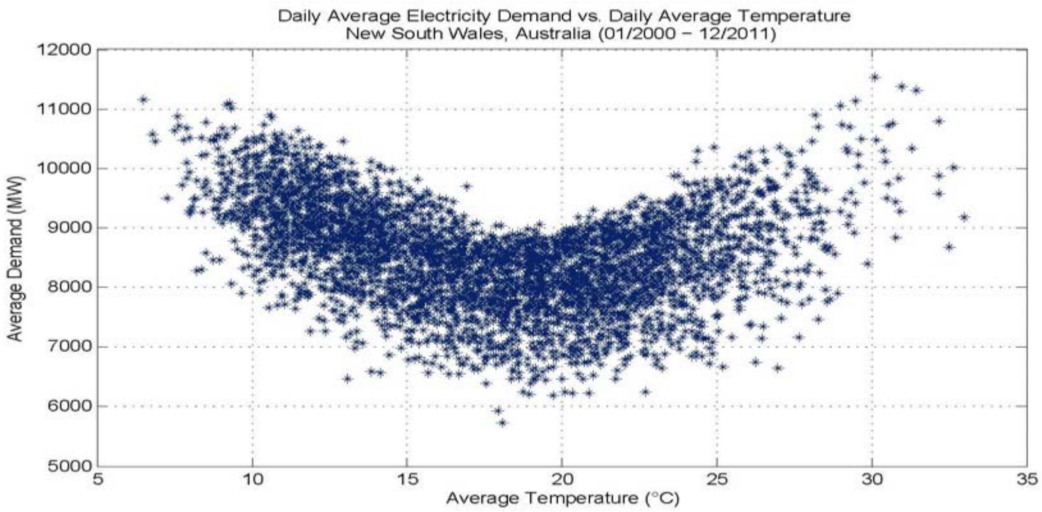
Abstract

The Master's Thesis underlying this paper was written at the Technical University of Munich in cooperation with Munich Re. It was sponsored by Munich Re and supervised by Prof. Dr. Matthias Scherer (TU Munich) and Dipl.-Math. Ralf Hungerbühler (Munich Re).

The intention of this paper is to build an appropriate model for the electricity demand and temperature in New South Wales (NSW), Australia. Because a strong correlation exists between temperature and electricity demand, the proposed model accounts for the interaction between both of them and uses temperature information to improve the modeling accuracy of the electricity demand.
The following figure shows the dependency structure between temperature and electricity demand, which is mainly shaped in this way because of the use of air-condition during hot days and intensive heating during cold days. One can clearly notice that a temperature around 18 °C leads to the lowest electricity demand for those observations. This is caused by the fact that human people feel comfortable at around 18 °C without having a need for cooling or heating. One can observe such a dependency structure at many places over the world. Therefore, the topics this paper deals with can easily be transferred to other locations in the world.



First, an overview of weather derivatives is given and the basic underlyings for temperature derivatives, namely Cooling Degree Days and Heating Degree Days, are introduced. Then different modeling and valuation approaches are mentioned.
The following section deals with a model for the daily average temperature of NSW. The model considers seasonality in the temperature during each year as well as variability in the variation. Finally, the autocorrelation between different days is also captured within the model.
Afterwards, a structurally related model for the daily average electricity demand is established. For this, an autoregressive model including temperature as an exogenous variable to account for the interaction between temperature and electricity demand is introduced.

Subsequently, the model is extended to generate half-hourly electricity demand values. Therefore, multiple linear regressions are used and further explanatory variables like the day of the week are taken into account.
Finally, a stylized financial contract whose pay-off depends on the temperature as well as on the half-hourly electricity demand is evaluated using Monte-Carlo simulations generated by the developed model.

1 Weather Derivatives

In the temperature market, especially with focus on the energy industry, traded temperature derivatives are mostly written on Degree Day Indices. Those are useful because they are designed to correlate well with the domestic demand for heating and cooling. The Cooling Degree Day Index is defined in the following way:

Definition 1: Cooling Degree Days (CDDs) (Brix, 2005)
The value of CDDs on a particular day i is defined as

$$CDD_i := \max(T_i - T_0; 0) = (T_i - T_0)^+,$$

where T_i is the average temperature on day i and T_0 is a baseline temperature. A CDD index over a period from τ_1 till τ_2, $\tau_1, \tau_2 \in \mathbb{N}, \tau_1 \leq \tau_2$ is defined as the sum of the CDDs over all days during that period, i.e.

$$CDD_{(\tau_1,\tau_2)} = \sum_{i=\tau_1}^{\tau_2} CDD_i.$$

Analogously, the Heating Degree Day Index is defined the other way around. For the baseline temperature normally a temperature of 18 °C is used, justified by the reasons explained in the abstract. Most of the temperature derivatives, like options or futures, are written based on those underlying indices, and therefore, one can think of different evaluation methods.

A common method is *Burn Analysis,* where the fair value of a contract is estimated from the distribution of historical payouts. Another method is to use *Simulation Technique.* Here, one builds a model for a specified index like CDD (*Index Modeling),* or one simulates the underlying time series, for example the daily mean temperature (*Daily Modeling*). In both cases Monte Carlo simulation is used for pricing. In this paper, the daily modeling method is used, as the stylized financial contract depends on the temperature as well as on the electricity demand. Therefore, in the following sections, a model for temperature and a model for electricity demand including temperature information are presented.

2 Modeling temperature of New South Wales

The main focus of this paper is to build a model to price a contract contingent on the average temperature and the electricity demand. Therefore, the model is kept as general as possible to stay in the same structural model class for both time series. The intention is to use a discrete ARMA/GARCH setting with some modifications. In a very general form, a temperature model has the following structure:

$$T_i = trend_i + s_i + \sigma_i T_i^{'}, \quad i \in \mathbb{N}.$$

Here, $trend_i$ captures a trend over a long period, i.e. many decades, and s_i describes the seasonal trend, which exists during every year. σ_i deals with the variability in the temperature variation over the season and $T_i^{'}$ describes the remaining temperature anomalies.

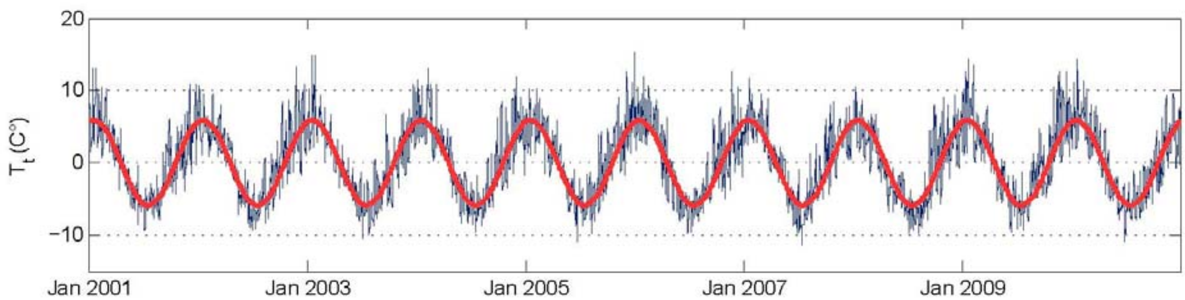
The daily mean temperature data used for the temperature analysis is delivered by the Australian Bureau of Meteorology for the station located at Bankstown Airport in New South Wales.
To account for the long-term trend $trend_i$ in the temperature observations, a linear trend function is fitted to the yearly mean temperature over 42 available years of data. This trend can be explained by the global warming phenomena. For the underlying 42 years of data, the yearly mean temperature differs by 0.64 °C.

To capture the seasonal trend, following the Ansatz of Benth (F.E. Benth, 2008), a truncated Fourier series is fitted to the trend-removed 15330 points of data representing 42 years without leap year days. The function has the following structure

$$s(t) = a_1 \sin\left(\frac{2\pi t}{365}\right) + a_2 cos\left(\frac{2\pi t}{365}\right)$$

and the parameters are estimated by using the standard least square method.

The following figure shows a sequence of detrended temperature data together with the fitted function $s(t)$ in red.



To account for the variability in the temperature variation over the season, again a truncated Fourier series, namely $\sigma^2(t)$, is used. The function

$$\sigma^2(t) = c + \sum_{k=1}^{2} c_k \sin\left(\frac{2\pi kt}{365}\right) + \sum_{l=1}^{2} d_l cos\left(\frac{2\pi lt}{365}\right)$$

is fitted to the calculated empirical variance values for each day of the year, i.e. to 365 values for the variance generated from 42 observations for each day. Therefore, the data is filled into a matrix to generate one column for every day of the year. According to the available data, every column has 42 values. Then the empirical variance for every column, i.e. for every day of the year, is calculated.
As a result, one observes a clear seasonal volatility effect in the temperature observations. During summer months the variation is higher than during winter months.

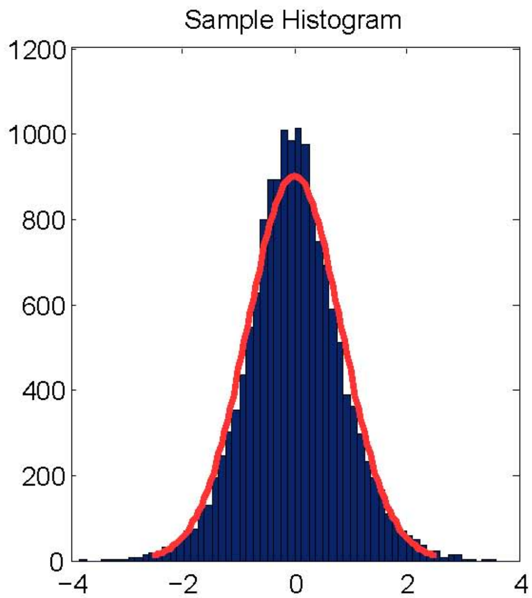
One ends up with the model for temperature anomalies to be

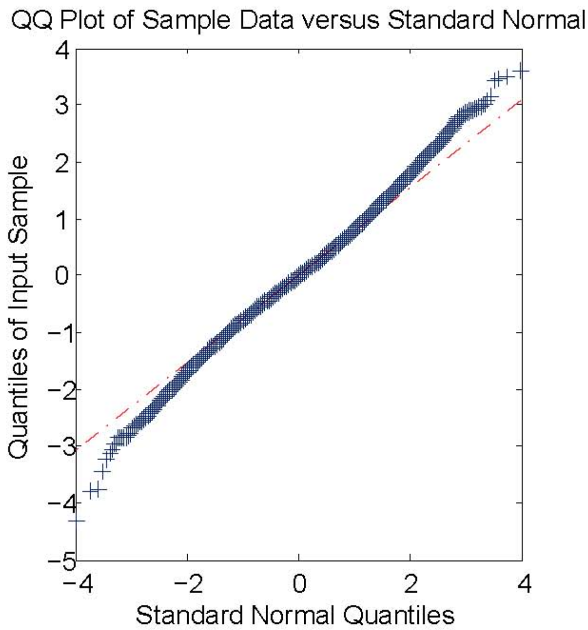
$$\frac{T_t - trend_t - s(t)}{\sigma(t)} = T_t^{'}.$$

Then an AR(1) process is used to capture the dependence of the remaining temperature anomalies $T_t^{'}$ to the values of the previous day, i.e.

$$T_t^{'} = \phi_1 T_{t-1}^{'} + \epsilon_t, \;\; t = 2, \dots, 15330.$$

The histogram and the qq-plot of the remaining innovations ε_t in the following figure suggest a good adequacy of the proposed model for the New South Wales data.





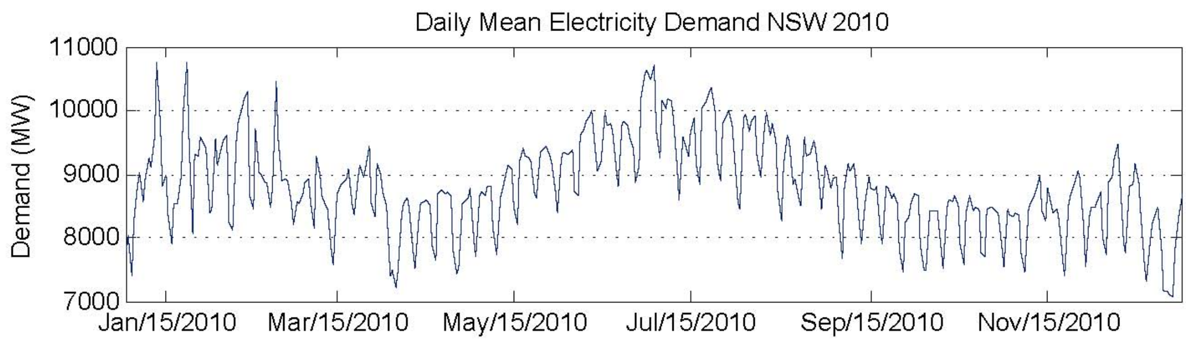
3 Modeling Electricity Demand for New South Wales

In this section, a structurally similar model to the temperature model is proposed for the daily mean electricity demand simulation of New South Wales. The main idea of this model is to include the knowledge of the temperature to enhance the electricity demand simulation. The daily mean electricity demand data comes from the Australian Energy Market Operator (AEMO) website, where also the half-hourly electricity demand for New South Wales, used in the following section, is published

At first, similar to the temperature model, a long-term trend is removed from the data. As the macroeconomic environment influences the electricity demand in a long-term view, a polynomial function of grade two is fitted to the data, i.e.

$$trend_t = a + bt + ct^2.$$

The following figure shows that one has to account for a weekly pattern additionally to a yearly cyclic seasonality for modeling electricity demand.



The weekly pattern is removed from the data by using the *Moving Average Technique* (Weron, 2006) to fit a deterministic weekly function s_t^{week} to the data. As a result, explicit values are estimated for each day of the week. The results are given in the following table.

Sun	Mon	Tue	Wed	Thu	Fri	Sat
-782.70	111.92	311.62	291.93	313.36	221.25	-467.38

One can clearly observe that the electricity demand is estimated to be the lowest every Sunday and lower than during weekdays on Saturday. This is consistent to the observations from the historical electricity demand data and those observations make sense because on weekends less electricity is needed by the industry.

The seasonality is again modeled by a truncated Fourier series but with more summands than the one used for the temperature model, i.e.

$$s(t) = \sum_{k=1}^{6}\left(a_k \sin\left(\frac{2\pi kt}{365}\right) + b_k cos\left(\frac{2\pi kt}{365}\right)\right).$$

To capture the variation in the seasonality, the same function for $\sigma^2(t)$ is used as in the temperature modeling case. The only difference now is that one has only 11 years of observations to estimate the daily variance compared to 42 years of temperature observations. In the electricity demand case one obtains a seasonal cyclic structure for the variation in the seasonality which has two local maxima within the year. Thus, variation in electricity demand is highest during summer months but also high during winter months. It is low during spring and autumn.

Summarizing the electricity demand model up to this point, the remaining demand anomalies $D_t^{'}$ are given by

$$\frac{D_t - trend_t - s_t^{week} - s(t)}{\sigma(t)} = D_t^{'}.$$

To include the temperature information in the next step, an extension of an AR-model, namely an ARX-model, is used for modeling the remaining dependency between the demand anomalies.

Definition 2: ARMAX Model (Weron, 2006)

For $t \in$ Z the autoregressive moving average model with exogenous variables $v^1, \dots, v^k$, or $ARMAX(p, q, r_1, \dots, r_k)$ can be compactly written as

$$\phi(B)D_t^{'} = \psi(B)\epsilon_t + \sum_{i=1}^{k} \beta^i(B)v_t^i,$$

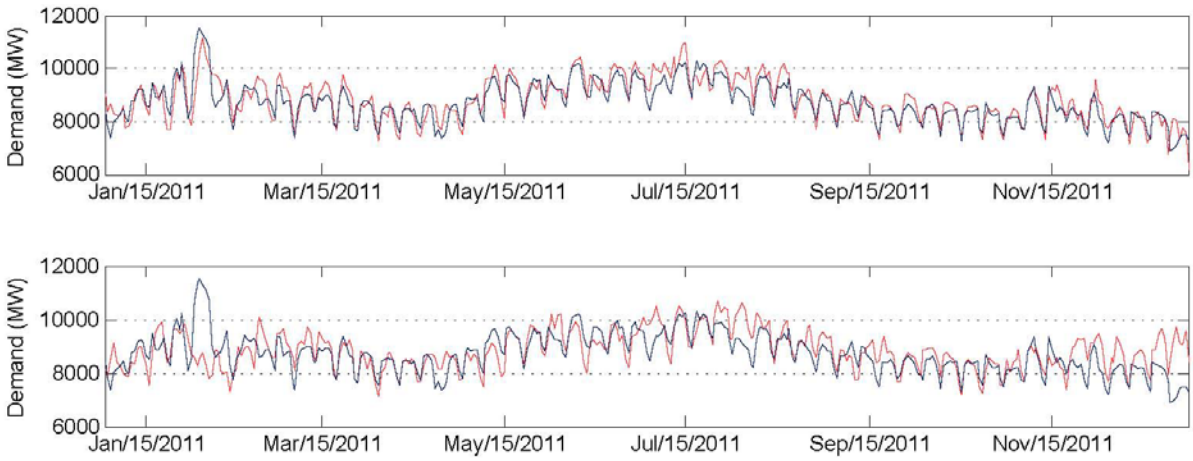
where the r_i are the orders of the exogenous factors. With B denoting the backstep operator, $\beta^i(B)$ is a shorthand notation for

$$\beta^i(B) = \beta_0^i + \beta_1^i(B) + \cdots + \beta_{r_i}^i(B^{r_i})$$

with β_j^ibeing the correspondent coefficients.

For the exogenous variable influencing the ARX(1)-process, statistically significant variables are used. The remaining temperature anomalies $T_t^{'}$ are taken into the model as well as their interaction with a summer and winter dummy variable. Therefore, the interaction terms are highly significant and one accounts for the smile shaped correlation structure, seen in the first figure of this paper, between temperature and electricity demand. Including the interaction between each season or the squared temperature anomalies does not improve the model significantly and is therefore not taken into account.

Taking a look at the modeling results in the following figure, one can clearly see the additional model accuracy when using the temperature information as exogenous variable in the electricity demand modeling procedure. The figure shows the out-of sample results for 2011 while the model was fitted to data from 1969 until 2011 for the temperature and from 2000 until 2010 for the electricity demand. The blue series represents the historically observed electricity demand and the red series shows one single simulation path with temperature influence (upper graph) and without temperature influence (lower graph).

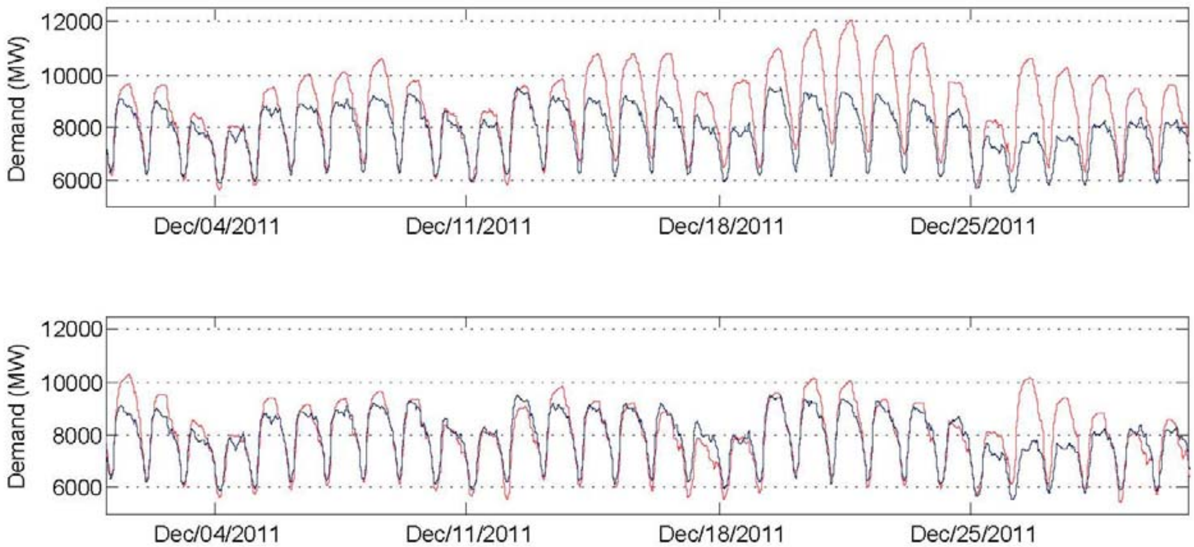


4 Modeling Half-Hourly Electricity Demand

For being able to evaluate different contracts which depend in their pay-off on the half-hourly electricity demand, the proposed model is extended to model half-hourly values in addition to daily average electricity demand values. The basic idea is to use linear regression technique to predict 48 model points for each day. Therefore, 48 linear regressions are calculated simultaneously using the daily average electricity demand coming from the previous model as explanatory variable. Additionally, dummy variables for the day of the week as well as for summer and winter are included into the regression. This ensures a very comfortable r-squared of 0.92 in mean over the 48 regressions.

In summary, the regression model has the following setting: The historically half-hourly electricity demand observations are grouped so that one gets 4018 observations for each half-hourly point in time to fit the regression. As basic configuration, a Saturday in autumn or spring is used. So all regression betas have to be viewed in comparison to a Saturday in autumn or spring.

As a result, the model captures the daily shape of electricity demand, i.e. low during the night and peaks during the day dependent on the day of the week and the season. The out-of-sample results for a few days in December 2011 are given in the following figure (in the upper graph without temperature influence and in the lower graph with temperature influence).



5 Monte Carlo Pricing and Further Applications

Finally, one is able to use the model to price various financial contracts written on the daily mean temperature, the daily mean electricity demand and/or the half-hourly electricity demand. Such contracts can help companies to insure against higher or lower electricity demand caused by abnormal high or low temperature evolutions. Due to the fact that the underlying series are modeled instead of pre-specified indices like the introduced Cooling Degree Days for example, the proposed model is very flexible for pricing all kinds of pay-off functions.

For further research, one can think of including the electricity price into the model. There should also exist a high correlation between the electricity price and the electricity demand as well as the temperature which could be modeled in a more sophisticated setting. Another approach to extend the model could be to incorporate weather forecasts to increase the accuracy of the forecasting simulations.

Major Reference List

Brix, S. J. (2005). *Weather Derivative Valuation.* Cambridge University Press.

Benth, F. E. et all (2008). *Stochastic modelling of electricity and related markets.* World Scientific.

Weron, R. (2006). *Modelling and Forecasting Electricity Loads and Prices.* Wiley.