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Papers

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The risk-free rate:
an inescapable concept?

Abstract

The notion of risk-free rate is essential to pricing risk in insurance, but it has been put under critics particularly when it comes to choose a benchmark for the discounting yield curve. The sovereign debt crisis has put in doubt the choice of government yield and the debate is open if choosing other benchmark would be more appropriate. In this paper, we review the concept and few alternative benchmarks including the Arrow-Debreu model. We conclude that the swap rates could be a good alternative given its less volatile behaviour.

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1. Introduction

The sovereign debt crisis has conveniently reminded us that it is not possible to obtain a risk-free performance from an investment. Nonetheless, in both economic theory and actuarial practice, the concept of a risk-free rate plays a central role. A bit like physicists who pre-suppose a frictionless universe in order to derive the laws of classical mechanics, economists use the concept, introduced by mathematicians, of risk neutrality in order to derive the equilibrium price of a risky financial asset, such as an option, or the cost of capital of a company or the Sharpe ratio to assess the risk/return performance of investments.

They calculate the real-world probability (also called the physical probability), P, of financial yield fluctuations, and the probability, Q, of a probabilistic universe in which investors are indifferent to the chances of profit and/or the risks of loss. The fair price is then identified as expected future value, calculated at risk-neutral probability and therefore adjusted by an interest rate that excludes risk premium, called the risk-free rate. It should be noted here that the risk-free rate is not a rate that does not fluctuate, but a rate that contains no counterparty (credit) risk.

The definition given on its French page by Wikipedia sums up well the practice in this field[1]:

"A risk-free rate in a specific currency and for a specific period is the interest rate observed on the government bond market for countries considered to be solvent and for intergovernmental organisations in the same currency and for the same period. The rate therefore implies an absence of credit risk and not the absence of any kind of interest rate risk, which remains present".

As we see in this definition, the return on domestically held short-dated government bonds is normally perceived as a good proxy for the risk free rate. However, theoretically this is only correct if there is no perceived risk of default associated with the bond. Government bonds are conventionally considered to be relatively risk-free to a domestic holder, because there is by definition no risk of default - the bond is a form of government obligation which is being discharged through the payment of another form of government obligation. This implies the risk of the government 'printing more money' (creating inflation) to meet the obligation, which is in fact a form of tax on savings. Come the time of sovereign debt crisis, it is only but natural that the credit risks run by the holders of Greek or Portuguese debt are raising the legitimate question of whether or not such a concept is relevant. We know that the risk-free rate does not actually exist in practical terms, so the issue is not whether or not such a rate exists in reality, but rather which approximation we should use to calculate an equilibrium price for insurance liabilities or risky assets. In fact, the advantage of using a risk-neutral valuation is that it enables us to compare assets or liabilities that would otherwise not be comparable. Risk-neutral valuation enables us to optimally manage a portfolio, whilst simultaneously allowing us to divide tasks, such as determining physical distribution (P) and finding the transformations needed to move to risk-neutral probability (Q), between the various experts involved. Determining a risk-free rate is therefore a necessary step towards being able to use the full power of the method, resulting in a better economic assessment of risks than would be achieved simply through studying the balance sheet. This is what makes this concept unavoidable, yet difficult to translate into real terms.

In this short article we look at various possible ways in which to construct a benchmark rate, from which we can determine the economic value of an insurance company's balance sheet and use as a basis to define a target performance for the company. We begin by proposing a number of alternatives, which we will then explore in the second section, before concluding with a proposed compromise that should hold also for a crisis period.

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[1] The English version of the definition in Wikipedia is much more complete and contains a theoretical discussion of the concept and of the proxies used in practice parallel to the one in this article.

2. Definition of a rate representing the risk-free rate

SCOR sets a performance target, C, for its capital above the 3-month risk-free rate. This performance objective is officially communicated to investors. In the current plan (Strong Momentum), the target is set at 1000 basis points (representing a yield of 10%) on average over a three-year cycle. This objective must be linked to the type of business conducted by the company, which generates a portfolio of assets and liabilities that are managed according to their interest-rate sensitivity. For example, SCOR's property damage liabilities (P&C insurance) have a duration of around 4.2 years. We therefore need to consider the difference between 3-month rates (T_{3m}), and 5-year rates, (T_{5y}), which represent a good approximation for Non-Life liabilities. Contract pricing should set a performance objective that complies with these various constraints. The technical performance, R, which is necessary to achieve this objective, should take account of these different parameters according to the following equation:

$$R = C - (T_{5y} - T_{3m}) \qquad (1)$$

We therefore need to find the benchmark values for the T's. There are various solutions, however these are only approximations and have their own advantages and drawbacks.

According to the Wikipedia definition, we can use the implicit interest rate for government bonds as calculated, for example, by the data provider Bloomberg (Bloomberg Fair Value, BFV) for the Euro. This value is based on an aggregate BFV curve for the different member countries of the Eurozone, naturally including Germany. A possible variant would involve taking an average of the curves by country, weighted by the company's reserve exposure to these countries. More generally, we can take an average weighted by reserve exposure to different currencies.

We could also say that, at the present time, these rates no longer give a good approximation of the risk-free rate, and that we should instead be using swap rates, which would be less affected by credit risk fluctuations than government bonds. The same was said from the interbank lending rate, but the crisis of 2008 has shown how fragile the money market could be in time of stress. This is why this option did not really become popular. However, the swap rates have been advocated by the regulatory authorities to remediate to the persistence of a very low government rate environment due to the aftermaths of the financial crisis.

An even more radical position would be to consider that implicit government bond rates and swap rates are equally affected by central bank policy, and that they therefore do not properly represent the risk-free rate. The asset performances of countries considered to be safe are vastly underestimated, and benefit from a bias due to their quality and therefore from a very high level of demand, whereas the asset performances of countries in difficulty are overestimated due to the increased risk aversion of investors, and therefore are suffering from a fall in demand for this asset type. A possible way out of this dilemma would be to use the implicit forward rate in current rate curves. The argument in favour of this approach rests on the fact that these rates are less sensitive than direct rates and implicitly contain investors' expectation that the rate curve will return to average.

One final alternative would be to abandon the rates emanating directly from the markets, instead adopting prospective rates based on a view of future economic developments. We could then estimate the discount factor based on the theoretical definition of the risk-free rate given by the Arrow-Debreu[2] model. This would be an unobservable estimate based on the "golden rule" that considers the risk-free rate to be a consequence of economic agents' risk aversion:

$$T = \delta + A * \beta \qquad (2)$$

where δ is the rate of time preference for the present (often taken at a value of 1%), A is the degree of risk aversion (actually the second derivative of the utility function) and β is the certainty equivalent of the per capita growth rate.

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[2] For a simple explanation of this model, see Eeckhoudt, Gollier and Schlesinger, 2005.

3. Estimation of the risk-free rate using the Arrow-Debreu model

The last alternative presupposes that the investor has an opinion on the future development of the economy. This is no longer just a matter of observing a market value, but is about exploring various possibilities that will enable the investor to determine a rate band according to the initial scenarios. In the current climate of uncertainty in terms of overcoming the crisis, there are two possible scenarios: the crisis is prolonged by a long period of deflation, such as Japan has experienced over the past few years; or there is an uncontrolled exit from the crisis along with an increase in inflation, designed to repay the debts that were accumulated to overcome the crisis in the first place.

To illustrate this idea, we will look at various economic scenarios and their impact on the rate calculated using the Arrow-Debreu model. We have just seen that this rate is unobservable and that it depends on estimations concerning the development of the economy. In Equation (2), we gave a possible value for δ; we still need to explain how to fix the variables A and β. For A, it is clear that the interest rate will rise in proportion to the level of risk aversion, because the certainty of future revenue can only be a strong incentive to get into debt now. The value of the A parameter is unknown. Its value is generally fixed in the literature at *3*. We will assume that A is a constant parameter here and will explore two values for this quantity: $A = 1$ and *3*. The value of *1* is justified because risk aversion means that, in the current crisis situation, people are not voluntarily committing to debt. The certainty equivalent is defined using the following formula:

$$\beta = E(g) - \frac{B}{2} * \sigma_g^2 \qquad (3)$$

where $E(g)$ is the expected value of the per capita growth rate, B is the "precautionary" coefficient (which equals $A+1$ if A is constant) and σ_g is the standard deviation of the per capita growth rate. These parameters may be chosen according to our views on future market developments. In *Table 1*, we explore, as previously indicated, two types of scenario, one Japanese-style scenario of lifeless growth and even deflation, over the long term, and one scenario of high inflation. These are the scenarios generally examined during stress tests.

Table 1: The real-world risk-free rate T_R in the Arrow Debreu model based on various different economic scenarios

Type of scenario	δ	A	$E(g)$	B	σ^2	β	T_R
Japanese Deflation I	1%	1	0.7%	2	2.2%	0.65%	**1.7%**
Inflation Scenario I	1%	1	3%	2	6.6%	2.56%	**3.6%**
Inflation Scenario I with uncertainty	1%	1	2%	2	6.6%	1.56%	**2.6%**
Japanese Deflation II	1%	3	0.7%	4	2.2%	0.60%	**2.8%**
Inflation Scenario II	1%	3	3%	4	6.6%	2.13%	**7.4%**
Inflation Scenario II with uncertainty	1%	3	2%	4	6.6%	1.13%	**4.4%**

Table 1 above sets out the real-world risk-free rate T_R values using equation (2). We need to calculate the nominal rate T, which is also defined by (2) (with the corresponding parameters), and which enables us to discount cashflows. To do this, we need to introduce inflationary expectations, $E(i)$, along with the volatility of this quantity, σ_i, and to calculate the certainty equivalent of inflation, β_i:

$$\beta_i = E(i) + \frac{A}{2} * \sigma_i^2 \qquad (4)$$

We also need to introduce inflation scenarios, *i.e.* to choose $E(i)$ and σ_i values compatible with the scenarios chosen in *Table 1*. These scenarios lead to the results set out in *Table 2*, which sums up well the dilemma in which we find ourselves when we want to use an unobservable model. The results vary more than just between single and double figures, i.e. between 3.7% and 10.6%, while the inflation scenarios are decidedly moderate. It is easy to imagine the variation impact that the choice of such rates would have, for example, on the results of an embedded value calculation with cashflow durations of around 10 years.

Table 2: The nominal risk-free rate T according to different inflation parameters

Type of scenario	δ	A	$E(i)$	σ_i^2	β_ι	T
Japanes Deflation I	1%	1	2%	3.6%	2.1%	**3.7%**
Inflation Scenario I	1%	1	3%	3.6%	2.1%	**5.6%**
Inflation Scenario I with uncertainty	1%	1	2%	6.6%	2.1%	**4.6%**
Japanes Deflation II	1%	3	2%	3.6%	3.2%	**5.0%**
Inflation Scenario II	1%	3	3%	3.6%	2.2%	**10.6%**
Inflation Scenario II with uncertainty	1%	3	2%	6.6%	2.2%	**6.6%**

Clearly, it is therefore not acceptable to base pricing calculations on such rates, which are justified purely theoretically and which, in any case, do not give a rate curve enabling us to properly discount the company's asset and liability cashflows. Moreover, we are reduced to finding an observable market solution that can be used as a benchmark to compare our results to those of our peers. This solution should fully satisfy the banking analysts monitoring insurance companies, since it will make the results much clearer.

4. Towards a market proxy for the risk-free rate

Earlier on we looked at several different possibilities, based on market observations, for defining a rate that could be used as a basis for the risk-free rate. In this section, we will examine and discuss the evolution of these possible rates. We will consider data on the implicit government bond rates, as published on Bloomberg (the BFV rates), as well as the swap rates published by the same data provider. We take three observation points: the first two around a year apart, 22 September 2010 and 7 September 2011, and a third, closer point of 28 November 2011. Between the last two dates, the sovereign debt crisis became more acute and had a non-negligible impact on interest rates, as we will see later on. Thus, we are covering a

sufficiently representative period. The calculations that use the risk-free rate do not update their parameters overly frequently, limiting themselves to time intervals of around one or two months. Nonetheless, a major change in the economic environment will always be followed by an update of the rates in our pricing systems.The numbers shown in Table 3 below present the rates as they appear on the Bloomberg terminal, with one exception: the rate weighted by the liability exposure of a major international insurer to the different countries of the Eurozone. We see that weighting according to the exposure of an insurance company gives very different results at all maturity levels. However, there is no difference for swap rates because Bloomberg does not give different rates by country. There is just one single swap rate curve for the entire Eurozone. The difference between possible benchmark rates for the Euro oscillates between 60 and 80 basis points, depending on the dates on which it is measured, which is a considerable

difference given the fact that these rates will be used to discount cash flows that may be very long. For certain currencies, such as the AUD (the Australian dollar), the difference may be as much as 100 basis points. This is no doubt due to a higher interest rate on this currency than on others.

In these tables, we see that the most stable rates are the swap rates. For example, the compound 5-year Euro swap rate only varies by 10 basis points between the three observations, whereas the compound government Euro rate varies by more than 50 points. As for the forward compound rate, this varies by 40 points. If we look at the differences between the 3-month and the 5-year rates set out in the following tables, the variations are smaller but still reach 40 points for government rates and 25 points for swap rates. Therefore, swap rates provide greater stability and offer a viable alternative to government rates in this volatile period.

Table 3: Evolution of government and swap rates

28.11.2011	Government Rates				Swap Rates		
	Gov, 3m	Gov, 5y	Fwd (4y, 1y)	Fwd (5y, 1y)	Swap 5y	Fwd (4y, 1y)	Fwd (5y, 1y)
EUR	0.29%	2.04%	2.42%	2.65%	2.10%	2.11%	2.33%
EUR (Weighted by exposure)	1.20%	2.99%	3.36%	3.57%	2.10%	2.11%	2.33%
USD or USD pegged	0.03%	0.92%	1.12%	1.42%	1.34%	1.43%	1.70%
GBP	0.39%	1.23%	1.44%	1.76%	1.72%	1.70%	1.90%
CAD	0.93%	1.49%	1.63%	1.80%	1.52%	1.51%	1.74%
CHF	0.02%	0.33%	0.41%	0.61%	0.68%	0.76%	0.99%
AUD	4.11%	3.34%	3.36%	3.57%	4.32%	4.48%	4.61%
JPY	0.11%	0.40%	0.47%	0.63%	0.53%	0.53%	0.65%
Compound Rates	**0.32%**	**1.53%**	**1.80%**	**2.06%**	**1.79%**	**1.82%**	**2.05%**
	0.73%	**1.96%**	**2.22%**	**2.48%**	**1.79%**	**1.82%**	**2.05%**

07.09.2011	Government Rates				Swap Rates		
	Gov, 3m	Gov, 5y	Fwd (4y, 1y)	Fwd (5y, 1y)	Swap 5y	Fwd (4y, 1y)	Fwd (5y, 1y)
EUR	0.53%	1.51%	1.74%	1.94%	1.99%	1.96%	2.20%
EUR (Weighted by exposure)	0.78%	1.98%	2.25%	2.45%	1.99%	1.96%	2.20%
USD or USD pegged	0.05%	0.93%	1.12%	1.41%	1.19%	1.30%	1.62%
GBP	0.45%	1.39%	1.60%	1.88%	1.73%	1.76%	2.01%
CAD	0.91%	1.46%	1.60%	1.83%	1.59%	1.62%	1.88%
CHF	0.02%	0.54%	0.66%	0.80%	0.90%	1.04%	1.27%
AUD	3.98%	3.93%	3.98%	4.12%	4.74%	4.84%	4.95%
JPY	0.11%	0.34%	0.39%	0.52%	0.45%	0.42%	0.54%
Compound Rates	**0.44%**	**1.33%**	**1.53%**	**1.77%**	**1.71%**	**1.74%**	**2.00%**
	0.55%	**1.54%**	**1.76%**	**2.00%**	**1.71%**	**1.74%**	**2.00%**

07.09.2011	Government Rates				Swap Rates		
	Gov, 3m	Gov, 5y	Fwd (4y, 1y)	Fwd (5y, 1y)	Swap 5y	Fwd (4y, 1y)	Fwd (5y, 1y)
EUR	0.53%	1.51%	1.74%	1.94%	1.99%	1.96%	2.20%
EUR (Weighted by exposure)	0.78%	1.98%	2.25%	2.45%	1.99%	1.96%	2.20%
USD or USD pegged	0.05%	0.93%	1.12%	1.41%	1.19%	1.30%	1.62%
GBP	0.45%	1.39%	1.60%	1.88%	1.73%	1.76%	2.01%
CAD	0.91%	1.46%	1.60%	1.83%	1.59%	1.62%	1.88%
CHF	0.02%	0.54%	0.66%	0.80%	0.90%	1.04%	1.27%
AUD	3.98%	3.93%	3.98%	4.12%	4.74%	4.84%	4.95%
JPY	0.11%	0.34%	0.39%	0.52%	0.45%	0.42%	0.54%
Compound Rates	0.44%	1.33%	1.53%	1.77%	1.71%	1.74%	2.00%
	0.55%	1.54%	1.76%	2.00%	1.71%	1.74%	2.00%

We note a certain degree of convergence of the different rates between September 2010 and September 2011, although the acceleration of the sovereign debt crisis from September to November clearly brought this difference back to September 2010 levels. This can be seen in the final two tables (Table 4 and Table 5), where we show the difference between 3-month rates and 5-month rates weighted by the insurance company's liability exposure, either by currency or by country. For example, the difference between the 3-month and 5-year rates falls from 1.29% in September 2010 to 0.89% in September 2011, rising again to 1.21% in November 2011.

Table 4: Evolution of the rate differential between the 3-month rate and the corresponding rate

	Delta vs. IR (Gov, 3m)	22.09.2010	07.09.2011	28.11.2011
Compounded by currency exposure	Gov, 5y	1.29%	0.89%	1.21%
	Fwd (Gov 4y, 1y)	1.57%	1.09%	1.47%
	Fwd (Gov 5y, 1y)	1.86%	1.33%	1.74%
	Swap 5y	1.52%	1.27%	1.47%
	Fwd (Swap 4y, 1y)	1.68%	1.30%	1.50%
	Fwd (Swap 5y, 1y)	1.92%	1.56%	1.73%
Compounded By country exposure	Gov, 5y	1.38%	0.99%	1.23%
	Fwd (Gov 4y, 1y)	1.67%	1.21%	1.49%
	Fwd (Gov 5y, 1y)	1.96%	1.45%	1.75%

Table 5: Evolution of the differential between the two weightings (countries - currencies)

	Country-currency split	22.09.2010	07.09.2011	28.11.2011
Gov IRs*	Gov, 3m	0.01%	0.11%	0.41%
	Gov, 5y	0.10%	0.22%	0.43%
	Fwd (Gov 4y, 1y)	0.11%	0.23%	0.42%
	Fwd (Gov 5y, 1y)	0.11%	0.23%	0.42%

*) This table does not show a swap rate alternative since Bloomberg gives only one swap for the Eurozone

Tables 4 and 5 clearly show that the differences between the various rates may be significant and may be heavily dependent on the politico-economic environment. This simple study has the merit of showing that the swap rates offer a greater level of stability and represent a good compromise between the different options based on market rates, even though their absolute value is slightly higher than that of government rates. It is significant that swap rates are among the candidates being studied by the EIOPA as substitutes for implicit government bond rates, because, un-like the latter, they offer a liquidity premium. This alone is not a sufficient argument, but their relative stability and their low level of variation compared to government rates plead in their favour. Moreover, the economic situation does not justify the use of government rates, which for some are clearly under-valued (rates of States benefiting from a quality premium) and, for others, thoroughly over-valued (States with debt problems). Bloomberg has the merit of presenting just one single swap rate for the entire Eurozone.

5. Conclusion

After these reflections, the reader may well be asking what is actually the point is of all this deliberation. After all, the risk-free rate is a theoretical construction that we are trying to place in a world from which it is impossible to eliminate risk. Once again, we need to find a way to implement more market practice-compatible methods of valuing the assets and liabilities of an insurance company. These methods are based on theoretical models with highly simplifying hypotheses. These simplifications become even more caricatured in a period of severe economic and financial volatility. It is, therefore, legitimate to question whether they can really be adapted to new data.

Despite all this, we do not think that the weaknesses justify abandoning methods that have proved effective in other periods, and which facilitate more efficient risk management. Nevertheless, these methods must be adapted to the situation at hand in order to properly assess the risks run by insurance companies, whilst maintaining a certain level of consistency. To return to the analogy used in the introduction, the dawn of Newtonian mechanics led to fundamental technological advances in the field of engineering, even though they basically ignore the friction that forms a crucial part of physical reality. Engineers were obliged to reintroduce this friction into their techniques, without losing the benefits of the theory. Actuaries have the same task before them as part of the economic assessment of risks.

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