SCOR Papers

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Modelling operational risk in the insurance industry

Abstract

One of the major new features of Solvency II is that insurance companies must now devote a portion of their equity to covering their exposure to operational risks. The regulator has proposed two approaches to calculating this capital requirement: a standard and a more advanced approach. The standard approach is a simplified calculation of a percentage of premiums or reserves. The advanced approach uses an internal model of risk that corresponds to the company's real situation. The EIOPA has encouraged insurance companies to adopt the internal model by structuring the standard approach such that it uses up much more equity, as we noted in Quantitative Impact Study 5 (QIS5). This paper proposes an approach that distinguishes frequency risks from severity risks. Frequency risks are defined as the risk of suffering small losses frequently. They are modelled by the Loss Distribution Approach. Severity risks represent the risk of large but rare losses. They are modelled by Bayesian networks.

Note: because we have modelled frequency and severity risks separately, the frequency risk model is much less sensitive to technical choices (adjusted distributions, aggregation).

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1. Using the Loss Distribution Approach to Model Frequency Risk

The Loss Distribution Approach consists in adjusting statistical distributions to loss data. More specifically, this means modelling i) the frequency of loss events and ii) their severity, then combining them to obtain the distribution of total losses. This approach is often used to model total insurance losses as part of the rate-setting or provisioning process.

To calibrate the model, historical loss data must be available.

Step 1: choose and calibrate frequency and severity distributions

For each risk k, the loss distribution will be modelled as follows:

$$S_k = \sum_{j=1}^{N_k} X_k^{(j)}$$

where N_k is the random variable representing the number of losses for each risk k

$X_k^{(j)}$ is the random variable representing the amount of loss j for the risk k

S_k is the sum of all losses for the risk k.

The Poisson, binomial and negative binomial distributions are the ones generally used for frequency.

There is a virtual consensus in the industry in favour of the maximum likelihood method for estimating the distribution parameters.

The choice of model is then validated by statistical testing.

Step 2: Building the total loss distribution

Monte-Carlo simulation gives a good approximation of the loss distribution function S_k.
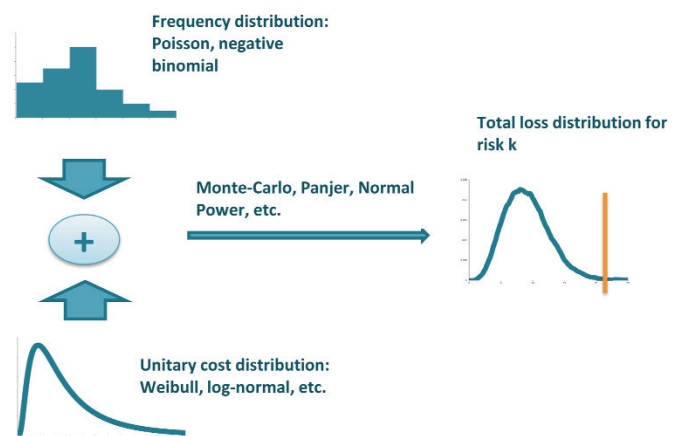

Practical problem: the data collection threshold

In general, operations staff cannot declare all losses; rather, a declaration threshold must be set. The observable data will then consist only of the losses that exceed this threshold. In our adjustments, we will have to take into account the fact that the data are truncated and modify the parameter estimates and curve fitting tests as a result. Otherwise, we will underestimate the frequency and adjust the distribution of individual costs incorrectly.

Individual cost distribution:

The data below the threshold are not reported but must be taken into account in estimating the distribution parameters. Specifically, these parameters will have to be estimated taking into account the lack of data to the left of the bars shown in the graph below.

This graph represents the cost of individual losses and the theoretical distribution readjusted to account for the truncated data:

Data collection threshold
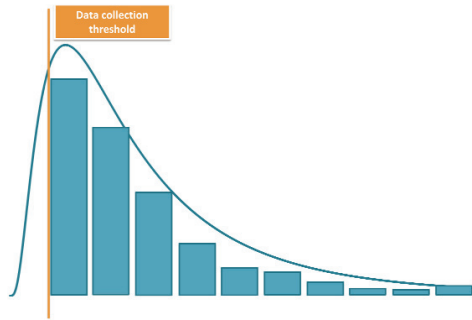
Let U be the loss data collection threshold.

Let X be the random variable representing the cost of non-truncated losses, i.e. losses including amounts that might be less than U, and F its distribution function.

Only the distribution $X \mid X \geq U$ generates observable data. This conditional distribution is the distribution of X truncated to the left of U.

Knowing that the observable data are beyond the threshold U, we can specify the truncated distribution function:

$$P(X < x \mid X \geq U) = \frac{P(U \leq X < x)}{P(X \geq U)} = \frac{F(x) - F(U)}{1 - F(U)}$$

$$for\ all\ x > U$$

$$P(X \leq x \mid X \geq U) = 0 \quad for\ all\ x \leq U$$

The graph below shows how a log-normal distribution $\ln(2;1)$ is deformed depending on where the data collection threshold is positioned.

— f(x)
— f(x|X>27)
— f(x|X>53)
— f(x|X>60)
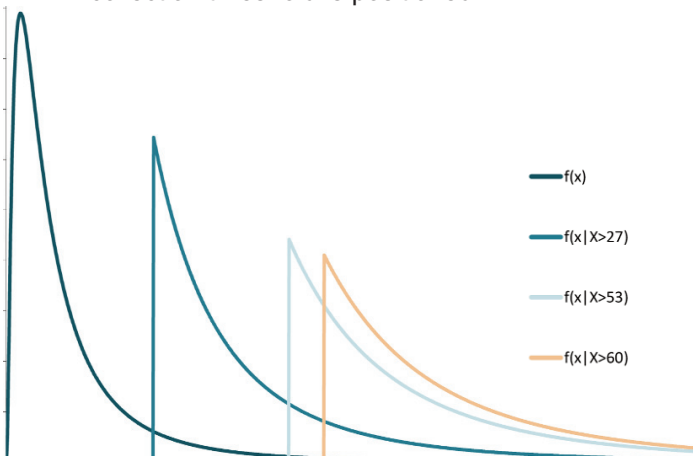
Note: certain distributions, e.g. exponential and Pareto, remain stable when truncated to the left.

The maximum likelihood method enables us to calibrate our distributions but does not yield an analytical solution.

Let $(x_1, x_2,...,x_n)$ be a sample of losses whose amount exceeds the threshold U. The likelihood condition at threshold U can be written as follows:

$$\prod_{i=1}^{n} \frac{f(x_i)}{P(X_i \geq U)} = \prod_{i=1}^{n} \frac{f(x_i)}{1 - F(U)}$$

The log-likelihood can then be written:

$$\sum_{i=1}^{n} \ln\left(\frac{f(x_i)}{1 - F(U)}\right) = \sum_{i=1}^{n} \ln(f(x_i)) - n \ln(1 - F(U))$$

The parameters are estimated in the traditional manner by maximising log-likelihood.

Frequency distribution:

When there is a data collection threshold, the observed loss frequency is underestimated and must be adjusted to take undeclared losses into account. For each observation period i, the number of losses, indicated n_i, is increased by the estimated number of losses under the threshold, indicated m_i, as follows:

$$n_i^r = n_i + m_i$$

The relationship between the number of losses under the threshold and the number of observable losses is the same as the relationship between the portion below the density curve of individual losses to the left of the threshold and the portion to the right of the threshold:

$$\frac{m_i}{n_i} = \frac{\hat{F}(U)}{1 - \hat{F}(U)}$$

where $\hat{F}$ is the truncated distribution function of X, whose parameters have been estimated with the preceding method.

From which we can derive:

$$n_i^r = n_i + m_i = n_i + \frac{n_i \times \hat{F}(U)}{1 - \hat{F}(U)} = \frac{n_i}{1 - \hat{F}(U)}$$

2. Modelling severity risk using the Bayesian method

The Bayesian approach consists in carrying out a qualitative risk analysis based on expert knowledge and in transforming it into quantitative analysis.

A Bayesian network is a probabilistic causal graph representing the existing knowledge in a given domain. It is made up of discrete random variables, called nodes, linked by ordered arcs that represent a causal relationship.

Specifically, we have studied the exposure–occurrence–severity model. The task consists in defining and modelling these three characteristic measures of risk, which are in turn influenced by variables called key risk indicators (KRI).

We present below the methods for evaluating the various elements of the Bayesian network.

a) Evaluating exposure

Exposure represents all of the elements of the company that are exposed to risk. It must be defined in such a way that the risk can materialise into an occurrence only once per year.

b) Evaluating occurrence

As the exposed elements are chosen such that there can be a maximum of only one loss, occurrence will be a binominal distribution B(n,p), where n is the number of exposed elements and p the probability to be estimated.

c) Evaluating severity

We assume that there has been an occurrence, and we identify the quantifiable variables (KRI) involved in the calculation of severity.

The structure of the Bayesian network is defined by experts via scenarios. The parameters of the Bayesian network can be determined empirically or based on expert knowledge.

Once the Bayesian network is constructed, the calculation algorithm needs to be defined.

Let $(X_1, X_2,...,X_n)$ be the elements exposed to the operational risk in question.

Let $P_i = P(\text{Exposure} = X_i)$ be the probability that the exposure indeed represents the elements X_i.

Let $PO_i = P(\text{Occurence} = \text{«Yes»} \mid \text{Exposure} = X_i)$ be the probability that the risk materialises into an occurrence, given that the exposure is X_i.

These two probabilities are known (they were estimated as previously described).

Let $PS_i = P(\text{Severity} \mid \text{Occurence} = \text{«Yes»}$ and $\text{Exposure} = X_i)$ be the severity distribution, given that a loss occurred on elements X_i.

The algorithm consists in carrying out the following steps in succession:

1) position Exposure at X_i and Occurrence at Yes in the Bayesian network and read the severity distribution :

$$PS_i = P(\text{Severity} \mid \text{Occurence} = \text{«Yes»} \text{ and } \text{Exposure} = X_i)$$

2) sample the number of losses F_i according to the binomial distribution $B(\text{nb}(X_i); PO_i)$.

3) for each loss from 1 to F_i, sample the severity according to the distribution PS_i.

4) find the sum of the F_i severities.

Repeat these four steps a large number of times, recording the sum of the severities each time. In this way, we obtain the distribution of total losses.

3. Sample application of the Bayesian method

We will now apply the Bayesian model to the risk of error in order execution in the financial markets. The risk is treated in a very simple fashion, because the objective is to show how the Bayesian approach works.

The first step in the Bayesian approach is to create a network, represented by a graph, and to define the distributions corresponding to each factor. These parameters can be reviewed during a back-testing phase.

Based on an analysis of the risk, we constructed the following graph:
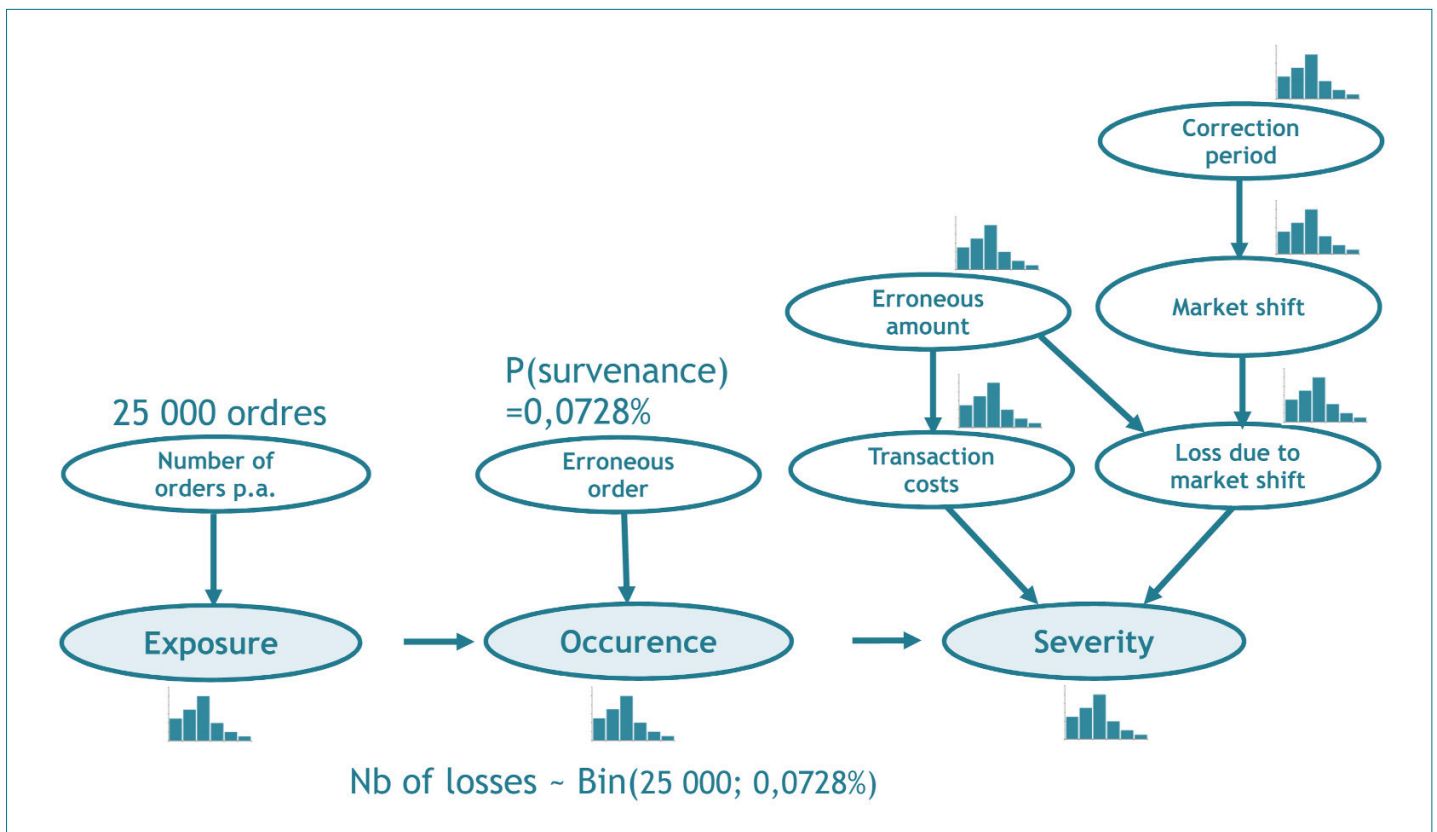
In the Bayesian approach, three elements are defined to model the operational risk: exposure, occurrence and severity.

Exposure: this must correspond to elements exposed to risk that can be transformed into a loss only once during the period. We have chosen to make orders these elements. Indeed, an order can be erroneous only once. We do not anticipate an increase in the number of orders in the coming year and the number of observed orders is on average 25,000 per year.

Occurrence: this is defined as an error in an order. On average, we have 18.2 erroneous orders per year, meaning that the probability a given order will erroneous is 18.2/25,000, or 0.0728%.

Severity: this is defined as the sum of the loss due to market shift plus transaction costs. We make the following assumptions about the factors entering into the calculation of severity:

• the amount of the error (expressed in millions of euros):

Error amount (in millions of euros)	
5	66%
15	18%
50	16%

• the correction period: : this is the time it takes to correct the error, i.e. from the time it takes place until a corrected order is placed:

Correction period (in days)	
0.125	66%
1	33%
90	1%

• transaction costs: we assume that transaction costs are equal to 0.25% of the amount of the transaction. The erroneous amount is executed the first time, then a second time to correct it. The costs will be calculated here by multiplying the erroneous amount by 0.5%.

• market shift: this is the change in rates observed during the time it takes to correct the order.

• Losses due to market shift are calculated by multiplying the erroneous amount by the amount of the rate shift.

Market shift	Correction period (in days)		
	0.125	1	90
2%	66%	62%	60%
5%	34%	37.99%	28%
30%	0%	0.01%	12%

Let's now run the algorithm described in the theoretical part:

The elements exposed to the risk are orders of the same type: $X_i = X$, which implies that $P_i = P(\text{Exposure} = X_i) = 1$.

We deduce from this the probability of an occurrence, keeping in mind that exposure is X_i :

$PO_i = P(\text{Occurence} = \text{«Yes»} \mid \text{Exposure} = X_i) = P(\text{Occurence} = \text{«Yes»}) = 0{,}0728\ \%$

The algorithm consists in carrying out the following steps in succession:

1) Calculate the severity distribution

$PS_i = P(\text{Severity} \mid \text{Occurence} = \text{«Yes»} \text{ and Exposure} = X_i)$

2) Sample the number of losses F_i according to the binomial distribution $B(nb(X_i);\ PS_i) = B(25\,000;\ 0.0728\ \%)$. Orders are independent and each order generates a Bernoulli distribution.

3) For each occurrence from 1 to F_i, sample the severity according to the distribution PG_i.

4) Find the sum of the F_i severities.

Repeat these four steps 10,000 times, recording the sum of the severities each time. In this way, we obtain a distribution of total losses and we can deduce a VaR of 99.5%.

We have carried out sensitivity tests on the various parameters of the Bayesian network. In each set of tests, we vary the parameters of a factor, while holding the parameters of all other factors constant.
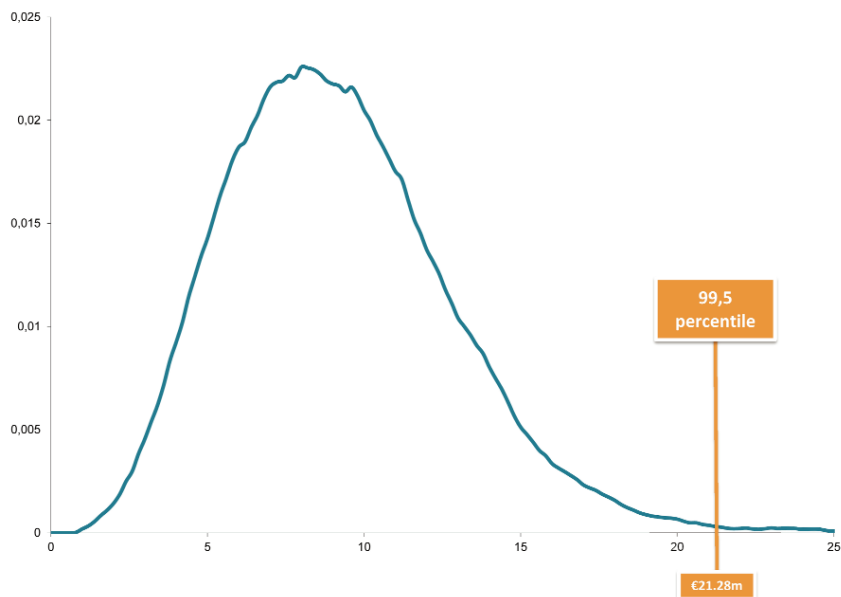
The Bayesian model presents numerous advantages:
-	it takes into account not only quantitative but also qualitative factors, which is not the case for most models;
-	it shows the links of causality between the variables. Risks are aggregated through the very construction of the network, obviating the need to estimate correlations;

-	it uses inference to detect risk reduction factors;
-	Risk Management can use it to implement action plans and observe their effectiveness.
The major disadvantage of Bayesian networks is that it takes a long time to set them up, because they require a detailed analysis of each risk.

4. References

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