SCOR
Papers

A game-theoretic approach to non-life insurance market cycles

By **Christophe Dutang**

Institut du Risque et de l'Assurance, Université du Maine, Le Mans

Winner of the French Actuarial Prize, 2012

Abstract

In this paper, we develop a noncooperative game to model a non-life insurance market. Our first goal will be to analyze the effects of competition between insurers through different indicators: the solvency level, the market share, the underwriting results. Secondly, we will seek to further understand the genesis of insurance market cycles. Insurance market cycles have troubled actuaries and academics for decades: this game-theory focus will allow us to shed a different light on the subject.

SCOR
Global Risk Center

1 Introduction

Insurance market cycles and the study of their causes have been puzzling actuaries for many years. S. Feldblum [12] discusses four main causes for the presence of underwriting through their aggregate effect. These causes are (i) actuarial pricing procedure, (ii) underwriting philosophy, (iii) interest rate fluctuations and (iv) competitive strategies. Feldblum compares contributions through out the 20th century on the topic, see also [19] and [29] for an overview.

Actuarial pricing procedures are subject to claim cost uncertainty, information lag (due to accouting, regulatory and legal standards). Such effects are likely to generate fluctuations around an equilibrium price, when extrapolating premiums. (See e.g. [28] and [6].) In addition, over-strict attitudes on the part of underwriters combined with a lack of coordination is an extra recipe for underwriting cycles. In particular, policies cannot be priced independently of the market premium, but neither can the market premium be driven by one's individual actions. This is called underwriting philosophy by Feldblum [12], and is also acknowledged by M. Jablonowski [16], who assumes that (i) insurers do not make decisions in isolation from other firms in the market, and (ii) profit maximization is not the exclusive, or even the most important, motivation of insurers. Interest rate deviations further increase the frequency and amplitude of market cycles, as they have an impact on the investment result and (indirectly) on the maximum rebate that underwriters can afford to attract presumably customers with a low-risk profile. E.C. Venezian [13] was among the first to demonstrate this effect. Finally, the competition level on most mature insurance markets is sufficiently high that any increase in market share can only be carried out by price decrease (due to very little product differentiation). The hunger for market share is driven by the expected reduction of claim uncertainty when increasing the policy number, which is motivated by the law of large numbers. This, coupled with capital constraints (e.g. [15]) and price inelasticity, forces insurers not to deviate too much from market trends.

Pure economic models suggest that the equilibrium premium is the marginal cost, as any upward deviation from this marginal cost will result in losing all the policies. This is not relevant to apply economic models of other industries to the insurance market because of the adverse selection and the inertia of the insurance demand. The celebrated Rothschild and Stiglitz model shows that at the equilibrium individuals with low-risk aversion choose full coverage, whereas individuals with high-risk aversion prefer partial coverage. However, this economic model cannot addres the insurance market cycle dynamics. G.C. Taylor [27] deals with underwriting strategies of insurers and provides first attempts to model optimal responses of an insurer to the market on a given time horizon. (See also [17, 11, 21] for extensions.) All these papers focus on one single insurer and in that way assume that insurers are playing a game against an impersonal market player, so that the market price is independent of their own actions.

In this paper, we wish to investigate the suitability of game theory for insurance market cycle modelling. Among earlier works using Noncooperative game theory to model the non-life insurance market, two kinds of models were pursued: the Bertrand oligopoly where insurers set premiums and the Cournot oligopoly where insurers choose optimal values of insurance coverage. M.K. Polbor [23] considers a Bertrand model in which rational consumers maximize their utility function and for which the equilibrium premium is the expected loss. R.M. Powers and M. Shubik [24] propose a

Cournot model with two types of players: policyholders who state the amount that they are willing to pay, and insurers who state the amount of risk they are willing to underwrite. Based on a clearing-house system to determine the market price, each player maximizes its expected utility. Assuming risk neutral insurers and risk averse consumers, the resulting premium equilibrium is larger than the expected loss.

None of these models can model the insurance cycles. In this paper, we propose a repeated nonco-operative game models that replicates the main insurance features and dynamics. Using the game theory, we extend the insurer-vs-market reasoning of [27]. We also extend the Bertrand model of [25] by considering a lapse model and an aggregate loss model for policyholders. The lapse model describes the policyholder behavior through a lapse probability which is a function of the premiums offered by the insurers. We also consider a solvency constraint function for insurers. C. Dutang, H. Albrecher and S. Loisel [10] show that incorporating competition when setting premiums leads to a significant deviation from both the actuarial premium and a one-insurer optimized premium. We show that although the repeated game models a rational behavior of insurers in setting premium, the resulting market premium is cyclical. The rest of the paper is organized as follows. Section 2 introduces the one-period model based on [10]'s model. Section 3 presents the dynamic framework of the repeated game and its application to the French motor market, before Section 4 concludes.

2 The one-period model

Consider I insurers competing in a market of n policyholders with one-year contracts (n is fixed). The policyholders are assumed to react to price changes (either stay with the present insurer or switch to one of the competitors), but do not have any other influence on the premium level (which is a realistic assumption, in particular for personal lines of business such as compulsory third-party motor liability). In view of the one-year time horizon and the randomness of claim sizes, this model focuses on non-life insurance products (i.e. products for which the claim event is NOT linked to the life of the policyholder).

The "game" for insurers is to set the premium for which policies are offered to the policyholders. Let $(x_1, \ldots, x_I) \in \mathbb{R}^I$ be a price vector, with x_j representing the premium of Insurer j. Once the premium is set by all insurers, the policyholders choose to renew or to lapse from their current insurer. Then, insurers pay occuring claims during the coverage year. At the end of the period, underwriting results are determined, and the insurer capital is updated: some insurers may be bankrupt. As we deal with a one-period model, for simplicity we do not consider investment results.

In the next subsections, we present the four components of the game: (i) a lapse model, (ii) a loss model, (iii) an objective function and (iv) a solvency constraint function. These four components are frequently considered by practitioners to be the critical factors for such a study. In the sequel, a subscript $j \in \{1, \ldots, I\}$ will always denote an insurer index, whereas a subscript $i \in \{1, \ldots, n\}$ denotes policyholder index. In the sequel, "insurer" is used when referring to players of the insurance game.

2.1 Lapse model

In this subsection, we present our lapse model which is designed as a compromise between reflecting the policyholders' behavior in a reasonable way, and still keeping mathematical tractability. Let n_j be the initial portfolio size of Insurer j (such that $\sum_{j=1}^{I} n_j = n$). It seems natural that the choice of policyholders for an insurer is highly influenced by the choice of the previous period. We assume that the choice of the (initial) n_j policyholders of Insurer j follows an I-dimensional multinomial distribution $\mathcal{M}_I(n_j, p_{j\to}(x))$ with probability vector $p_{j\to}(x) = (p_{j\to1}(x), \ldots, p_{j\to I}(x))$. The probability $p_{j\to k}(x)$ to choose an insurer depends on the price vector x, concretely, the differences of premiums. Empirically, the probability to lapse $p_{j\to k}(x)$ (with $k \neq j$) is generally much lower than the probability to renew $p_{j\to j}(x)$. To our knowledge, only the UK market shows lapse rates above 50%, cf. [8].

In the economics literature, $p_{j\to k}$ is considered in the framework of discrete choice models. In the random utility maximization setting, D. McFadden [20] proposes multinomial logit and probit probability choice models. In this paper, we choose a multinomial logit model, because of its simplicity. Working with unordered choices, we arbitrarily set the insurer reference category for $p_{j\to k}$ to j, the current insurer. We define the probability for a customer to go from insurer j to k given the price vector x by the multinomial logit model

$$p_{j\to k}(x) = \begin{cases} \dfrac{1}{1 + \sum\limits_{l \neq j} e^{f_j(x_j, x_l)}} & \text{if } j = k, \\[4mm] \dfrac{e^{f_j(x_j, x_k)}}{1 + \sum\limits_{l \neq j} e^{f_j(x_j, x_l)}} & \text{if } j \neq k, \end{cases} \tag{1}$$

where the sum is taken over the set of insurers $\{1, \ldots, I\}$ and f_j is a price-sensitivity function. We consider two types of price functions

$$\overline{f}_j(x_j, x_l) = \bar{\mu}_j + \bar{\alpha}_j \frac{x_j}{x_l} \quad \text{and} \quad \widetilde{f}_j(x_j, x_l) = \tilde{\mu}_j + \tilde{\alpha}_j (x_j - x_l). \tag{2}$$

The first function $\overline{f}_j$ assumes a price-sensitivity according to the ratio of proposed premium x_j and competitor premium x_l, whereas $\widetilde{f}_j$ works with the premium difference $x_j - x_l$. Parameters μ_j, α_j represent a base lapse level and price-sensitivity, respectively. We assume that insurance products display positive price elasiticity of demand $\alpha_j > 0$. One can check that $\sum_k p_{j\to k}(x) = 1$.

The portfolio size $N_j(x)$ of Insurer j for the next period is a random variable determined by the sum of renewed policies and (new) policyholders coming from other insurers. Hence, $N_j(x)$ is a sum of I independent binomial variables $(B_{kj})_k$ with parameters $n_k, p_{k\to j}(x)$

$$N_j(x) = B_{jj}(x) + \sum_{k=1, k \neq j}^{I} B_{kj}(x). \tag{3}$$

Note that $(B_{kj})_j$ are not independent variables as $(B_{k1}, \ldots, B_{kI})$ is a multinomial random variable. This assumption is in contrast with the standard models in classical ruin theory, where the portfolio size is assumed constant over time (see e.g. [1] for a recent survey and [18] for an attempt to have a premium-dependent portfolio size).

2.2 Loss model

Let Y_i be the aggregate loss of policy i during the coverage period. We assume no adverse selection among policyholders of any insurers, i.e. Y_i are independent and identically distributed (i.i.d.) random variables, for all $i = 1, \ldots, n$. Let us assume a simple frequency – average severity loss model

$$Y_i = \sum_{l=1}^{M_i} Z_{i,l},$$

where the claim number M_i is independent of the i.i.d. claim severities $(Z_{i,l})_l$ of Policyholder i. Therefore, the aggregate claim amount for Insurer j is

$$S_j(x) = \sum_{i=1}^{N_j(x)} Y_i = \sum_{i=1}^{N_j(x)} \sum_{l=1}^{M_i} Z_{i,l},$$

where $N_j(x)$ is the portfolio size defined in Equation (3). The aggregate claim amount is still a compound distribution of the same kind, since Y_i are assumed i.i.d. random variables. Indeed we have

$$S_j(x) = \sum_{i=1}^{M_j(x)} Z_i, \text{ with } M_j(x) = \sum_{i=1}^{N_j(x)} M_i,$$

where $(Z_i)_i$ are i.i.d. claim severities and $M_j(x)$ denotes the total number of claims of Insurer j. For the severity distribution for Z_i, we consider the lognormal distributions $LN(\mu, \sigma^2)$. For the frequency distribution for M_i, we consider the Poisson $P(\lambda)$ and the negative binomial $NB(r, p)$ distributions, leading to a distribution of $M_j(x)$ as Poisson $P(N_j(x)\lambda)$ and negative binomial $NB(N_j(x)r, p)$. We denote these two loss models as PLN and NBLN, respectively.

2.3 Objective function

In the two previous subsections, we presented two components of the insurance markets: the lapse model (how policyholders react to premium changes) and the loss model (how policyholders face claims). We now turn our attention to the underwriting strategy of insurers, i.e. on how they set premiums.

In Subsection 2.1, we assumed that price elasticity of demand for the insurance product is positive. Thus, if the whole market underwrites at a loss, any actions of a particular insurer to get back to profitability will result in a reduction of his business volume. This has two consequences for the choice of the objective function: (i) it should involve a decreasing demand function of price x_j given the competitors price vector $x_{-j} = (x_1, \ldots, x_{j-1}, x_{j+1}, \ldots, x_I)$ and (ii) it should depend on an assessment of the insurer break-even premium π_j per unit of exposure.

The parameter π_j corresponds to the estimated mean but depends on the assessment of loss expectation by Insurer j. We thus define π_j as

$$\pi_j = \omega_j \overline{a}_{j,0} + (1 - \omega_j)\overline{m}_0, \tag{4}$$

where $\overline{a}_{j,0}$ is the actuarial premium based on the past loss experience of insurer j, $\overline{m}_0$ is the market premium, available for instance, via rating bureaus or through insurer associations and $\omega_j \in [0,1]$ is the credibility factor of insurer j.[1] ω_j reflects the confidence of insurer j in its own loss experience: the closer to 1, the more confident insurer j is. Note that π_j takes expenses into account implicitly via the actuarial and the market premiums.

We choose the demand function as

$$D_j(x) = \frac{n_j}{n}\left(1 - \beta_j\left(\frac{x_j}{m_j(x)} - 1\right)\right), \tag{5}$$

where $\beta_j > 0$ is the elasticity parameter and $m_j(x)$ is a market premium proxy. The demand $D_j(x)$ is not restricted to $[0, n_j/n]$, and thus D_j targets both renewal and new business. In this form, $D_j(x)$ approximates the expected market share $\mathbb{E}(N_j(x))/n$ presented in Subsection 2.1. As the elasticity parameter β_j is positive, a premium increase (of Insurer j) will result in a decrease of the demand for insurance. The market proxy used in Equation (5) is the mean price of the other competitors

$$m_j(x) = \frac{1}{I-1}\sum_{k \neq j} x_k.$$

The market proxy aims to assess other insurer premiums without specifically targeting one competitor. The market proxy can be interpreted as the premium of an ideal medium competitor. Consequently, Insurer j will not target the cheapest, the most expensive or the leading insurers.

Now we can state our objective function. We suppose that Insurer j maximizes the expected profit of the next year's policies which we here define in the multiplicative form

$$O_j(x) = \frac{n_j}{n}\left(1 - \beta_j\left(\frac{x_j}{m_j(x)} - 1\right)\right)(x_j - \pi_j), \tag{6}$$

i.e. the product of the demand D_j and the expected profit per policy, representing a company-wide expected profit. Thus, maximising the objective function O_j leads to a trade-off between increasing premium to favour higher projected profit margins and decreasing premium to defend the current market share. Note that O_j has the desirable property of being infinitely differentiable with respect to x.

2.4 Solvency constraint function

Another key feature of the model is a solvency constraint the goal of which is to require insurers to hold a certain amount of capital in order to protect policyholders against adverse collective claim experience. Therefore, in addition to maximizing a certain objective function, insurers must satisfy a solvency constraint imposed by the regulator. A reasonable criterion for finding the minimum capital requirement is linked to deviations of the aggregate losses from its expected value, concretely the

---

[1] Rating bureaus or rating agencies are organizations collecting statistical data from insurers in order to publish market information for both insurers and policyholders. The credibility factor is the weight given to individual loss experience in contrast to collective loss data.

difference of a high-level quantile and the mean of the loss distribution. For simplicity, this quantity is taken to be a linear function of the standard deviation of the loss distribution. In practice, the solvency capital is also required on a prospective basis : we take the simplifying assumption to use only the in-force policy number. Thus, we define the solvency constraint function as

$$g_j^1(x_j) = \frac{K_j + n_j(x_j - \pi_j)(1 - e_j)}{k\sigma(Y)\sqrt{n_j}} - 1,$$ (7)

where k is the solvency coefficient chosen to approximate a 99.5% quantile and e_j denotes the expense rate. In the following, we choose $k = 3$, see [10] for more details. The numerator corresponds to the sum of the current capital K_j and the expected profit on the in-force portfolio, whereas the denominator approximates the required capital. The constraint $g_j^1(x) \geq 0$ is equivalent to $K_j + n_j(x_j - \pi_j)(1 - e_j) \geq k\sigma(Y)\sqrt{n_j}$, but g_j^1 is normalized with respect to capital, providing a better numerical stability.

In addition to the solvency constraint, we need to impose bounds on the possible premium. A first choice could be simple linear constraints as $x_j - \underline{x} \geq 0$ and $\overline{x} - x_j \geq 0$, where $\underline{x}$ and $\overline{x}$ represent the minimum and the maximum premium, respectively. However, the following equivalent reformulation is numerically more stable:

$$g_j^2(x_j) = 1 - e^{-(x_j - \underline{x})} \geq 0 \text{ and } g_j^3(x_j) = 1 - e^{-(\overline{x} - x_j)} \geq 0.$$

The minimum premium $\underline{x}$ could be justified by a prudent approach by regulators while the maximum premium $\overline{x}$ could be set, e.g., by a consumer rights defense association. In the sequel, we set $\underline{x} = \mathbb{E}(Y)/(1 - e_{min}) < \overline{x} = 3\mathbb{E}(Y)$, where e_{min} is the minimum expense rate. Summarizing, the constraint function $g_j(x_j) = (g_j^l(x_j))_{1 \leq l \leq 3}$ for Insurer j is

$$\{x_j, g_j(x_j) \geq 0\} = \{x_j \in [\underline{x}, \overline{x}], K_j + n_j(x_j - \pi_j)(1 - e_j) \geq k_{995}\sigma(Y)\sqrt{n_j}\}.$$ (8)

2.5 Premium equilibrium

We consider two solution concepts for our game: the *Nash* equilibrium for which it is assumed that insurer actions are taken simultaneously, and the *Stackelberg* equilibrium for which actions take place sequentially. (See e.g. [14] and [22].) For the Stackelberg concept, it is assumed there is (at least) one leader acting before the so-called followers to define the game sequence. As described in [10], the Nash is the most appropriate concept for modelling competition in the absence of a clear leadership. We give now the definition of a Nash equibrilium.

Definition (Nash equilibrium). *For a game with I insurers, with payoff functions O_j and action set X_j, a Nash equilibrium is a premium vector $x^\star = (x_1^\star, \ldots, x_I^\star)$ such that for all $j = 1, \ldots, I$, $x_j^\star$ solves the subproblem*

$$\sup_{x_j \in X_j} O_j(x_j, x_{-j}^\star).$$

where x_j and x_{-j} denote the action of insurer j and the other insurers' actions, respectively. The action set X_j of Insurer j may be parametrized as $X_j = \{x_j, g_j(x_j) \geq 0\}$.

A Nash equilibrium can hence be interpreted as a point at which no insurer has an incentive to deviate, given the actions of the other insurers. In our insurance game context, we refer to a Nash equilibrium as a premium equilibrium. According to Proposition 2.1 of [10], the premium equilibrium $x^\star$ exists and is unique.

3   The dynamic framework: application to the French motor market

In practice, insurers do not play once but play an insurance game over several years as they gather new information on incurred losses, available capital and competition level. In this section, we present the dynamic framework based on the one-shot game of the previous section. Firstly, we give arguments in favor of the chosen dynamic model, compared to other possible dynamic game models. Secondly, we present the dynamic game, some properties and numerical illustrations.

3.1   Dynamic game models

Dynamic games is a complex topic compared to one-shot games. From T. Basar and G.J. Olsder [2], extending a static game to a dynamic game consists not only of adding a time dimension t for the control variable x but also requires the definition of a state equation $(\gamma_{t+1} = f(\gamma_t, \dots))$ and a state variable γ_t. The purpose of the state equation and variable is to "link" the information between players, see Definition 5.1 of [2]. Depending on which information the players have about the state variable, different classes of games are defined: open-loop (knowing only the first state γ_1), closed-loop (all states γ_t up to time t), feedback (only the current state γ_t). Computational methods for dynamic equilibrium generally use backward equations, e.g. Theorem 6.6 of [2] for feedback strategies and Theorem 6.10 in a stochastic setting. This method does not correspond to the insurance market reality for two main reasons: (i) premium is not set backwardly, the claim uncertainty is a key element in insurance pricing, (ii) the time horizon is infinite rather than finite.

A class of discrete-time games, first introduced by L. Shapley [26], use a finite-state space where a transition probability models the evolution of the current state depending on player actions. As the set of possible strategies (a series of pure or mixed actions) is huge, [26] focuses only on strategies depending on the current state. These games are referred to as Markov games. Although a Markovian property for our insurance game may be appropriate, we do neither limit our strategy space to a finite set nor use a finite-state space.

Finally, repeated games study long-term interactions between players during the repetition of one-shot finite games. The horizon either infinite or finite plays a major role in the analysis of such games, in particular where punishment strategies and threats are appropriate. Most of the theory (i.e. Folk theorems) focuses on the set of achievable payoffs rather than the characterization of the equilibrium. Folk theorems demonstrate that wellfare outcomes can be attained when players have a long-term horizon, even if it is not possible in the one-shot game, see e.g. [22]. Our game does not belong to this framework for several reasons since our strategic action sets evolve over a time, the action set is not finite and stochastic perturbations complete the picture.

We choose a repeated game but with infinite action space, such that at each period insurers set new premiums depending on past observed losses. In other words, the Nash equilibrium is computed

at each period. Our repeated game does not enter the framework of dynamic games as presented in [2], but shares some of the properties of Markov games and classical repeated games. Our approach is similar to [5] where they study the interbank system.

3.2   Game sequence

In this subsection, we describe the repeated game framework. Now insurers aggregate information as the time goes on. For period $[t, t+1[$, we denote the premium by $x_{j,t}^{\star}$, the gross written premium by $\text{GWP}_{j,t}$, the portfolio size by $n_{j,t}$ and the capital by $K_{j,t}$.

Let d be a positive integer such that the time window $[t-d, t-1]$ will be used to compute market and actuarial premiums used in the break-even premium (4). At the beginning of each time period, the average market premium is determined as

$$\bar{m}_{t-1} = \frac{1}{d} \sum_{u=1}^{d} \underbrace{\frac{\sum_{j=1}^{N} \text{GWP}_{j,t-u} \times x_{j,t-u}^{\star}}{\text{GWP}_{.,t-u}}}_{\text{market premium for year } t-u},$$

which is the mean of last d market premiums. With current portfolio size $n_{j,t-1}$ and initial capital $K_{j,t-1}$, each insurer computes its actuarially based premium as

$$\bar{a}_{j,t-1} = \frac{1}{1 - e_{j,t}} \frac{1}{d} \sum_{u=1}^{d} \underbrace{\frac{s_{j,t-u}}{n_{j,t-u}}}_{\text{avg ind loss}},$$

where $s_{j,t}$ denotes the observed aggregate loss of insurer j during year t. Thus, break-even premiums are $\pi_{j,t-1} = \omega_j \bar{a}_{j,t-1} + (1 - \omega_j) \bar{m}_{t-1}$.

In this setting, objective $O_{j,t}$ and constraint functions $g_{j,t}$ are also time-dependent. The objective function in the dynamic model is given by

$$O_{j,t}(x) = \frac{n_{j,t-1}}{n} \left(1 - \beta_{j,t-1} \left(\frac{x_j}{m_j(x)} - 1 \right) \right) (x_j - \pi_{j,t-1}),$$

and the solvency constraint function by

$$g_{j,t}^{1}(x_j) = \frac{K_{j,t} + n_{j,t-1}(x_j - \pi_{j,t-1})(1 - e_{j,t-1})}{k_{995} \sigma(Y) \sqrt{n_{j,t-1}}} - 1.$$

Note that the characteristics of insurers evolve over time notably through the break-even premium $\pi_{j,t-1}$, the expense rate $e_{j,t-1}$, the portfolio size $n_{j,t-1}$ and the sentivity parameter $\beta_{j,t-1}$.

The game sequence for period $[t, t+1[$ is as follows

1. The insurers maximize their objective function subject to the solvency constraint

$$\sup_{x_{j,t}} O_{j,t}(x_{j,t}, x_{-j,t}) \text{ such that } g_{j,t}(x_{j,t}) \geq 0.$$

2. Once the premium equilibrium vector $x_t^\star$ is determined, customers randomly lapse or renew. We get a realization $n_{j,t}^\star$ of the random variable $N_{j,t}(x^\star)$.

3. Aggregate claim amounts $S_{j,t}$ are randomly drawn according to the chosen loss model and the portfolio size by $n_{j,t}^\star$. We get a new aggregate claim amount $s_{j,t}$ for period t.

4. The underwriting result for insurer j is then computed as $UW_{j,t} = n_{j,t}^\star \times x_{j,t}^\star \times (1 - e_{j,t}) - s_{j,t}$.

5. Finally, we update the capital by the following equation $K_{j,t+1} = K_{j,t} + UW_{j,t}$.

This game sequence is repeated over T years. To reflect bankruptcy, insurers are pulled out of the market when they have either a tiny market share ($< 0.1\%$) or negative capital. Furthermore, we remove players from the game when the capital is below the minimum capital requirement (MCR), whereas we keep them if capital is between MCR and solvency capital requirement (SCR).
Let $I_t \subset \{1, \ldots, I\}$ be the set of insurers at the beginning of year t and $R_t \subset \{1, \ldots, I\}$ the set of removed insurers at the end of year t. If some insurers are removed, i.e. $\mathrm{Card}(R_t) > 0$, then corresponding policyholders randomly move to other insurers according to a I_{t+1}-dimensional multinomial distribution. Say from $l \in R_t$ to $j \in I_{t+1}$, insured randomly move with multinomial distribution $\mathcal{M}_{I_{t+1}}(n_{l,t}, p_{l \to}^-(x_t^\star))$, where the probability vector $p_{l \to}^-(x_t^\star)$ has jth component given by

$$ p_{l \to j}^-(x_t^\star) = \frac{p_{l \to j}(x_t^\star)}{1 - \sum_{k \in R_t} p_{l \to k}(x_t^\star)}. $$

When there are no more insurers, i.e. $\mathrm{Card}(I_{t+1}) = 0$, the game ends, while if there is a single insurer, i.e. $\mathrm{Card}(I_{t+1}) = 1$, the game continues and the survivor insurer set the highest premium.
In the current framework, we make the following implicit simplifying assumptions: (i) the pricing procedure is done (only) once a year (on January 1), (ii) all policies start at the beginning of the year, (iii) all premium are collected on January 1, (iv) every claim is (fully) paid on December 31 and (v) there is no inflation and no stock/bond market to invest premium.
In practice, these assumptions do not hold: (i) pricing by actuarial and marketing departments can be carried out more frequently, e.g. every 6 months, (ii) policies start and are renewed throughout the year, (iii) premium is collected throughout the year, (iv) claims are settled every day and there are reserves for incurred-but-not-reported claims and (v) there are inflation on both claims and premiums, and the time between the premium payment and a possible claim payment is used to invest in stock/bond markets. However, we need the above simplifications to have a sufficiently simple model.

3.3 Facts and figures of the French market

Now, we focus on the French motor market. This market has had a long history dating to the Greeks, see e.g. [7]. More recently, during the 90s, the insurance market experiences various privatizations and a decline of state involvements. Nowadays, insurers and mutuals are facing a fierce competition with banks, and especially on the P&C market. The motor market (both personal and corporate lines) represents roughly half of the P&C market.

Time serie models have been applied on French macroeconomic data to explain the French motor market. For instance, C. Blondeau [3] points out the existence of a 6-year insurance cycle using cointegrated time series. Blondeau shows the long term dependency between interest rate fluctuation, the gross domestic product and the combined ratio. which emphasizes the role of capital, incurred losses, inflation on the premium level. C. Bruneau and N. Sghaier [4] also study this market at an aggregate level as well as line-of-business levels. The authors want to test the validity on the French market: extrapolation pricing, rational expectation, capacity constraint. They use vector error correction models (VECM). They validate (only) the extrapolation pricing and the capacity constraint thesis and estimate a period length of 5.18 years for the French motor market. On a personal time serie of the French market premium between 1971 and 2007, we estimate a cycle period of 8.71 years using a basic auto-regressive AR(2) model, see Figure 1.
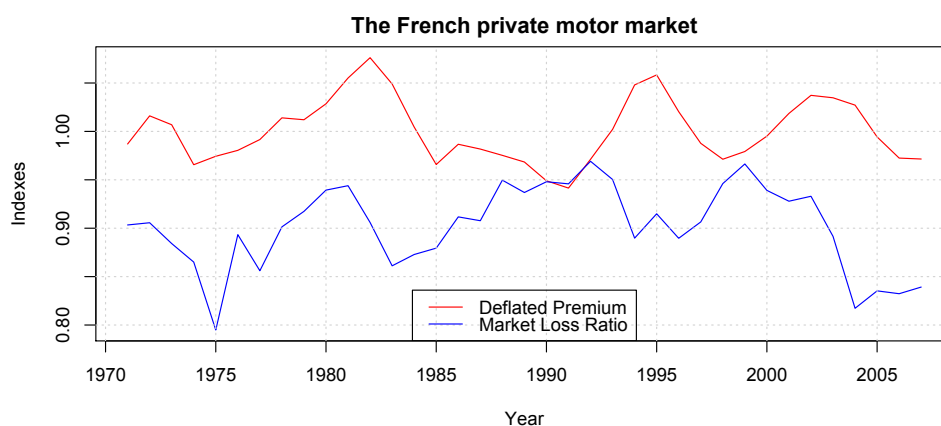
Figure 1: The French motor cycle

3.4   Game-theoretic modelling

We now consider the application of our insurance game on the French motor market. Unlike [9] where only three insurers are modelled, we model the top 10 insurers of the French market. As explained at the beginning of this section, objective and solvency constraint functions depend on parameters evolving over time: the portfolio size $n_{j,t}$, the capital $K_{j,t}$, the break-even premium $\pi_{j,t}$. Doing this, we want to mimic the real economic actions of insurers on a true market, where each year insurers update their tariff depending on last year's experience of the company.

Furthermore, we want to take into account the portfolio size evolution over time. As $n_{j,t}$ will increase or decrease, the insurer j may become a leader or lose leadership. Hence, depending on market share (in terms of written premium), we update the lapse parameters $(\alpha_{j,t}, \mu_{j,t})$, the expense rate $(e_{j,t})$ and the sensitivity parameter $(\beta_{j,t})$ on three sets of values. There is only one parameter not evolving over time: the credibility factor ω_j which is set to a common value of $\omega_j = 9/10$ in our numerical experiments.

We run our game for the four combinations of the two loss models (PLN and NBLN) and the two types of price-sensitivity functions ($\overline{f}_j$ and $\widetilde{f}_j$). On each of the 1000 simulations over $T = 25$ periods, we determine a market premium path m_t by averaging the premium equilibrium $x_t^\star$. We plot on Figure 2 some market premium paths (dashed lines) and three quantiles (solid lines) for the $\widetilde{f}_j$-NBLN model (the premium value is scaled so that 100 corresponds to the pure premium $E(Y)$ and expense rates ranges from 11% to 24%). The two plotted random paths show a cyclic behavior, whereas the three quantiles remain stable over time. On each random path, we can fit an AR(2) model $M_t = m + a_1(M_{t-1} - m) + a_2(M_{t-2} - m) + \epsilon_t$. If $a_2 < 0$ and $a_1^2 + 4a_2 \leq 0$, the fitted AR(2) is p-periodic with $p = 2\pi \arccos(a_1/(2\sqrt{-a_2}))$. Otherwise, the AR(2) is not periodic. On the Figure 3, we plot the histogram of fitted periods for the $\widetilde{f}_j$-NBLN model.
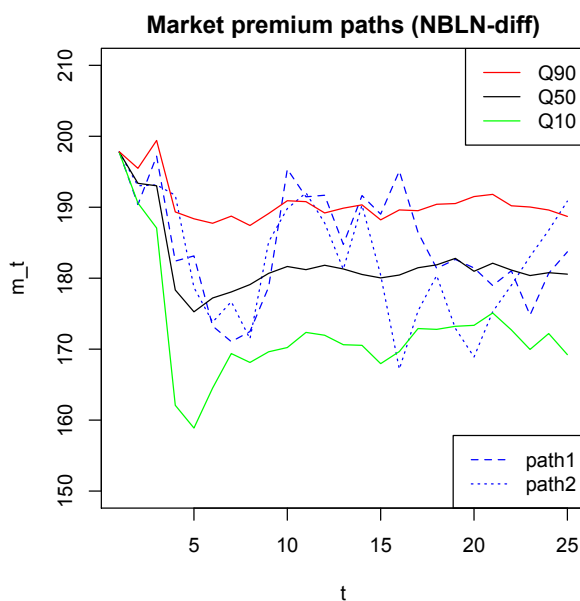
Figure 2: Market premium paths

In Table 1, we present some statistics of fitted cycle periods for the four different models: the minimum, the first quartile, the median, the mean, the third quartile, the maximum, the percentage of non-cyclic paths and the standard deviation. We observe that a quarter of market premium paths are cyclical, when $\widetilde{f}_j$ is used, whereas for $\overline{f}_j$ only 4% or 7% are not cyclical. Furthermore, the loss model seems to increase cycle periods since for NBLN loss models, quantiles are above the corresponding quantiles for PLN loss models.

Histogram of cycle periods (NBLN-diff)
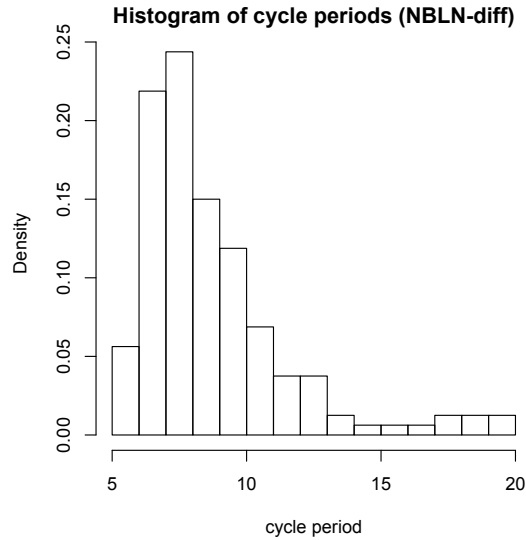
Figure 3: Cycle periods

	Min.	1st Qu.	Median	Mean	3rd Qu.	Max.	NA's	Std. Dev.
$\widetilde{f}_j$-PLN	4.618	6.193	6.738	7.354	7.735	22.74	27%	2.433
$\widetilde{f}_j$-NBLN	5.178	6.558	7.543	9.28	9.341	53.54	29%	7.277
$\overline{f}_j$-PLN	5.42	6.639	7.234	7.742	8.114	18.1	4%	1.912
$\overline{f}_j$-NBLN	5.852	7.367	8.405	9.621	10.26	33.02	7%	3.987

Table 1: Cycle periods.

4   Conclusion

Based on [10], this paper assesses the suitability of noncooperative game theory for insurance market modelling. We extend the one-player model of [27] and subsequent extensions which are based on optimal control theory. To our knowledge, the use of a repeated of noncooperative game to model non-life insurance markets is new in the current literature. First, this game-theoretic approach is the first to account for the effect of competition on insurer solvency. The proposed rational game shows that the most significant part of solvency relies on the ability of insurers to sell contracts (i.e. premium risk). This is opposite to classic risk theory where the collection of premiums is fixed per unit of time and the main risk is the randomness of losses. Secondly, this game also sheds new light on the presence of cycles in non-life insurance markets. Since for a range of parameters the market premium appears to be cyclical, we add a new argument in favor of a rational explanation (i.e. competition and loss uncertainty) for the presence of insurance cycles.

The game can be extended in various directions. A natural second step is to consider adverse

selection among policyholders. In practice, insurers do not propose the same premium to all customers. Considering two risk classes of individuals would be an interesting extension of the game. A second extension is to model investment results as well as loss reserves, which both play a major role in long-tail business. We could also consider reinsurance treaties for players in combination with a catastrophe generator. This will be the topic of future studies.

References

[1] ASMUSSEN, S. AND ALBRECHER, H., *Ruin Probabilities*. World Scientific New Jersey, 2nd edition (2010).

[2] BASAR, T. AND OLSDER, G., Dynamic Noncooperative Game Theory. SIAM (1999).

[3] BLONDEAU, C., *Les déterminants du cycle de l'assurance de dommages en france*. In Actes de XIXe Journées Internationales d'Economie (2001).

[4] BRUNEAU, C. AND SGHAIER, N., *Les cycles de souscription de l'assurance non-vie en france*. Bulletin Français d'Actuariat, 8(15):38–66 (2008).

[5] COHEN-COLE, E. AND PATACCHINI, E., *Systemic risk and network formation in the interbank market*. Working paper, CAREFIN, University Bocconi (2010).

[6] CUMMINS, J. D. AND OUTREVILLE, J. F., *An international analysis of underwriting cycles in property-liability insurance*. The Journal of Risk Finance, 54(2):246–262 (1987).

[7] CUMMINS, J. D. AND VENARD, B., *Handbook of international insurance*. Springer (2007).

[8] DREYER, V., *Study the profitability of a customer*. Master's thesis, Université Louis Pasteur de Strasbourg (2000).

[9] DUTANG, C., *Etude des marchés d'assurance non-vie à l'aide d'équilibres de Nash et de modèles de risques avec dépendance*. PhD thesis, ISFA, Université Claude Bernard Lyon I (2012).

[10] DUTANG, C., ALBRECHER, H., AND LOISEL, S., Competition among non-life insurers under solvency constraints: A game-theoretic approach. European Journal of Operational Research, 231(3), (2013).

[11] EMMS, P., HABERMAN, S., AND SAVOULLI, I., *Optimal strategies for pricing general insurance*. Insurance: Mathematics and Economics, 40(1):15–34 (2007).

[12] FELDBLUM, S., *Underwriting cycles and business strategies*. In CAS proceedings (2001).

[13] FIELDS, J. A. AND VENEZIAN, E. C., *Interest rates and profit cycles: A disaggregated approach*. Journal of Risk and Insurance, 56(2):312–319 (1989).

[14] FUDENBERG, D. AND TIROLE, J., *Game Theory*. The MIT Press (1991).

[15] GRON, A., *Capacity constraints and cycles in property-casualty insurance markets*. RAND Journal of Economics, 25(1) (1994).

[16] JABLONOWSKI, M., *Earnings cycles in property/casualty insurance: A behavioral theory*. CPCU Journal, 38(3):143–150 (1985).

[17] KLIGER, D. AND LEVIKSON, B., *Pricing insurance contracts - an economic viewpoint*. Insurance: Mathematics and Economics, 22(3):243–249 (1998).

[18] MALINOVSKII, V. K., *Competition-originated cycles and insurance companies*. work presented at ASTIN 2009 (2010).

[19] MARKHAM, F. J., *An investigation into underwriting cycles in the South African short-term insurance market for the period 1975 to 2006*. Technical report, University of the Witwatersrand (2007).

[20] MCFADDEN, D., *Econometric Models of Probabilistic Choice*. In Structural Analysis of Discrete Data with Econometric Applications, chapter 5. The MIT Press (1981).

[21] MORENO-CODINA, J. AND GOMEZ-ALVADO, F., *Price optimisation for profit and growth*. Towers Perrin Emphasis, 4:18–21 (2008).

[22] OSBORNE, M. AND RUBINSTEIN, A., *A Course in Game Theory*. Massachusetts Institute of Technology (2006).

[23] POLBORN, M. K., *A model of an oligopoly in an insurance market*. The Geneva Paper on Risk and Insurance Theory, 23(1):41–48 (1998).

[24] POWERS, M. R. AND SHUBIK, M., *On the tradeoff between the law of large numbers and oligopoly in insurance*. Insurance: Mathematics and Economics, 23(2):141–156 (1998).

[25] REES, R., GRAVELLE, H., AND WAMBACH, A., *Regulation of insurance markets*. The Geneva Paper on Risk and Insurance Theory, 24(1):55–68 (1999).

[26] SHAPLEY, L., *Stochastic games*. Proc. Nat. Acad. Sci. U.S.A., 39(10):1095–1100 (1953).

[27] TAYLOR, G. C., *Expenses and underwriting strategy in competition*. Insurance: Mathematics and Economics, 6(4):275–287 (1987).

[28] VENEZIAN, E. C., *Ratemaking methods and profit cycles in property and liability insurance*. Journal of Risk and Insurance, 52(3):477–500 (1985).

[29] WEISS, M., *Underwriting cycles: a synthesis and further directions*. Journal of Insurance Issues, 30(1):31–45 (2007).

Each year SCOR rewards the best academic papers in the field of actuarial science with the Actuarial Prizes. These prizes are designed to promote actuarial science, to develop and encourage research in this field and to contribute to the improvement of risk knowledge and management. The juries are composed of academics and insurance, reinsurance and financial professionals. The winning papers are selected using criteria including an excellent command of actuarial concepts, high-quality analysis instruments, and subjects that could have a practical application in the world of risk management.

Actuarial Prize

SCOR Global Risk Center

SCOR Papers, edited by SCOR, are one of the tool supporting the SCOR Global Risk Center.
The SCOR Global Risk Center gathers and analyses the most interesting resources about risks. It operates as a dual resource center, based both on data and resources produced by SCOR itself, and on all other resources available selected specifically by SCOR. Publications will be available in English, French and/or German.
SCOR Global Risk Center is available at www.scorglobalriskcenter.com or on SCOR's website – www.scor.com.