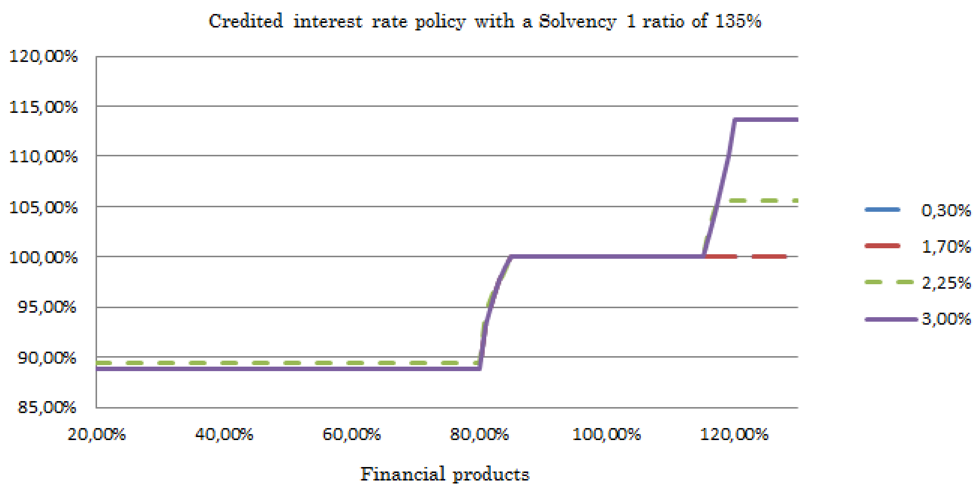



**FIGURE 20 –** Credited interest rate policy with a Solvency 1 ratio of $135\%$

The results obtained from our two exercises in modelling are both satisfactory and consistent with the choices made when the decision matrix and membership functions were determined.

## 3 Results : Fuzzy logic contributions to behavioural modelling

To grasp the results of the study and perform an in-depth analysis of the contributions of fuzzy logic to behavioural modelling, we compare the results derived from the ALM model obtained using a traditional model of behaviour with the results obtained using fuzzy logic.

### 3.1 The contributions of fuzzy logic to modelling economic surrenders

We determined a Best Estimate value for the company's liabilities using two models :

- **The standard model** : economic surrenders are determined using the ACPR function [12].
- **The fuzzy model** : economic surrenders are determined using the tools of fuzzy logic.

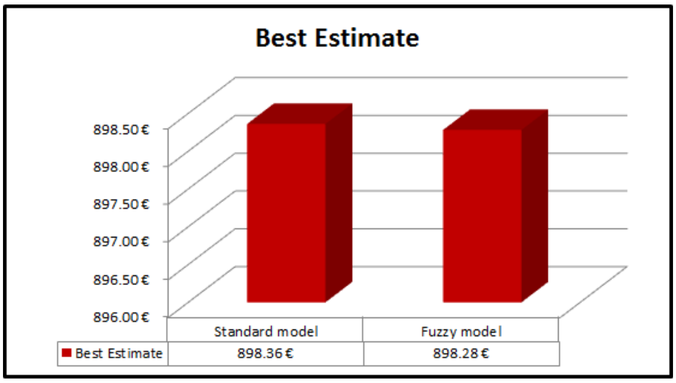



**FIGURE 21 –** Best Estimate for each model

Despite using a different modelling of economic surrenders, the Best Estimate of the fuzzy model is actually similar to the best estimate derived from the standard model. In other words, all parameters and assumptions being equal, the calibration of our decision matrix leads to results that are comparable to the ACPR surrender function that we used.

The "detailed" vision of economic surrenders expressed by fuzzy logic is consistent with the vision of economic surrenders expressed via the ACPR function, since the average behaviour of policyholders is similar. The ACPR is in some ways positioned on an average individual, while our decision matrix is positioned over several profiles of individuals, which leads to globally similar behaviour in the projections.

The real difference between the two models is how the parameters are set for the surrender assumptions. Via the methodology specific to fuzzy logic, the economic surrenders model was calibrated using simple and clearly formulated assumptions. Consequently, one of the principal contributions of fuzzy logic is that it resolves some of the limitations of the ACPR function, *i.e.*, an insurer

12. The ACPR's surrender function is calibrated on the average parameters recommended by the regulator.

that wants to use the ACPR law can justify and provide arguments for the calibration of its choice using fuzzy logic.

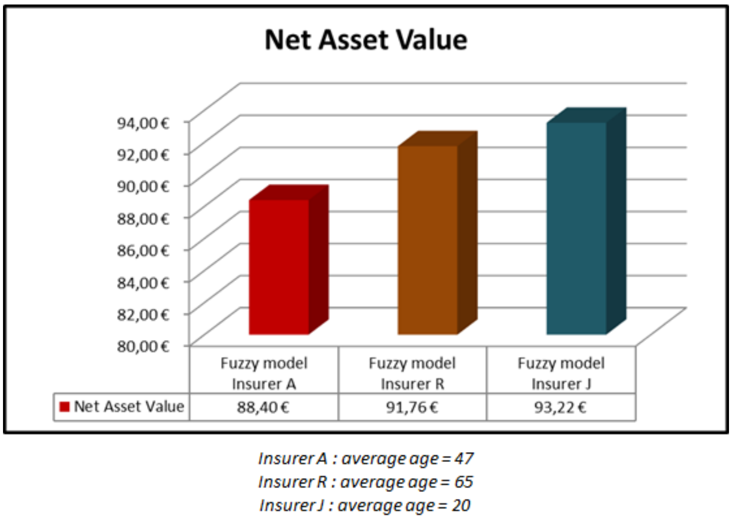
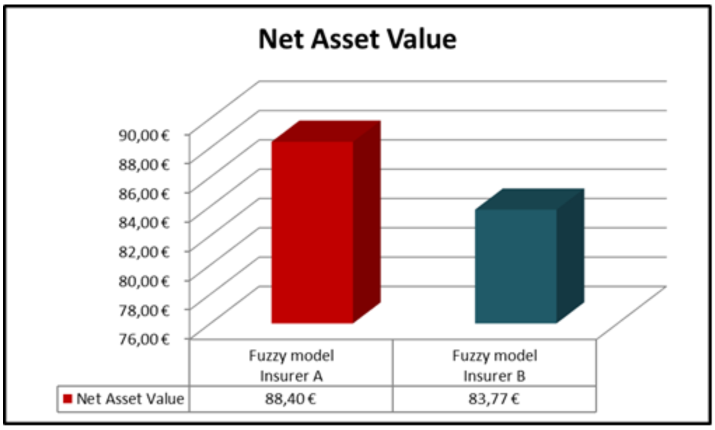



**FIGURE 22 –** The average age of the portfolio's sensitivity

The contributions of the fuzzy logic approach can also be seen when we study the sensitivity of the fuzzy model results based on the average age of the portfolio of policyholders[13]. In fact, a more refined modelling of surrender behaviours underscores an NAV gap between three companies where the average policyholder age differs. The ACPR surrender function, positioned on an average individual, would not have captured the differences between the three companies[14]. To capture the singularity of each company using the standard model, it is necessary to calibrate $3$ ACPR functions (one for adults, one for retirees, one for young people). Conversely, this would render the problem of calibrating this function even more complex.

13. Note that this time we present not the Best Estimate but the Net Asset Value (economic capital). Net Asset Value can be viewed as the wealth that belongs to the insurer. Net Asset Value is used more often when we are looking at things from the insurer's perspective. But the reasoning remains the same. Every change in Net Asset Value is explained by a change in Best Estimate.

14. Net Asset Value would have been identical for all three companies.




*Insurer A : central scenario*
*Insurer R : "reactive" scenario*

**FIGURE 23 –** Sensitivity of decision matrix

Fuzzy logic, via the decision matrix, offers flexibility in calibration and adjustment that is relatively simple to use compared with the ACPR's surrender function. For example, if Company B, whose management does not have the same perception of its policyholders as Insurer A in that it considers young policyholders to be more proactive, then all it has to do in order to accommodate stronger or higher proactivity on the part of young policyholders is to modify the decision matrix. In conformity with the assumptions inscribed in the decision matrix, the change has a direct impact on the insurer's future NAV (see figure 23).

Our fuzzy logic module has been built to be directly applicable to atypical portfolios. Fuzzy logic thus stands out as a tool that is capable of generalizing the ACPR surrender function to all life insurance liabilities and of assigning or easily modifying the sensitivities depending on the policyholder profiles. In addition, the insurer can add other linguistic variables, which in its opinion are characteristic of the behaviour of its own policyholders without adding to the complexity of the calibration or the justification of the modelling.

Accordingly, fuzzy logic truly adds value to the modelling of economic surrenders. It should not be considered as a different approach that is the opposite of the surrender function recommended by the ACPR ; instead, it should be understood as an extension that allows us to resolve certain problems, such as justification, calibration and generalization.

## 3.2 Contributions of fuzzy logic to the modelling of the credited interest rate policy

Now we turn our attention to the contribution of fuzzy logic to modelling the credited interest rate policy. This entails, once again, comparing the Best Estimate value that is derived from the standard model with the Best Estimate value produced by the fuzzy model. As a reminder, in the standard model the credited interest rate target is fixed because the insurer is looking every year to credit $100\%$ of the competing rate, while in the fuzzy model management is targeting a percentage of the competing rate on the basis of the approximate assessment it has made of the company's solvency, its investment income, and the PPE.

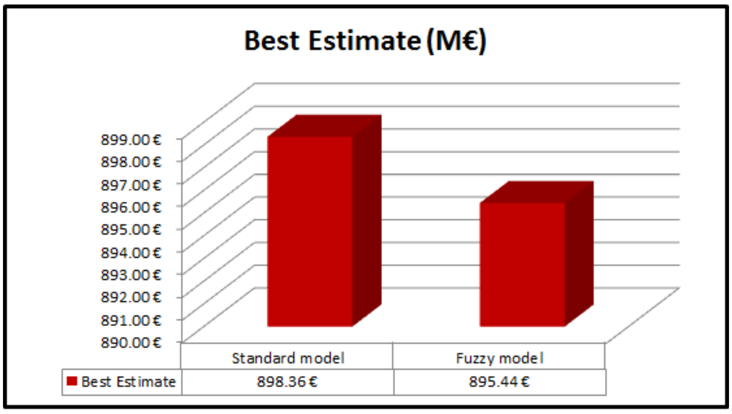



**FIGURE 24 –** Best Estimate for each model

By integrating the "fuzzy logic"-type credited interest rate policy into our model, we are able to decrease the value of the insurer's obligations and, consequently, increase its economic capital/NAV by the same amount.

In other words, adopting fuzzy logic leads to better management of the credited interest rate without making the modelling more complex. This should come as no surprise, actually. If the results are better in the model with fuzzy logic, it is essentially because, in addition to the modelling itself, the assumptions driving the two models are different. Indeed, here we are comparing a static policy with a dynamic one which, by its very construction, optimizes management's decision to maximize future wealth. It is important to note that it is possible in the standard model to transpose management decisions in the form of a constraint optimization problem and obtain results that are similar to, or even better than, the fuzzy logic model.

The real value of the fuzzy modelling of management action resides in the resulting calibration of the decision matrix. Fuzzy methodology puts assumptions related to calibration in "plain language" and thus gives management the possibility of expressing its management policy in a straightforward, qualitative way while also distancing itself from the mathematical modelling that the decision matrix hides. A strong point of fuzzy logic, the decision matrix is thus the key that allows us to draw a concrete link between management's actual policy and the modelling behind it.

## 4 Conclusion

Fuzzy logic is an innovative theory in insurance, offering a robust conceptual mathematical framework that is able to reflect expertise in a model. This theory provides an alternative to every modelling problem linked to inadequate data.

In the case of economic surrenders, our study has demonstrated that the fuzzy logic passage allows us to generalize and justify the calibration of surrenders, while also justifying the demands of the regulator, i.e., the ACPR function. So fuzzy logic offers the first solutions to the limitations associated with attempts to model economic surrenders. However, the benefits of fuzzy logic will not be forthcoming without a real effort on the part of insurers to understand surrender behaviour. Indeed, let's not forget that the calibration of the decision matrix must be the result of a serious assessment of policyholder behaviour, and that its justification and rationale must occupy a central place within the framework of its use. On this subject, there are two options available to help insurers improve their understanding of policyholders : an underwriting questionnaire designed to give the insurer an idea of the profile of the new policyholder [15] and more in-depth work on the part of insurers to analyse the reasons for policyholder surrender [16].

As for the modelling of management actions, fuzzy logic offers an interesting compromise between optimization, complexity, and a faithful representation of reality. But above all, this theory allows management to provide a straightforward and consistent explanation of its management policy in a forecast model. This latter point may be of particular interest in the future regulatory framework of Solvency 2, where insurers are asked to assess their own capital requirements by taking into consideration, among other things, the risks associated with management decisions. With this in mind, the **decision matrix**, which is the cornerstone of fuzzy logic, stands out as an efficient and pragmatic steering tool for management. Using this instrument, management can easily test the robustness of its decisions under situations of stress and thus come up with an optimal management policy that satisfies the companys own criteria for assessing its appetite for risk.

Fuzzy logic makes genuine contributions to solving actuarial problems. Our analysis of the results has shown that these contributions can go beyond the subjects considered in this study thanks to a theory that is rich but above all not exploited in the insurance industry. **This is why we really encourage ongoing work and applaud the rise of fuzzy logic in forecast modelling in insurance.**

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